



THE MASS RATIO DISTRIBUTION OF SPECTROSCOPIC BINARY STARS

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THE IMPORTANCE OF $f(q)$

- M_A and M_B and the mass ratio $q=M_B/M_A$ distributions
- Binary formation mechanisms? e.g. random pairing, $f(q)$ constant, q depends on M_A ?
 - we already know that multiplicity is function of M_A
 - and possibly $f(q)$ also
- Evolution of binary systems? e.g. twins population?
- Comparison between populations or families of stars e.g. PRGs and normal G-K giants;
long and small periods

SPECTROSCOPIC BINARIES

$$f(M) = \frac{(M_B \sin i)^3}{(M_A + M_B)^2} = \frac{K_A^3 P}{2\pi G} (1 - e^2)^{3/2}$$

For exoplanet:

$$f(M) \approx \frac{M_B^3 \sin^3 i}{M_A^2} \quad (\text{since } M_B \ll M_A).$$

M_A known \rightarrow one can obtain $M_B \sin i$

As we can assume i is randomly distributed, **if M_A is known**, one could thus use the distribution of

$$Y = f(m)/M_A = q^3/(1+q)^2 \sin i$$

to determine the distribution of $f(q)$

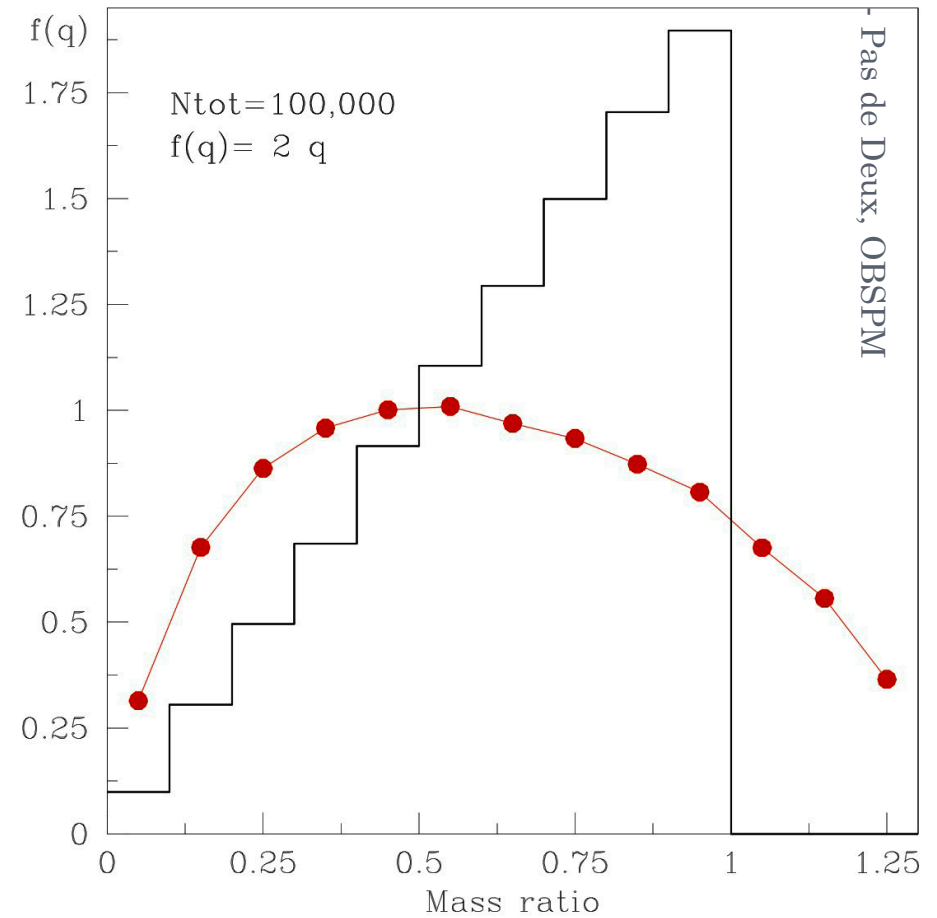
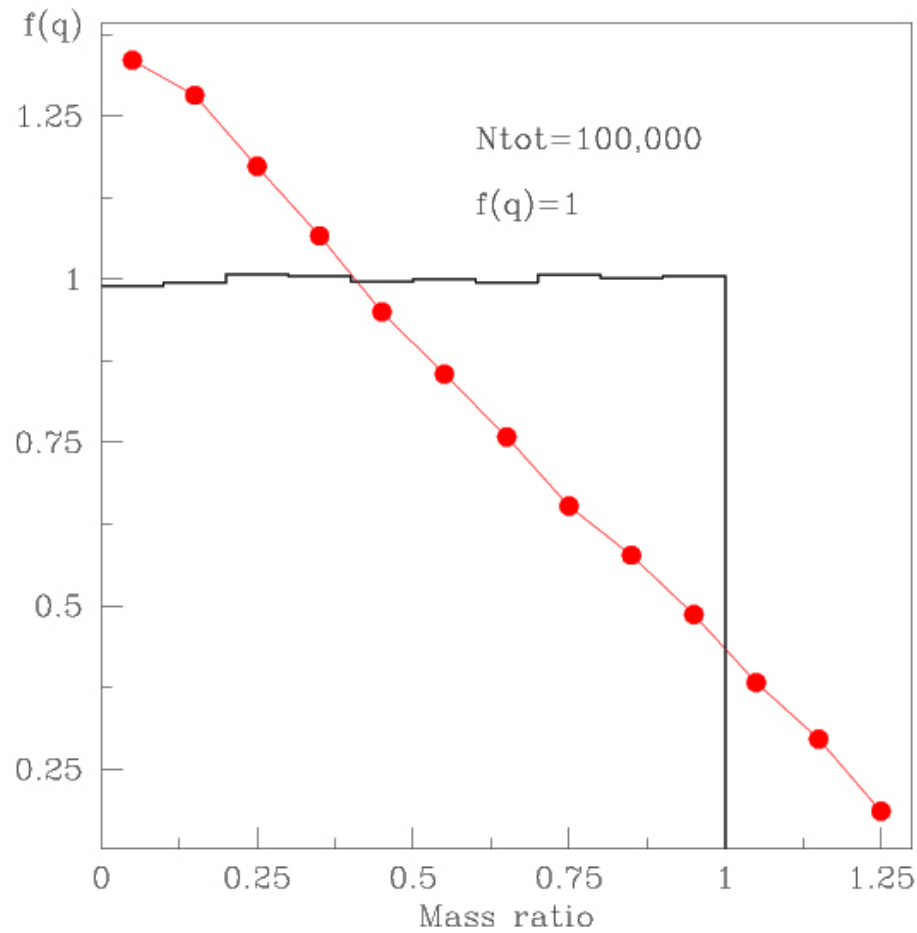
THE WRONG WAY

