Limitation of the TTV technique for the detection of non-transiting planets

G. Boué[1,2], M. Oshagh[1,3], M. Montalto[1], N.C. Santos[1,3]
1. Centro de Astrofísica, Universidade do Porto, Rua das Estrelas, 4150-762 Porto, Portugal.
gwenael.boue@astro.up.pt,
2. ASD, IMCCE-CNRS UMR8028, Observatoire de Paris, UPMC, 77 avenue Denfert-Rochereau, 75014 Paris, France,
3. Departamento de Física e Astronomia, Faculdade de Ciências, Universidade do Porto, Portugal

Introduction

A single transiting planet produces periodic dimming of the luminosity of its star with the same period as its orbit. However, if a planet belongs to a multiplanetary system, the planet-planet interactions perturb its orbit and the signal becomes aperiodic. The deviations from exact periodicity are called transit timing variations (hereafter TTVs). For arbitrary planetary configurations the signal is weak and difficult to detect, unless the perturber is massive enough. However, if the system is in or close to a mean motion resonance (hereafter MMR), the gravitational interactions are enhanced, and even an Earth-like planet can be detected [1, 2]. In that case, the main frequency of the TTV signal is associated to the "great inequality" (in reference to the Jupiter-Saturn system close to the 5 : 2 MMR) [3, 4]. The corresponding period is of the order of a year for a 3-day transiting planet. Thus, such systems should have just started to be observable by Kepler. However, the inversion of TTV signals close to MMRs is not unique, and it may be difficult to characterize a non-transiting perturber [5, 6, 7, 8]. Such difficulties have already been experienced with the Kepler-19 system [4]. Here, we present the issue raised by the long period of the signal, and show its degeneracy.

1. Amplitude of TTV produced by an Earth-mass planet

1.1 *Amplification at MMR*

In this section, we highlight the role of MMR in the detection of terrestrial planets. For that purpose, we consider an Earth-mass planet perturbing a transiting Jupiter-mass planet. The orbit of the transiting planet is supposed to be initially circular with a period of 3 days. Here, we consider only the effect of the period ratio between the perturber and the transiting planet. To show the effect of second and higher order MMRs, at least one of the two eccentricities should be non-zero, we thus set the eccentricity of the pertuber at 0.1. The system is followed over $N = 365$ transits corresponding to a time span of 3 years, the duration of Kepler's primary mission.
Figure 1a shows the standard deviation of TTV signals in that system [1]

$$\sigma = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(t_j - t_0 + P_1 j)^2}\ , \tag{1}$$

where P_1 and t_0 are chosen to minimize σ, and t_j, $j \geq 1$, are the midtransit times of the N transits. P_1 represents the best estimation of the period of the transiting planet. As expected, the amplitude of the signal decreases as the orbital period of the perturber gets longer, and it is enhanced at period commensurabilities [1, 2, 9]. For comparison, the detectability threshold, ranging from 10 sec to 1 min for accurate data, is displayed in the figure as an horizontal strip. It is then clear that the TTV signal produced by an Earth on a transiting planet with a period of 3 days is only possible if the system is in the vicinity of a MMR. However, it should be stressed that the amplitude of the signal scales linearly with the period of the transiting planet. Thus, if the perturbations are produced on a wider orbit, the detection should be easier. This point is discussed below.

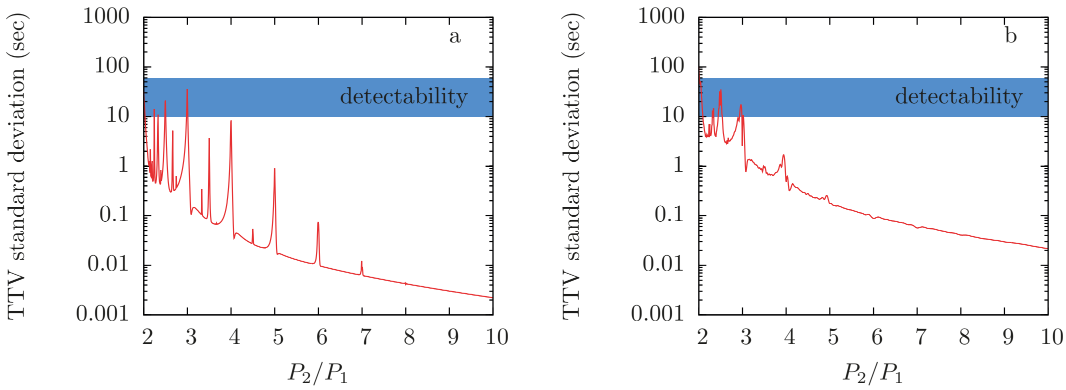

Figure 1: Standard deviation of TTVs produced by an Earth mass planet on a Jovian planet transiting a Solar mass star during 3 years. (a) The period of the transiting planet is 3 days. (b) The transiting period is 30 days. The stripe shows the range of typical detectability thresholds.

