Approximation of the gravitational potential of a non-spherical object: application to binary and triple asteroid systems.

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Introduction

Nowadays more than 200 asteroids have been identified as multiple. These systems are found all over the Solar System (Near-Earth, Main-Belt and Trojan asteroids, Trans-Neptunian objects). The study of the stability of these multiple systems is quite complex, because of their irregular shapes, and requires a full body - full body approach. More specifically, in the case of satellites of asteroids (when one of the body is much greater than the others), the shape of the main object has a major influence on the motion of the small bodies. This perturbation is more important than the perturbation of the Sun or of the planets as shown for example in [4, 9]. The introduction of this shape effect in a numerical integration requires the knowledge of many coefficients of the spherical harmonic expansion of the gravitational potential; their computation is performed using the software SHTOOLS [19] and a shape model. This procedure is explained in section 1.
Once the coefficients of the gravitational potential are determined, the dynamics of the asteroid satellites is described thanks to the software NIMASTEP [6], for the full short periodic motion, and, thanks to a home-made program (developed by J. Frouard), for the averaged long periodic motion.
Two applications are presented in section 2: the search for mean-motion, gravitational and secular resonances in the triple system (87) Sylvia, explaining its present and future configuration, and the validation of orbit for the binary system (41) Daphne.

1. Approximation of the gravitational potential of a non spherical asteroid

The gravitational potential due to the non-sphericity of an asteroid is classically developed in a spherical harmonic expansion (see e.g. [14]). In order to compute the coefficients of this expansion, we use a shape model for the asteroid and routines of the software SHTOOLS [19].
A shape model is a polyhedron with triangular surface facets that represents the whole surface of an asteroid. It can be determined by lightcurve inversion, or radar imaging or multi-data inversion (e.g., KOALA [2, 13]). Shape models of asteroids obtained through inversion techniques are available online on the database DAMIT (Database of Asteroid Models from Inversion Techniques) [7]. We assume here that the asteroids have an homogeneous structure (i.e., the density is constant inside the object), so that shape models represent the whole volume of the asteroid and not only its surface.
We deduce the coefficients of the spherical harmonics development (usually up to degree and order 10) from the shape models, using the software SHTOOLS. Practically, we take as input the file containing the shape model (a list of vertices and of the links between them) and we compute the coefficients in four steps : first the vertices are written in spherical coordinates. Second, an harmonic expansion is realized on the shape of the asteroid using the function `SHExpandLSQ` of SHTOOLS based on a least squares minimization to obtain the spherical harmonics coefficients related to the shape of the asteroid but not yet to its potential. Third, the expansion is used to compute a 2-dimensional map equally sampled in latitude and longitude (using the function `MakeGridDH`), and fourth, the function `ClimPlus` gives the expected coefficients of the gravitational potential.
Obviously, this procedure does not allow to get all the coefficients up to any degree and order. The number of obtained coefficients depends on the number of points given by the shape model.

2. Application to binary and triple asteroidal systems

2.1 *Search for instability zones in the triple system (87) Sylvia (J. Frouard and A. Compère)*

In 2005, Marchis et al. [16] discovered a second moon rotating around the asteroid (87) Sylvia, a large main-belt asteroid, already identified in 2001 as a binary [1]. It has been the first triple asteroid discovered. The two satellites (named Romulus and Remus) are much smaller than Sylvia and evolve in nearly circular and co-planar orbits. These satellites are sufficiently small and distant from Sylvia to be considered as point mass satellites. A first study of this system was published by [18], showing that classical secular resonances were present when Sylvia was modeled by a sphere but disappeared when a 2nd degree gravity field was added.

By our technique, we can detect other types of resonances in the neighborhood of this system in the non-spherical case [9]. We compute the coefficients of the spherical harmonics development for Sylvia up to the 4th degree and order using the shape model of [11] (these coefficients are given in [9]). A rapid approximation of the forces acting on the system shows that the shape of Sylvia is much more important than any other contribution, so the influence of the Sun and of the planets has not been taken into account for the full short term integrations.

We integrate the equations of motion in two different ways in order to distinguish between short and long periods. First we look for mean-motion (between the mean longitudes of the satellites) and gravitational (between the mean longitude of a satellite and the spin angle of Sylvia) resonances with a non-averaged model Sylvia-Romulus-Remus using NIMASTEP, in which we fix the rotation rate of Sylvia around its principal moment of inertia. Second, we search for secular resonances with a model Sun-Sylvia-Romulus-Remus, averaged over the mean longitudes.