- *“There is always an easy solution to every human problem – neat, plausible, and **wrong**.”*

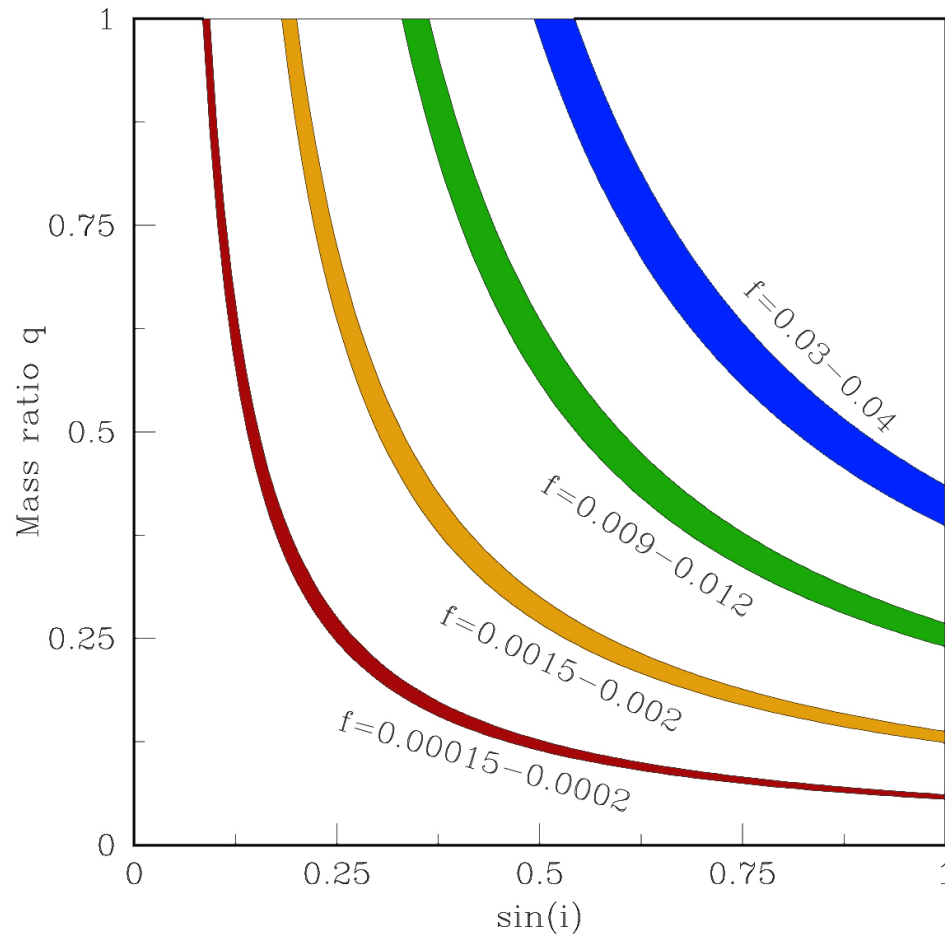
H.L. Mencken, 1917

- Simplest way: replace $\sin^3 i$ by $\langle \sin^3 i \rangle$
e.g. Aitken 35; Trimble 90; Trimble 09

ASSUMING MEAN $\sin^3 i$ DOES NOT WORK!



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e.g. Mazeh & Goldberg 92

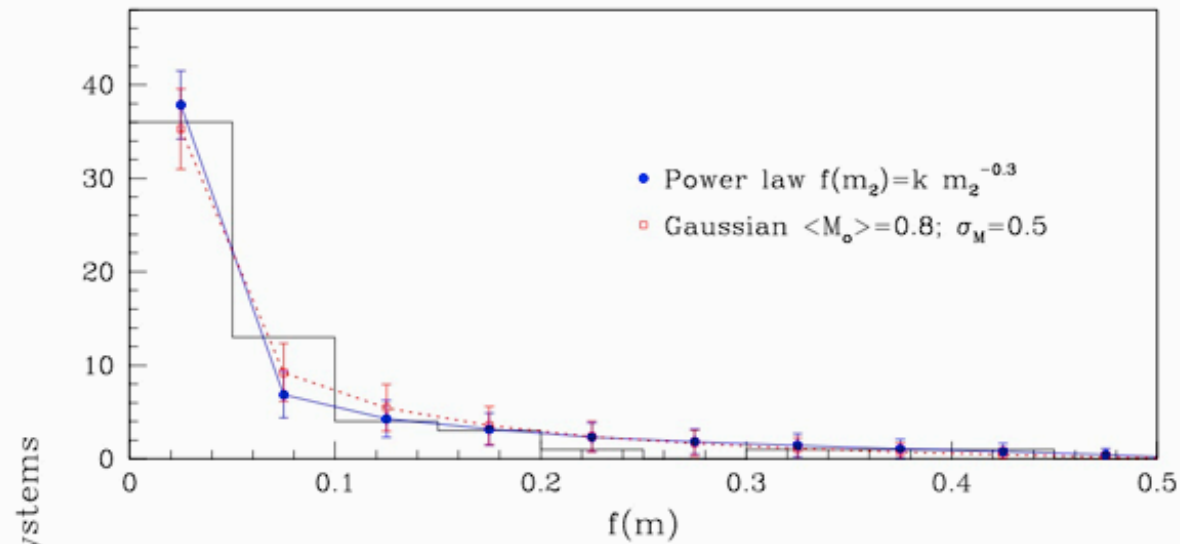
Errors arise because for a given $f(m)$, i and q are not independent anymore and so the mean cannot be the same as when the full range of i is allowed

