Resolved Binaries in the Kuiper Belt: Optimally Scheduling Observations For Orbit Determination

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1. Introduction

In the Kuiper belt, as in any astrophysical setting, the presence of binaries offers a valuable route to advancing understanding of the overall population (e.g., [1]). Dynamical masses can be determined directly from binary mutual orbits and the statistics of binary orbital properties can be used to constrain the origin and subsequent history of the population (e.g., [2, 3, 4]). Binaries can even be used to probe for the presence of objects fainter than current detection limits (e.g., [5, 6]). Of course, the first task is to determine the mutual orbits. For Keplerian motion (two point masses orbiting their common barycenter, with no external perturbations), seven independent parameters are needed to completely describe the orbit. An observation resolving the components of a binary to yield relative astrometry at an instant in time provides two independent constraints: the relative separation and position angle, or equivalently, relative positions in two Cartesian axes. In principle, four such observations could provide eight independent constraints, more than enough to completely determine the orbit. But in practice, constraints from separate observations are rarely so independent of one another that only four observations are needed. The period and geometry of a binary orbit being unknown *a priori*, it is not possible to schedule four observations in advance such that they reliably sample diverse longitudes around the orbit. Also, if the observer and the binary system remain fixed in space relative to one another over the time of observations, there is an unresolved degeneracy between an orbit solution and its mirror image through the sky plane (e.g., [7]).

Most Kuiper belt binaries are much fainter than 20th magitude, and most have sky plane separations much less than an arcsecond, so very few telescopes can resolve them. Only the Hubble Space Telescope (HST) and a handful of 8 to 10 m ground-based telescopes equipped with laser guide star adaptive optics systems have the required combination of spatial resolution and sensitivity. In such a resource-limited environment, it is essential to make the most efficient possible use of the available facilities. This paper describes our strategy for using information from existing observations to optimally schedule subsequent observations for purposes of orbit determination. More details about this ongoing observational program can be found at http://www.lowell.edu/~grundy/tnbs.

The key to optimal scheduling is knowledge of the probability density function (PDF) in seven-dimensional orbital element space. The orbital element PDF describes what parts of orbital element space are consistent with the observational data, and how likely it is that the real orbit lies in a particular region. The extent of this probability distribution can be progressively whittled away by obtaining additional observations, until it becomes so restricted that the orbit can be considered to be known. Reprojecting the PDF from orbital element space to an observer's instantaneous sky plane at a specified time produces a two-dimensional sky plane PDF describing where the secondary could appear relative to the primary at that time. Orbital motion causes this PDF to evolve as a function of time. The best time for a follow up observation is when the sky plane PDF is most broadly extended, or in other words, when the orbits consistent with the data map to the widest expanse on the sky. At such a time, a single observation will permit rejection of as much of the integrated probability in the orbital element PDF as possible. This paper describes one approach to the problem of estimating the orbital element PDF and using it to optimally schedule follow up observations, first described by Grundy et al. [8].

2. Estimating the Orbital Element Probability Density Function

The simplest algorithm for assessing the orbital element PDF is directly sampling orbital element space, either via a grid search or by generating random orbits in a Monte Carlo fashion. Any orbit so generated can be compared with available observational data and its probability assessed via χ^2 analysis. The problem is that solutions are often very localized and 7 dimensional orbital element space is vast. It is very inefficient to explore via such cumbersome methods. In practical terms, almost all of the orbits tested will be excluded by the data, so it will take a prohibitively huge investment of computer time to map out the orbital element PDF this way.

The available observational data can be used to inform the sampling of orbit element space such that regions not containing solutions consistent with the observational constraints are not explored. A simple way to generate a random orbit that avoids at least some irrelevant parts of orbital element space is to choose one of the available sky plane observations and generate a random three dimensional point in space consistent with that observation, i.e., along the line of sight defined by the observation. From that point, a random system mass plus a random three dimensional motion vector uniquely define an orbit (which need not be bound) guaranteed to be consistent with that particular observation. This approach is much more efficient than picking orbits completely at random. But it is possible to do even better by using data from more than one observation, if they are available. For instance, choosing random three dimensional points consistent with a pair of observations requires just a random mass to uniquely define an orbit consistent with both observations, if both points were obtained during a single orbit and were taken within a short enough span of time to indicate the direction of motion. Three observations can be used with the Thiele-Innes algorithm to achieve an even higher efficiency when three or more observations are available [9, 10]. As more observations are used, the process of sampling the PDF becomes more efficient, but accounting for the biases introduced by more sophisticated sampling schemes becomes increasingly challenging.

