The mass-ratio distribution of spectroscopic binaries

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Introduction

The distribution of orbital elements, and in particular the orbital period and the eccentricity, can reveal much about the formation mechanisms of binary systems as well as their subsequent evolution. In the same vein, the distributions of the masses of the two components, M_1 and M_2, or similarly, of M_1 and the mass ratio, $q=M_2/M_1$, are clues to critical questions related to binaries: did the binaries form through random pairing (in which case the mass of the individual components would be drawn from the IMF)? Does the mass ratio distribution depend on the primary mass (as does apparently the multiplicity of stars)? How will the systems evolve (some systems being possible only if the mass ratio is close to one, i.e. the system is made of quasi-twins)? How do families of stars compare to each other?
It is thus a reasonable thing to do to try to estimate for different samples of binary stars the distribution of their mass ratios and many attempts have been done in the literature. I will hereby review some of the pitfalls of such endeavours.

1. Spectroscopic binaries and exoplanets

We can distinguish between single- and double-lined spectroscopic binary (SB1 and SB2), depending on whether we can see the radial velocity variations of one or both components. The distinction will be due to the magnitude difference between the components, as too large a difference impedes the detection of the secondary star. Although the exact value of this limit will depend on the observational setting (e.g. wavelength range, exposure times, telescope size, method used), for main-sequence primaries a system will generally appear as a single-lined system if $q < 0.65$. For giants, the limit moves very close to 1. By observing in the near-IR, Mazeh et al. [1] have, however, shown how they could transform a sample of SB1 into SB2, thereby obtaining directly the distribution of q. The main difference between SB1 and SB2 as far as we are concerned indeed, is that for the first ones, we do not have a direct handle on the mass ratio in the system, contrarily to the second case.

For SB1, astronomers have to use the binary mass function, which is derived from the observed orbital period P, eccentricity e and radial-velocity amplitude K, through the relation

$$f(m) = K^3 P (1-e^2)^{3/2} / 2\pi G \text{, where } G \text{ is the gravitational constant.}$$

Unfortunately for us, this quantity has a rather complicated dependence on the mass ratio, and, even worse, also depends on the unknown inclination of the system on our line of sight, i:

$$f(m) = \frac{(M_2 \sin i)^3}{(M_1+M_2)^2} = M_1 \frac{q^3}{(1+q)^2} \sin^3 i$$

Exoplanets discovered by the radial velocity method are nothing else than SB1 in which $M_2 << M_1$, with M_1 generally known from stellar models, so that the knowledge of *f(m)* directly translates into the knowledge of $M_2 \sin i$. This explains why the radial velocity method provides a lower limit on the mass of the planet, M_2.

The above dependence of *f(m)* on q and i is the key to the problem at hand. It also shows what is the difficulty in deriving the mass-ratio distribution (MRD) from the observables. If we are limiting ourselves to a well-defined and well-behaved sample of SB, we can assume that for each system, we know the primary mass M_1. Thus, one knows the distribution of $Y= f(m)/ M_1$, $\Phi(Y)$, which is now only a function of q and i. We can also make the reasonable assumption that the inclination i is randomly distributed (there is no reason to have a connection between the *apparent* inclination of the system on our line of sight and the *physical* properties of the system). The problem thus consists in using the distribution of Y to obtain the sought distribution of q. Several methods exist to this aim.

2. Obtaining the MRD

2.1 *The wrong way*

As beautifully expressed by Mencken [2] "*there is always an easy solution to every human problem: neat, plausible, and wrong,*" and there is no reason to believe that astronomy is any different from daily life. To solve the above-mentioned problem, many authors have used – and some are unfortunately still using [3]! – a simplistic method that has been shown on several occasions [4–8] to provide incorrect results. This simple method assumes that one can replace in the definition of Y, $\sin i$ with a mean value, corresponding to either 0.589 or 0.679 (assuming that $f(i) \sim \sin i$ or $\sin^2 i$, respectively). This, however, will provide wrong results: an initially uniform distribution will for example result in a decreasing *f(q)*, while an increasing *f(q)* produces a Gaussian-shaped one. The reasons for this behaviour are well known and have been described in the above-mentioned papers. Here, I would like again to stress against using this simplistic and incorrect approach. It is also worthwhile to note that the same reasons why this method does not work explain why we cannot infer the mass of an exoplanet from its $M_2 \sin i$ value, as was recently rediscovered [9].