Similar for exoplanets: a *posteriori* distribution of $\sin i$ is dependent on M_B distribution (see also Ho & Turner 11)

Error is also due to the shape of $f(\sin^3 i)$, cf. Halbwachs 87

FUNCTIONAL FORM FITTING

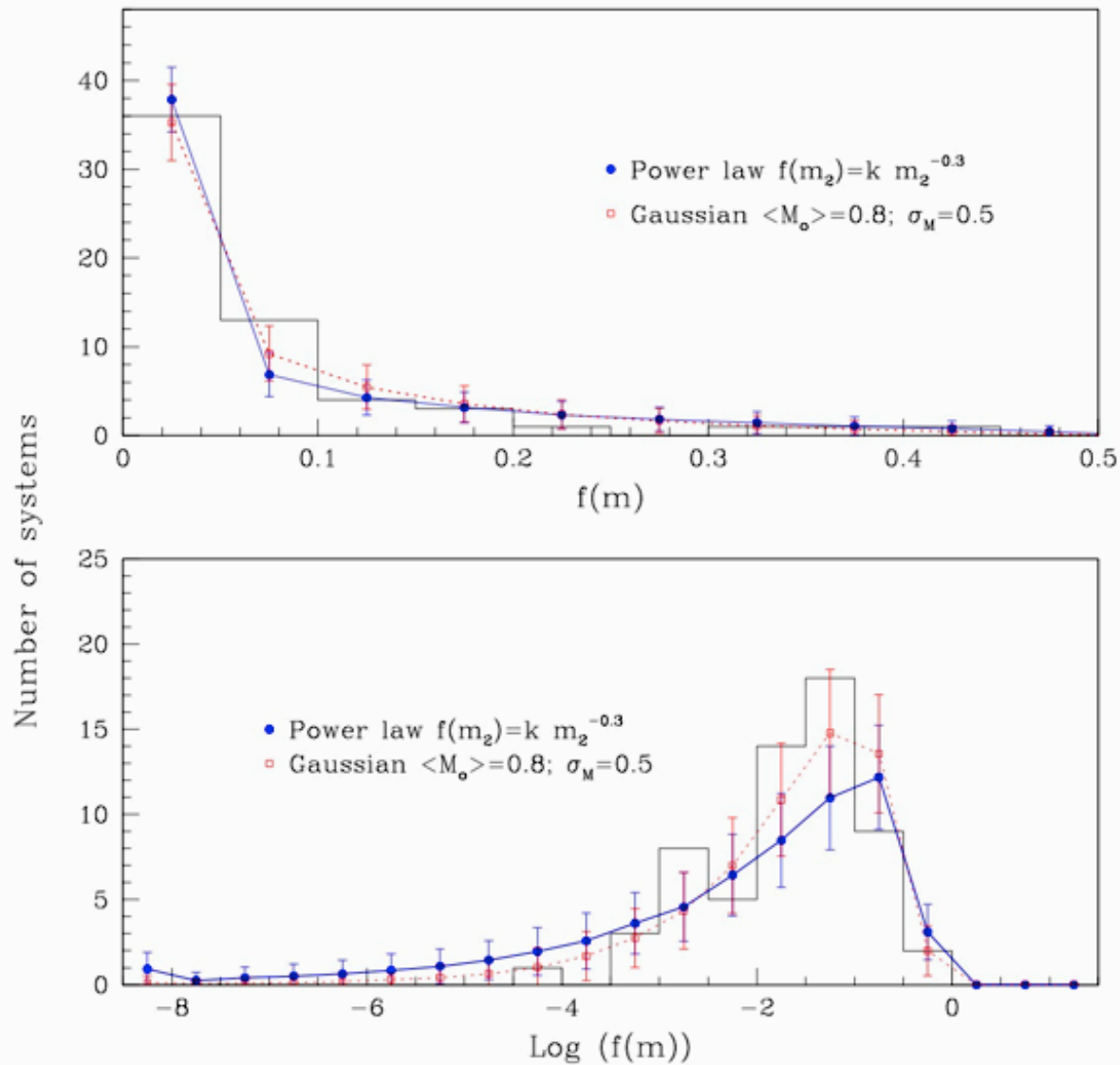
- Instead, one could assume a $f(q)$ and then compute the $f(Y)$ and compare to observed one – using a minimisation method
(Jaschek & Ferrer 72; Halbwachs 87; see also Tabachnik & Tremaine 02 for exoplanets)
- Disadvantage: need to assume functional form and is thus very limited
- Advantage: not tempted to see spurious peaks
- **Important (although obvious) remark:** one should not compare to distribution of $f(m)$ but distribution of $\log f(m)$, cf. wide dynamic range



- 60 orbits (for 53 Am systems)
- Fit distribution of mass function
- $M_1 = 2 M_\odot$
- Assume functional forms: gaussian and power law

INCORRECT!