1.2 *Increasing the period of the transiting planet*

As mentioned above, the amplitude of TTV signals scales linearly with the period of the transiting planet. Thus, increasing that period should enhance the signal. Figure 1b shows the results when the period of the transiting planet is increased by a factor 10 with respect to the one of Fig. 1a. As one can notice, the lower envelop of the TTV signal is indeed amplified by a factor 10. However, some of the resonant peaks have been reduced (see the 4:1 MMR, for example), and for the others, the amplitude remained almost unchanged. This is due to the fact that the signal, close to a MMR, is dominated by a long period term (the great inequality). Indeed, all the periods in TTV signals scale linearly with the period of the transiting planet. Thus, increasing the latter by a factor 10 expands also the long period by a factor 10. This is then longer than the observation time span (3 years). In the linear fit (1), the large oscillation is not averaged out, the coefficient P_1 becomes the sum of the "real" transiting period and the initial slope of the TTV signal. The residual amplitude is smaller than expected. To actually recover the factor 10 in the increase of the TTV signal at the MMRs, one should observe the system at least 15-20 years.

2. Degeneracy of the signal

Here we assume that a TTV signal has been detected. As discussed above, at or close to a MMR, the signal is dominated by a long term oscillation. To illustrate that point, we plot in Fig. 2 different TTV signals associated to different MMRs, with a gaussian noise of 20 sec to simulate observations. The associated initial conditions are gathered in Tab. 1. They all have a large signal to noise ratio and are thus easily detectable. However, since the signals are dominated by one single sinusoid, their shapes are very similar. The last column in Tab. 1 gives the reduced chi-square after a fit by a pure sine for each simulations. They are all lower than or of the order of 1.30. To highlight the degeneracy issue, the initial conditions have been chosen to reproduce the same amplitude and the same period for each curve. In particular, a 0.9 Earth-mass planet in a 1:2 MMR can produce a signal comparable to a 4 Saturn-mass planet in the 1:4 MMR (see Tab. 1).

Conclusion

The TTV signal produced by an Earth-mass planet is highly enhanced near MMRs. It thus seems that such conditions are optimal for the detection of low-mass planets. However, if the transit period is of

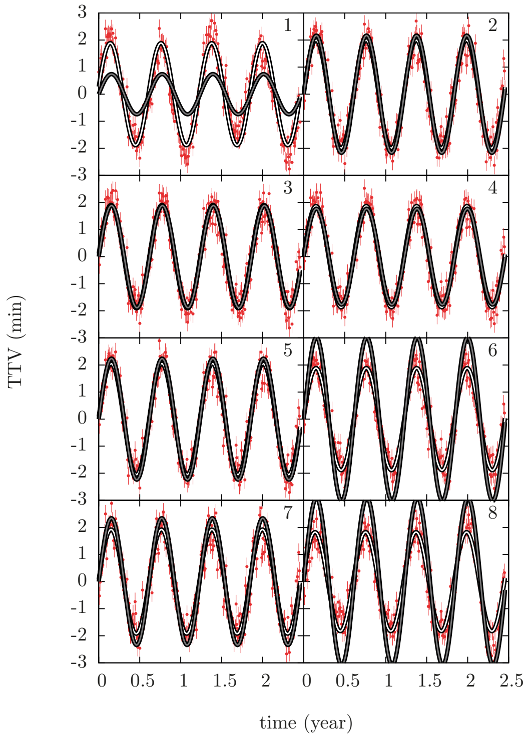

Figure 2: Example of TTV signal produced near period commensurabilities. A gaussian error of 20 seconds has been added to the signal to model the observation uncertainties. The white curve represents the highest amplitude term obtained by frequency analysis. The gray curve is the analytic approximation Eq. (21) of [10]. The number at the upper right corner of each subfigure corresponds to the set of initial conditions as given in table 1.

Table 1: Initial conditions, mass and eccentricity of the perturber used to perform the simulations of the Figure 2. ε and $\sqrt{\chi_r^2}$ are respectively the distance to the MMR, and the reduced chi-square after a fit by a sine function.

set[a]	MMR	m_2 ($M_\oplus$)	e_2[b]	ε	$\sqrt{\chi_r^2}$
1	1 : 2	0.9	0.087	0.0132	1.36
2	4 : 9	24.5	0.120	0.0034	1.15
3	3 : 7	21.1	0.100	0.0044	1.13
4	2 : 5	8.6	0.102	0.0067	1.02
5	3 : 8	17.2	0.160	0.0045	1.07
6	1 : 3	49.7	0.100	0.0134	1.25
7	3 : 10	95.2	0.194	0.0045	1.30
8	1 : 4	394.0	0.115	0.0134	1.31

[a] The initial conditions are in order of increasing semi-major axis.
[b] The eccentricity e_2 is chosen to produce low values of the reduced chi square $\sqrt{\chi_r^2}$.

the order of 3 days, the signal remains at the limit of detection for all second and higher order MMRs. For a transiting planet with a period ten times larger, the amplitude of the signal is increased by a factor ten. However, the period of that signal scales also linearly with the transit period, and it becomes longer than the duration of Kepler's primary mission. An Earth-mass planet can thus produce detectable TTVs but only at low order MMRs, i.e. in a compact system. Finaly, in systems close to MMR, the TTV signal is dominated by a single sinusoidal oscillation. Thus, if the pertuber is not detected by transit, the problem is highly degenerated. It may be difficult to characterize non-transiting planets at MMR, unless the residual oscillations in the TTV are detectable.

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