For the short-term integrations (with the non-averaged model), we used the chaos indicator MEGNO, designed by [3]. This indicator relies on the numerical integration of a tangent vector, which is computed from the variational equations of motion of the satellites. For stable quasi-periodic orbits, the mean MEGNO converges towards 2 and for chaotic motion, tends towards infinity. For orbits close to a stable periodic orbit, the mean MEGNO converges towards 0. An example of chaotic map is shown in Figure 1 where we compute the mean MEGNO for different values of the semi-major axis of the satellites. The position of the actual system is plotted in white on the map. The map shows different kinds of instability zones (in black in the figure). Under the value of 400 km for the semi-major axis of Remus (a_{rem}), the small satellite quickly collides with Sylvia. At the value of $a_{rem} = 440$ km, we find a 3:1 gravitational resonance for Remus inducing a strong chaos. The other instability zones correspond to mean-motion resonances between the two satellites. This study has been validated by a frequency map analysis [15].

Besides mean motion and gravitational resonances, other secular resonances (called evection resonances) are found close to the actual position of the satellites. For this analysis, we use the averaged equations, which allow to clearly differentiate the secular dynamics of the satellites and are much faster than the complete ones. We detect, near Romulus, the presence of low-order evection resonances (between the longitude of pericenter of Romulus and the longitude of the Sun) that can induce a strong chaos. This is checked by the chaotic diffusion of the orbits, by computation of the change of their frequencies over time using the frequency analysis [15]. These evection resonances are able to rapidly increase the eccentricity of Romulus over 0.18 (the initial eccentricity is 0.001).

We can also follow the long-term evolution of the satellites driven by tidal and BYORP effects (assuming synchronous satellites and an increasing evolution for the semi-major axis of the satellites) (see [10] for the description of the last effect). The system will first cross the evection resonance inducing an important growth of the eccentricity and inclination of the satellites (especially for Romulus).

All these results are developed in details in [9] and prove the efficiency of our combined technique.

2.2 *Validation of orbits for the binary system (41) Daphne (B. Carry and A. Compère)*

A second application of our method concerns the binary asteroid (41) Daphne. Daphne is a large main-

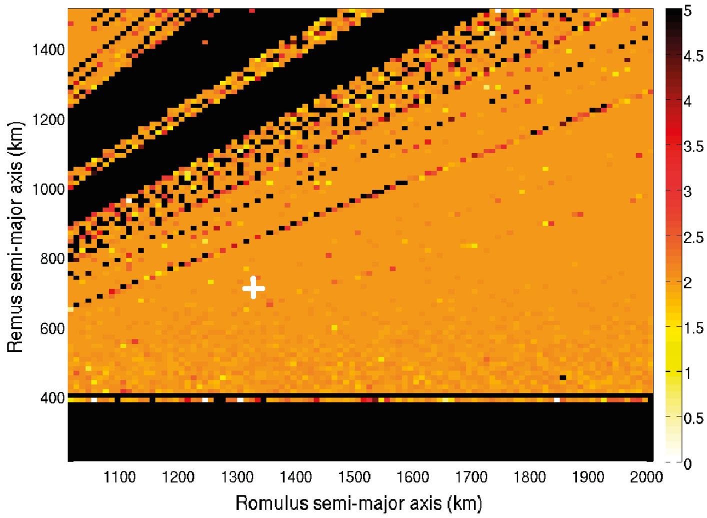

Figure 1: MEGNO map. The integration time is 20 years and the initial conditions are given in [9]. The values of MEGNO are cut to 5 in order to properly distinguish the structures. The white cross is the position of the actual system (the actual semi-major axes of the satellites are 1356 km for Romulus and 706 km for Remus). Numerical simulations are performed thanks to the local computing resources (Clusters ISCF and URBM-SYSDYN) at the University of Namur (FUNDP, Belgium).

belt asteroid with a very irregular shape. In March 2008, a small satellite S/2008 (41) 1 was discovered [5], with a short orbital period (about 1.1 days). The mass ratio between the two objects is extreme (about 10^6 [17]). Using these informations and the observations of the satellite, we test the keplerian orbits around Daphne that fits the observations. Choosing a point (position and velocity) of one of these selected orbits as initial condition, we perform numerical integrations of the motion of the satellite using two different shape models for Daphne. The first one is the convex model obtained by lightcurve inversion [8, 12], and the second one is the non-convex model derived with KOALA method. For both models, the coefficients of the harmonic expansion are computed by the method explained above. We perform forwards integrations to study the stability of the orbit (similarly to our work on Sylvia) and backwards ones to check if the orbit is consistent with the observations. Depending on the results, the orbit is validated or another initial Keplerian orbit is chosen and the mechanism is restarted. This work is still in progress and is very promising.

Conclusion

The knowledge of the gravitational field is required to conduct fine studies of the motion of a satellite or a probe orbiting an asteroid. Indeed, the acceleration caused by this effect on the small body is usually much more important than the other perturbations (e.g., Sun, Jupiter). Shape models can be used to approximate this gravitational potential; we have presented here a way to compute this field with the help of the software SHTOOLS and two applications of this technique have been presented. We have showed that, combining the shape models and different methods of integration, we are able to produce substantial informations about the dynamics of specific double or triple systems, especially in the case of satellites of asteroid.

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