(as seen when compare to $\log f(m)$)

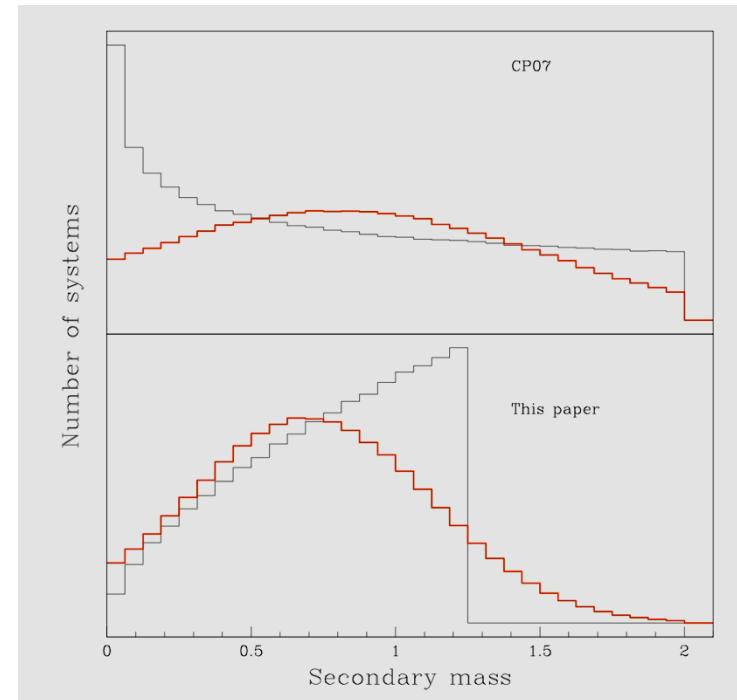
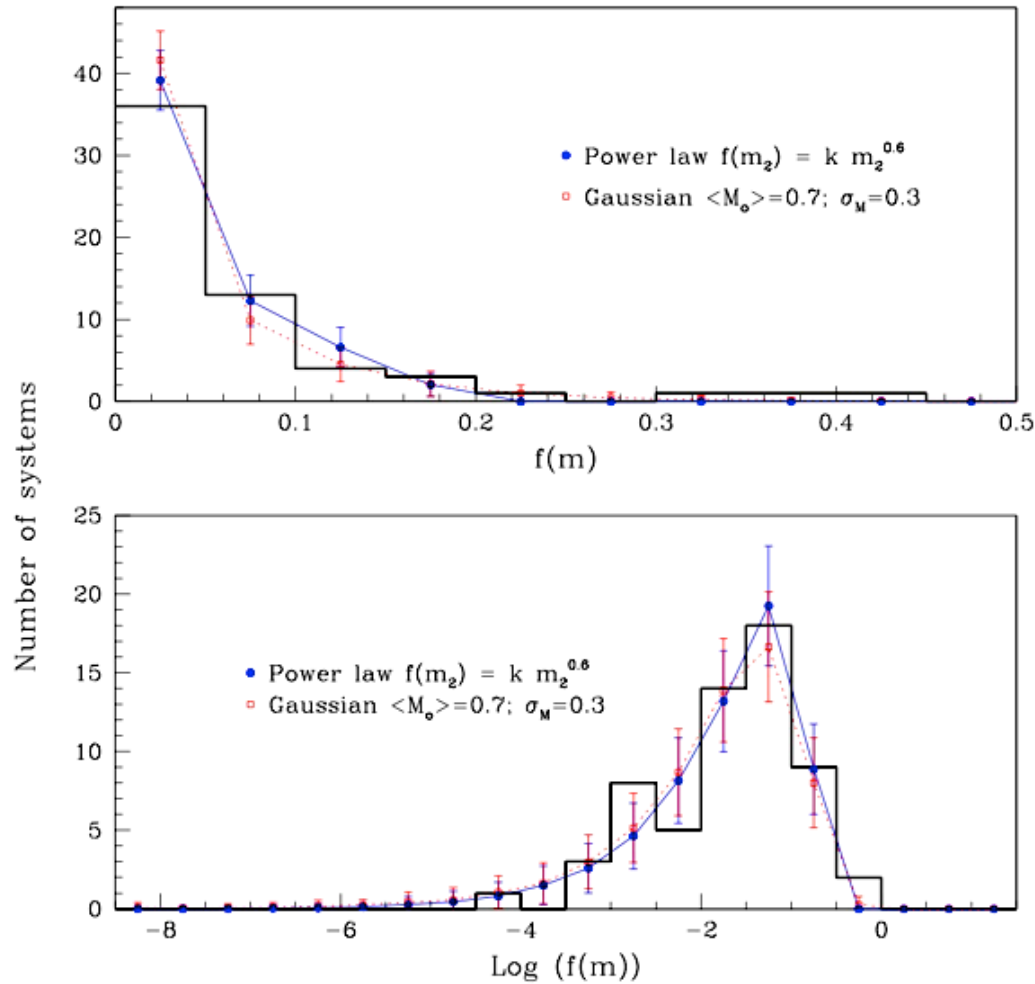


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REVISITING CP07

- Fit the $\log f(m)$ distribution
- Power law with positive index!
- Need to limit to $M_2 < 1.25 M_\odot$
- Gaussian is narrower