2.2 *Functional fitting*

The most obvious way to derive the MRD is of course to assume it in the first hand. One can indeed postulate that the MRD takes a given – most likely rather simple – functional form, that depends on a few parameters, and one can then use a minimisation technique to derive the most appropriate values of these parameters in order to reproduce the observed $\Phi(Y)$ [4,10; see also 11 for an application on exoplanets]. Although this method is rather limited as one need to make some (educated) guess for *f(q)*, it has the advantage that one will not – in principle – be tempted to over-interpret the data. There is an obvious caveat, however. When making the minimisation, one should not compare to the distribution of *f(m)* as, given the large dynamical range of values over which the latter spreads most of the points will be concentrated in one single bin and the comparison will be meaningless, leading to wrong conclusions. A better choice is done when comparing with the distribution of log *f(m)* for example (or any other binning that spreads the data in similarly populated bins). See [12] for an analysis of the kind of mistakes this can lead to.

2.3 *Inversion methods*

Of course, the best would be to be able to directly inverse the observed distribution of *f(m)* to derive the MRD. If we define $Q=q^3/(1+q)^2$, we have $Y=Q \sin i$, and thus $\Phi(Y) = \int f(q)\Pi(Y \mid q)\,dq$, where $\Pi(Y \mid q)$ represents the conditional probability to observe Y when the mass ratio q is known [6]. We can easily show that $\Pi(Y \mid q) = (3Q^{1/3}Y^{1/3}\sqrt{Q^{2/3} - Y^{2/3}})^{-1}\Theta(Q - Y)$, where $\Theta(x)$ is the Heaviside step function. The above integral can be inversed using the Richardson-Lucy iterative scheme, or any other equivalent method, as was done by several authors [e.g. 5–8, 13].

Here again, I would like to stress three caveats. The first one is that the above formulation is only valid when *f(i)* ~ sin *i*, and that all values of *i* are possible, *a priori*. However, depending on the precision of the observations – and the orbital periods sampled – it may not always be possible to detect all the binaries with the smallest inclinations. It is thus necessary to check how reliable this assumption is by e.g. performing Monte Carlo realisations or applying the technique on an associated sample of SB2 (as was done e.g. by [12]), and providing *a posteriori* confirmation of the obtained MRD by using the corresponding functional fit and comparing the obtained distribution of log *f(m)* with the observed one. The second caveat is that – depending on the size of the sample under scrutiny – one should be very careful at the level of details one could trust. Figure 1 shows nine Monte Carlo realisations of the inversion method on two different MRD (a uniform one and another one which consists of 3 separated peaks) for samples of different sizes: *N*=100 and 1000. It is obvious that for the smaller samples, one can almost obtain any kind of distributions. This should clearly serve as a warning at what should be trusted. For a sample of *N*=1000, although there are some minor fluctuations, we are clearly able to recover the main features of the initial distributions. Unfortunately, there are not many significant samples of 1000 SB available yet for study. Brown [14] has recently made similar remarks in the context of exoplanets. The third caveat – and the most tricky – is that in order to reproduce the **real** MRD, which is the one we are looking after, it is important to make sure that the sample we are considering is free from any observational biases, or at least that we think we understand these and can correct for them. This is far from an easy enterprise and, although it was done successfully in the past [e.g. 13, 15, 16], it generally leads to samples of rather small sizes.