Our implementation is based on two observations taken within a single orbit. We call these an “anchor pair” of observations. Some effort is required to obtain a suitable anchor pair (see Section 4; if no such pair is available, this method is not applicable). To generate a random orbit consistent with the anchor pair, random points on the sky plane are chosen consistent with the two observations and their uncertainties (assumed to be Gaussian). Random radial distances are generated along the two lines of sight as well. These should be drawn from a probability distribution that encompasses the range of distances relative to the primary where the secondary could have actually been at those times. Since this range is not actually known *a priori*, it is necessary to be conservative and choose a sufficiently wide distribution to ensure the actual location is included. Since an excessively wide distribution would result in many wasted orbits unlikely to satisfy other observations and thus contribute usefully to the estimated PDF, a reasonable compromise seems to be to use a Gaussian distribution centered on the primary with σ equal to a few times the maximum observed sky plane separation. Finally, a random system mass is chosen from a uniform distribution in log(mass), bounded by limits discussed in Section 4. This two-point boundary value problem is then solved to obtain the orbital elements, using the “*p*-iteration” method of Herrick and Liu (e.g., [11, 12]). To account for the goodness of agreement to the observational data and for the biases introduced by sampling in range and sky plane positions rather than in the orbital elements themselves, a weight W is computed for the orbit as described by Virtannen et al. [13, 14] and Virtannen and Muinonen [15],

$$W = \frac{\sqrt{\left|\det\left(\Phi^T \Lambda^{-1} \Phi\right)\right|}\, e^{-\frac{\chi^2}{2}}}{x_1 y_1 z_1 x_2 y_2 z_2 \left|\det J\right|}. \qquad (1)$$

Here Φ is a matrix of partial derivatives of sky plane observations with respect to the orbital elements, Λ is the covariance matrix of the observational uncertainties, χ^2 is the usual measure of goodness of fit computed as Σ((orbit-data)/errors)^2, x_1 through z_2 are the values of the Gaussian sampling function at the location of the spatial points randomly selected to correspond to each of the two anchor points 1 and 2, and J is a matrix containing partial derivatives of each of the orbital elements with respect to each of the sampling variables. For a collection of orbits, the weights are normalized such that their sum is unity. The normalized weights indicate how much each orbit contributes to the ensemble representation of the PDF in orbital element space. The structure of this representation tends to be dominated by the orbits having the highest weights. Some orbits have weights too small to contribute appreciably to the PDF. These can be discarded. Virtannen et al. [14] discuss formal criteria for how many orbits are needed to properly represent the PDF. In our experience, it is often sufficiently well characterized for optimal scheduling purposes well before those criteria have been met [8].

3. Optimal Follow-up Time

Given a collection of Monte Carlo orbits representing the orbital element PDF, the next task is to use them to assess optimal timing for the next observation. The Monte Carlo orbits are reprojected to the sky plane as a function of time. In doing this, the weights need to be adjusted by a factor of $|\det K|^{-1}$ and then renormalized to maintain a total of unity, in order to account for the mapping from one set of variables to the other [14]. Here K is a 6×6 matrix of partial derivatives of each of the sky plane (and radial) Cartesian coordinates and their time derivatives with respect to each of the orbital elements (excluding the period). Each orbit becomes a point on the sky plane with a time dependent location and weight. At a given time, the mean sky plane separation between each possible pair of points i and j (where $i \neq j$), weighted by $\sqrt{W_i^2 + W_j^2}$, is computed to assess the astrometric precision needed to distinguish between two random orbits selected from the collection. Times when this average separation is largest are the most useful times to perform a follow up observation. Additional scheduling constraints (such as telescope/instrument availability, solar elongation, pointing restrictions, etc.) can readily be accommodated, by looking at the times of greatest average separation after all other scheduling constraints have been satisfied. Likewise, observation times can be scheduled to maximize the probability of observing near periapse and/or minimum separation, particularly useful for constraining the eccentricity of the orbit.