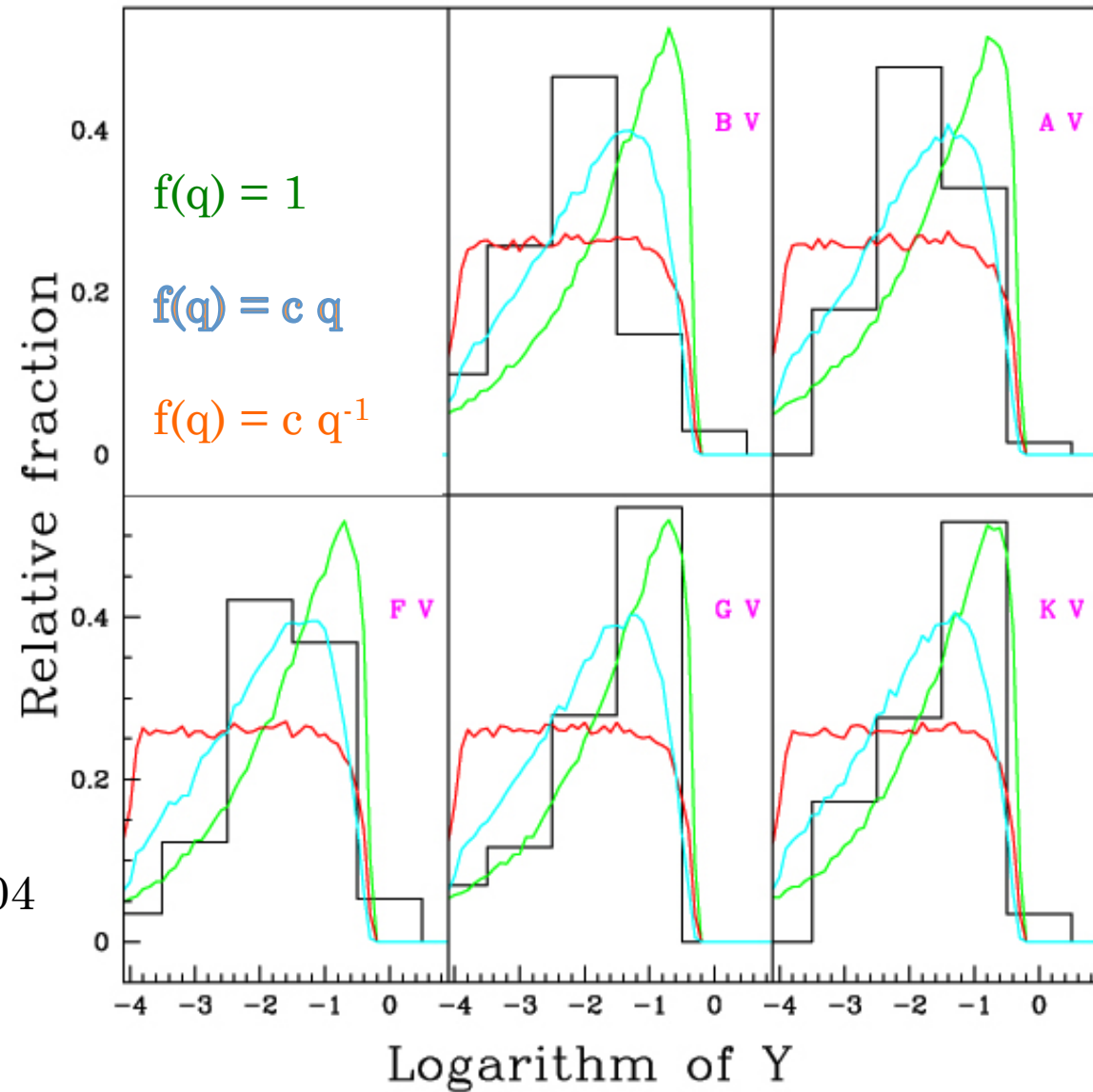


S_B^9 DATABASE

Spectral Type	Number of systems	$\langle q \rangle$
B V	101	0.3789
A V	67	0.4686
F V	57	0.6164
G V	43	0.5673
K V	29	0.6105

Pourbaix, Jancart & Boffin 04

Distribution of Y



INVERSION METHOD

From $Q=q^3 / (1 + q)^2$, we have $f(m) = M_A Q^3 \sin^3 i$

Thus $Y(Q) = Q \sin i$ is available from observations. The distribution $\psi(Q)$ we are looking for is thus given by

$$\Phi(Y) = \int_0^\infty \Psi(Q) \Pi(Y|Q) dQ.$$

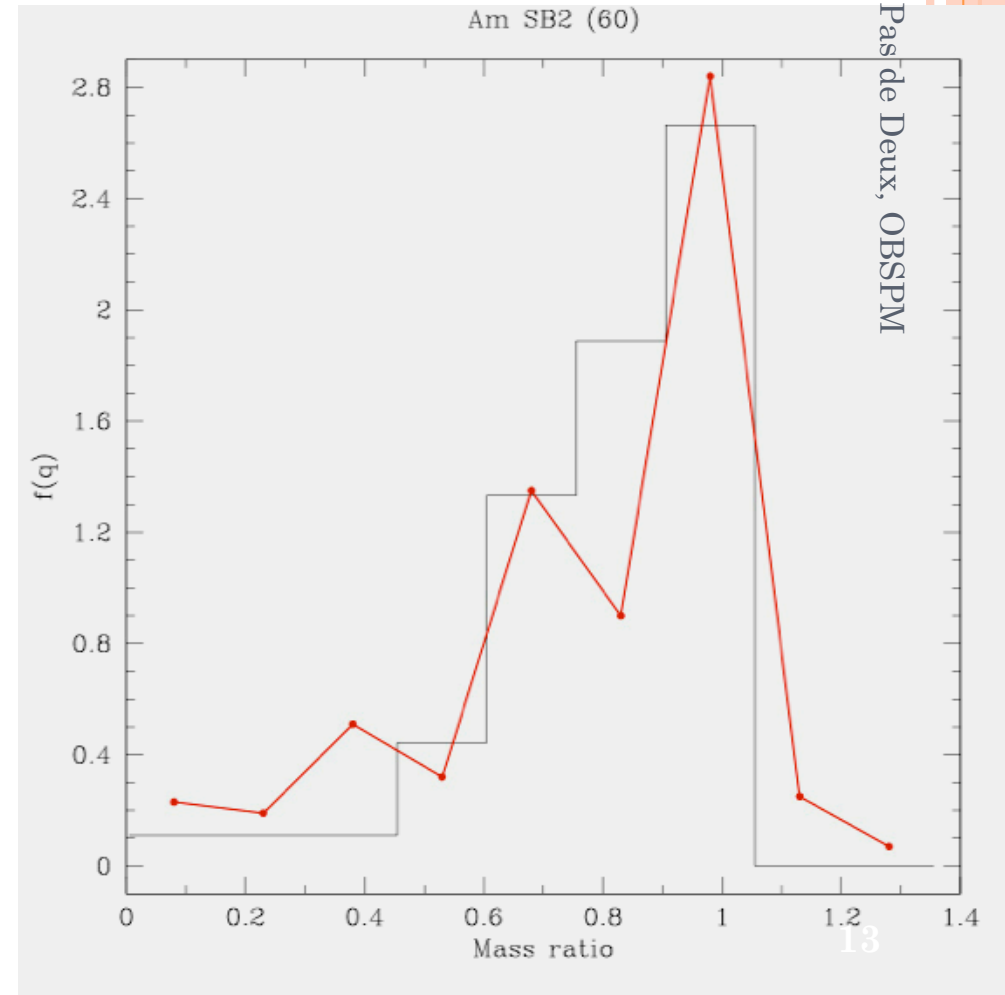
As $f(i) = \sin i$, this reduces to an Abel equation.