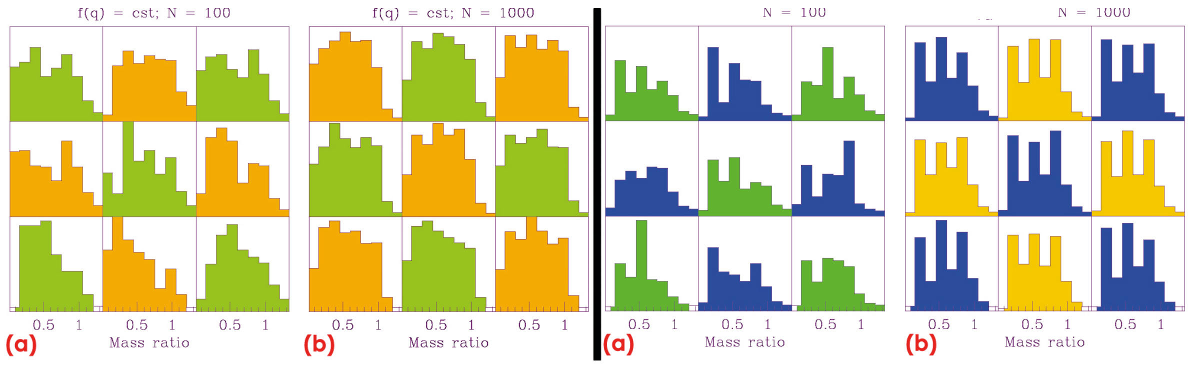

Fig. 1: Sets of nine random realisations of inversion of a uniform MRD (left) and of a 3-peaked one (right) are shown for samples of two different sizes: *N*=100 (a) and *N*=1000 (b).

The GAIA link

The GAIA survey will provide us, after 5 years of observations, with a large new sample of spectroscopic binaries – perhaps a few millions orbits will be obtained – allowing us to perform a detailed statistical analysis and creating many subsamples to study different effects. The survey will be homogeneous and well-defined, which will make it more easy to correct for the observational biases. However, GAIA will suffer from very poor precision in the radial velocity. Moreover, this precision will worsen very quickly as a function of the magnitude of the star as well as its spectral type (the error on one measurement being of a few km/s for bright G-K stars to several 10-20 km/s for faint A-F stars). This means that many systems with rather large semi-amplitudes will be missed. I illustrate this in Figure 2, where I show what would be the derived MRD (in black) assuming (just for illustration purposes) that the real MRD is uniform: many systems with small q would be missed and one would need to correct for this. The kind of binaries as a function of eccentricity, period or inclination that would be missed is shown in the right panel. These are the effects that will need to be corrected for.

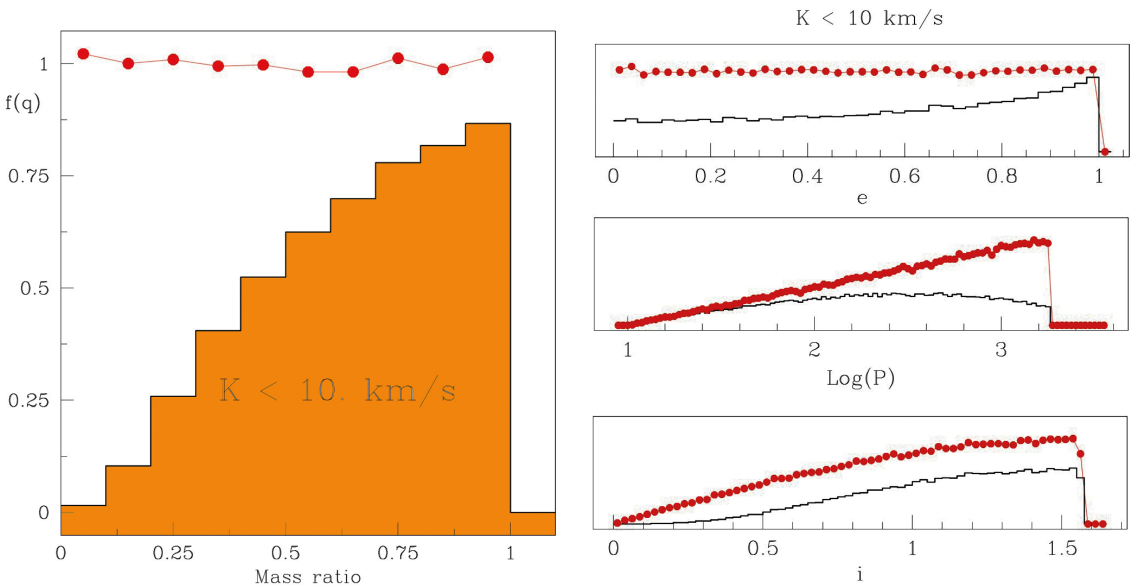

Fig. 2: Effect of the poor precision of GAIA on the derived MRD (left) as well as the systems missed because of their eccentricity, period, or inclination, respectively (right).

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