4. Obtaining Suitable Anchor Observation Pairs

The need for a suitable pair of anchor observations within a single orbit was mentioned in Section 2. How can such a pair of observations be obtained before the orbital period is even known? The initial discovery observation contains information that can be used to get a rough idea of what the orbital period might be to help in scheduling an anchor observation pair. We use the observed magnitudes of the two objects along with plausible ranges of albedos and densities to get a range of plausible masses (typically spanning around two orders of magnitude; this mass range is also used to bound the distribution used to generate random masses described in Section 2). The observed separation at discovery offers an additional key piece of information. As noted by Grundy et al. [8], for randomly oriented orbits, this separation tends to be similar to the semimajor axis of the orbit. The exact distribution of semimajor axis versus discovery separation depends on the orbital eccentricity, which is also unknown *a priori*, but for a broad variety of eccentricity distributions, the actual semimajor axis remains within a factor of two of the discovery separation more than 80% of the time (note that in Grundy et al. [8] Fig. 1 the actual distribution shown is a half-Gaussian centered at $e = 0$, with $\sigma = 0.5$, contrary to what the caption says). From these rough estimates of the mass and semimajor axis, a rough estimate of the period can be computed and used to inform scheduling of potential anchor observations.

5. Complications

With real observations of real binaries, various complications can arise. These can be handled within the framework described in this paper at a cost of additional computer processing. For instance, if the anchor observations are so widely separated that it is not clear what the direction of motion is, it is still possible to use them as described in Section 2. But in addition to generating random positions and masses, it becomes necessary to also generate a random single bit direction of motion (around the shorter way or the longer way) before solving the two point boundary value problem to derive the unique orbit corresponding to that set of random numbers. Orbits with the wrong direction of motion that happen to be consistent with the available data will contribute to the Monte Carlo representation of the PDF, but the scheduling of follow-up observations will eventually eliminate that part of the PDF, just as other parts of the PDF that do not contain the true orbit are eventually eliminated by further observations.

Two binary components of equal brightness present another kind of difficulty, one that has affected many of our transneptunian binaries (examples include (79360) Sila-Nunam, (148780) Altjira, 1999 OJ_4, (275809) 2001 QY_{297}, and 2003 QY_{90}). In these cases, it is desirable to shorten the time interval between the anchor observations to ensure that the identities of the two bodies are not accidentally mixed up between the two epochs. In practice, it can help to schedule a set of three including one pair separated by a very short interval (to guard against being fooled by an unexpectedly short orbital period) followed by another observation somewhat later. However, the identities of primary and secondary (as defined in the anchor set) are ambiguous in observations obtained much later or much earlier. For each of those observations, we assign an identity bit, with 0 corresponding to arbitrarily assigned identities and 1 to the reverse. When generating random orbits to sample the PDF, random values (0 or 1) are also chosen for these bits. As with the previous case, this means we generate orbits for both correct and incorrect permutations of the identity bits, and the incorrect permutations can potentially contribute to the Monte Carlo representation of the PDF in regions of orbital element space far from the actual orbit. But just as before, follow up observations will eventually eliminate these false solutions.

These examples illustrate how a broad range of ambiguities can be overcome by generating more random orbits sampling both sides of an ambiguity. This additional sampling comes at a cost of more computer time, and a corresponding expansion of the populated region of the PDF. Additional observations, scheduled using the same methods, can eventually resolve the ambiguities and enable the orbit to be determined, although the number of observations required grows with the ambiguities.

6. Conclusion

Since 2005, we have been using this methodology in scheduling observations of numerous transneptunian binaries with HST, Keck, and Gemini telescopes (e.g., [8, 16, 17]). This experience has shown that for binaries unaffected by complications such as described in Section 5, optimal scheduling usually enables determination of at least the period, eccentricity, and semimajor axis of the mutual orbit with only four additional observations following a binary discovery observation. Two mirror solutions sometimes remain for the other orbital elements, since the slow heliocentric motion of transneptunian objects means that it often takes many years for sufficient parallax to accumulate to distinguish between mirror orbit solutions. Nuisance factors such as similar-brightness components, poorly timed anchor observations, or optimal follow-up times precluded by telescope scheduling constraints can necessitate additional observations. The most observations we have needed thus far to determine a binary mutual orbit was a total of 14 observations of (79360) Sila-Nunam, an equal-brightness pair in a tight orbit viewed edge-on, with several early observations not optimally scheduled, as well as a gap of about two years in telescope availability interrupting the sequence of observations.

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