One can thus solve it, either by numerically computing it (need smoothing) or using the Lucy-Richardson inversion algorithm

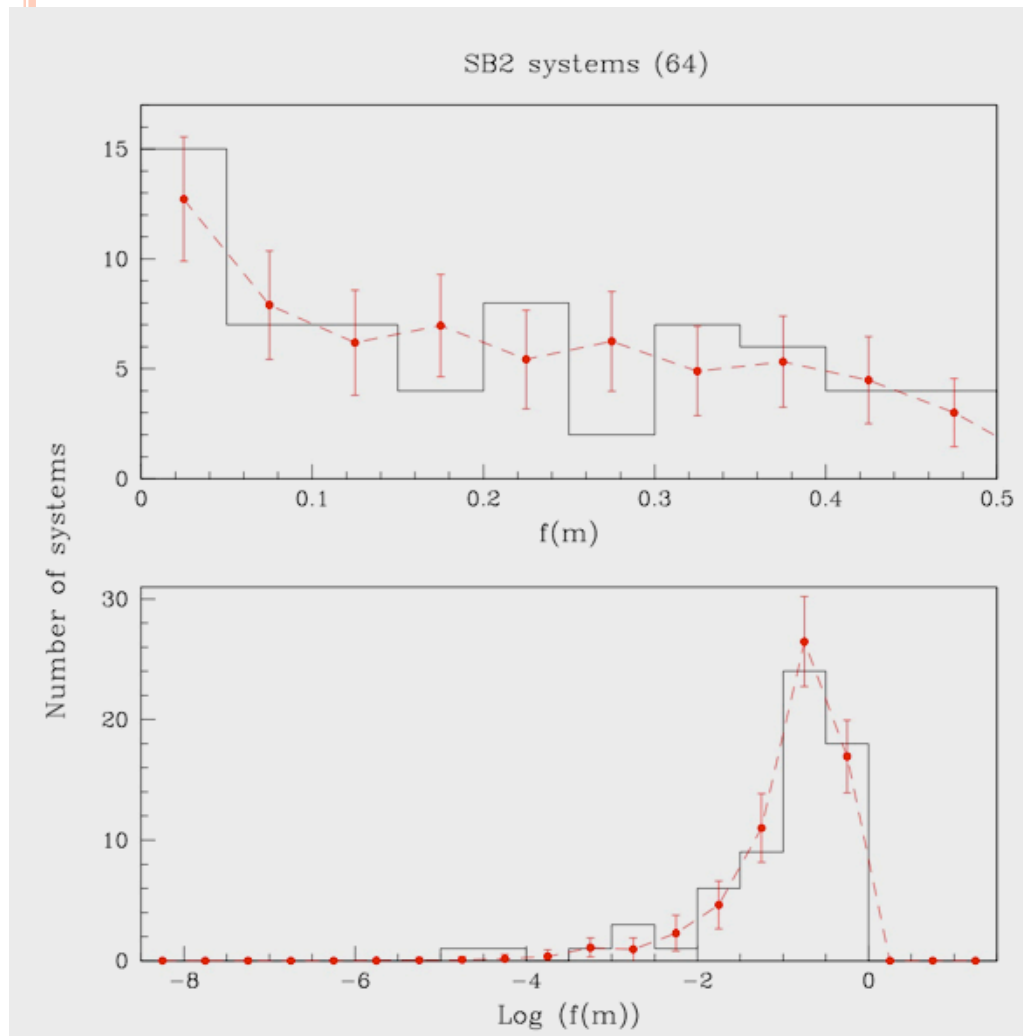
Boffin+ 92, 93, Cerf & Boffin 94, Mazeh & Goldberg 92

BACK TO AM STARS: EXTENDING THE SAMPLE

- Literature search → created a new catalogue to have more orbits
- 162 orbits of Am stars : 98 SB1 and 64 SB2
- For SB2, we directly have q
- I apply Richardson-Lucy inversion method – check with SB2

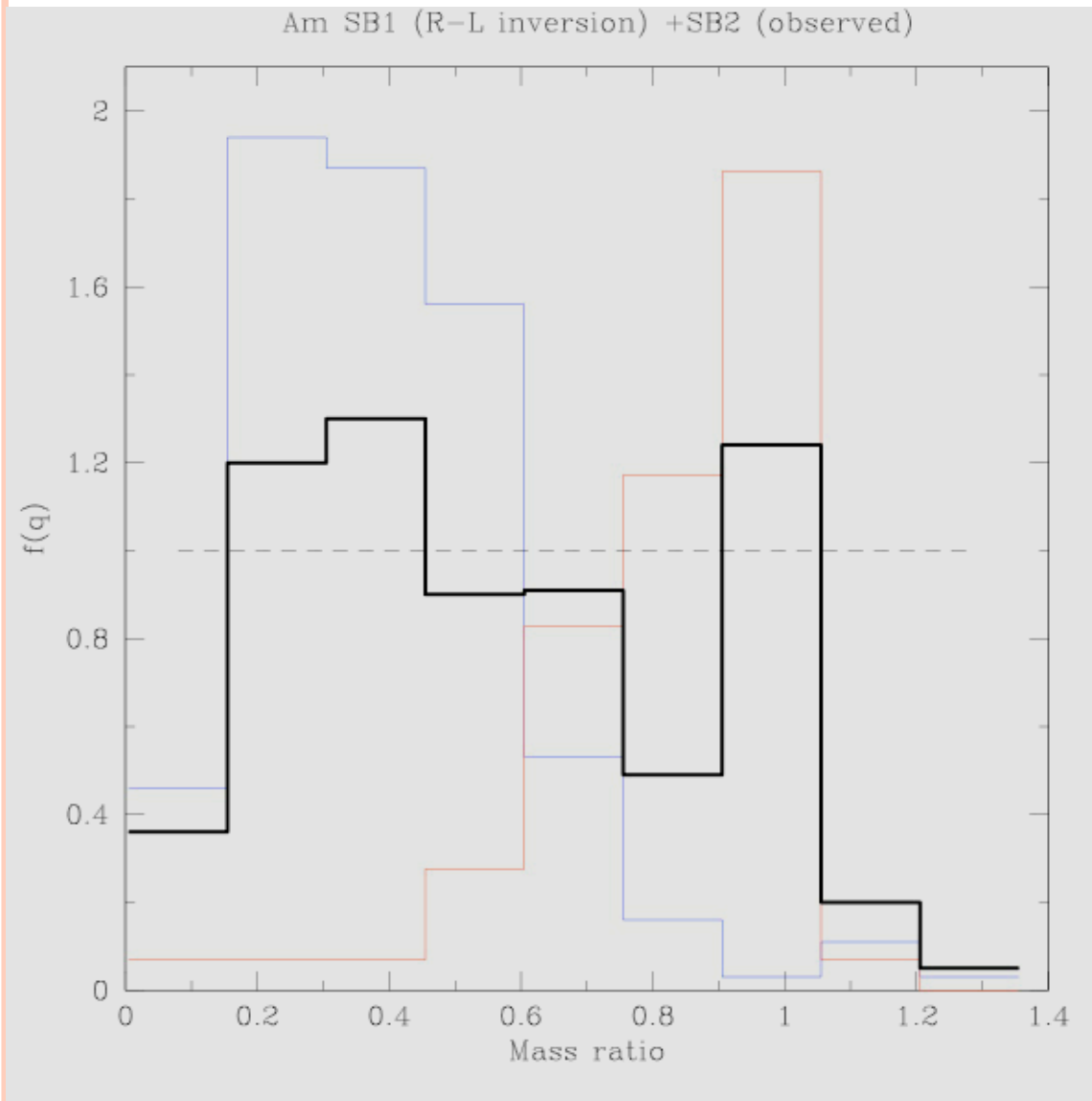


EXTENDED AM STAR SAMPLE



- With SB2, I can also check the methodology, i.e.
 - random i
 - M_1 constant ($=2 M_\odot$)
 - fitting $\log f(m)$
- works!
- can apply to the whole sample of SB1

MASS RATIO DISTRIBUTION



- SB1 : R-L method
 - SB2 : direct
 - Or SB1 + SB2 : R-L method
- compatible
- Final $f(q)$ will depend on ratio between SB1 and SB2
 - Observational biases difficult to assess!

OBSERVATIONAL BIASES

- Magnitude selection (Öpik effect)
- Detection limit K_D , where $K < K_D$ are not found
 - Typically $K_D > 3-4 s_{RV}$
- Orbits too long cannot be obtained (no solution but also K too small)
- One need to be aware of these and, if sure we understand them, correct them, or be sure we do not need to care about them.

THE SOLAR-LIKE SAMPLE – NEARBY AND IN CLUSTERS

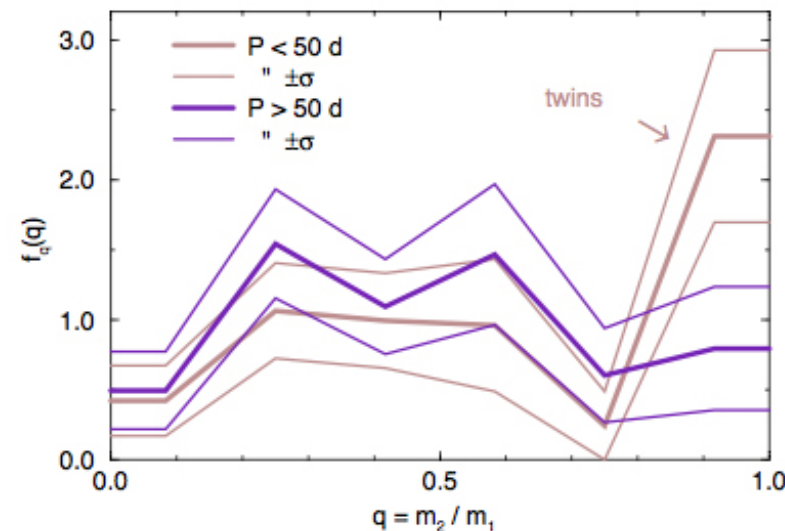
f(q) for binaries with $P < 10$ y

For G-K-M primaries, 2 modes:

- $0,1 < q < 0,7$,*

With a "brown dwarf desert" ($q < 0,1$) vanishing for $P > 2-3$ years

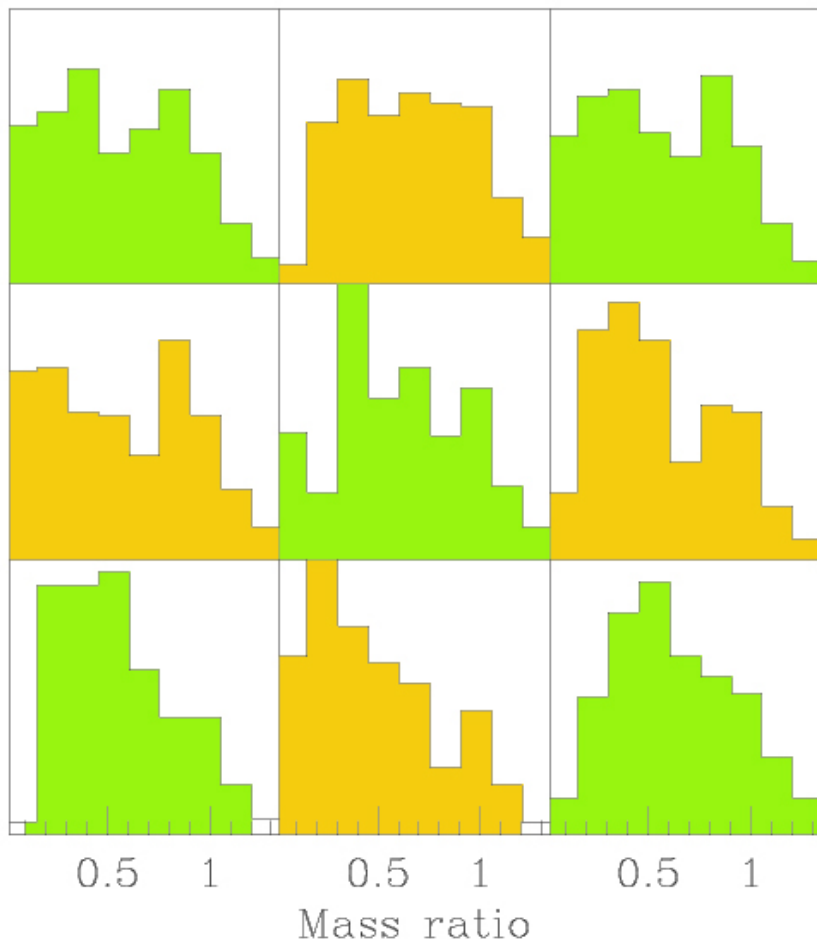
- $q > 0.8$, with a peak around $q=1$ ("twins"). Vanishing when P increasing.*



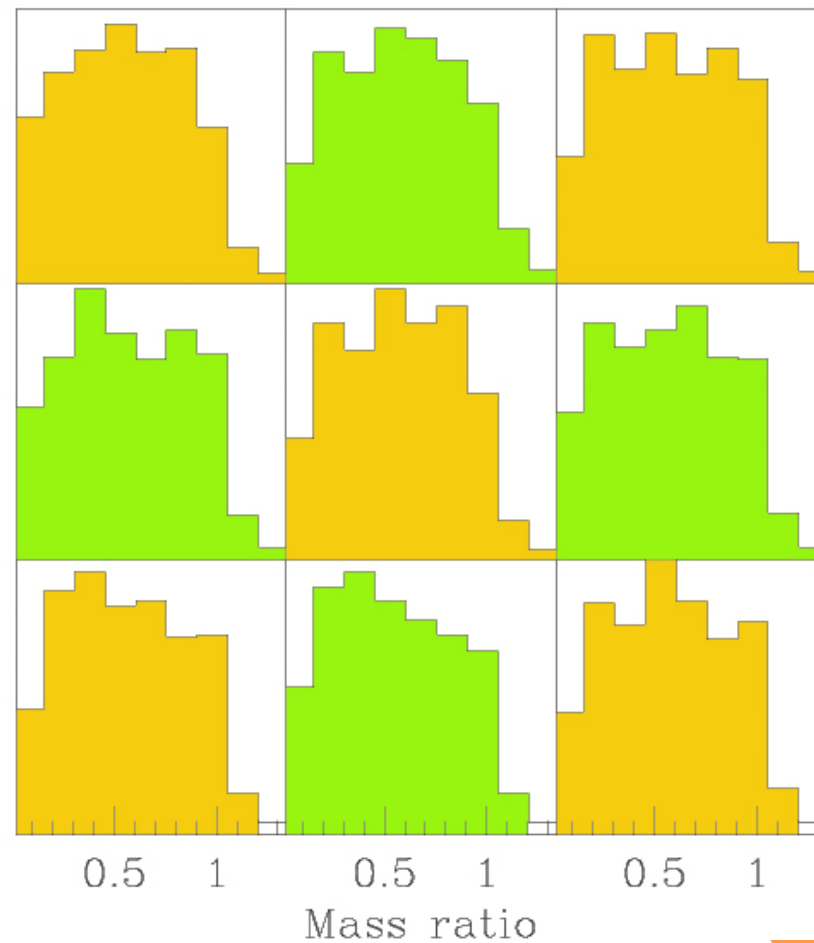
Halbwachs+ 03

EFFECT OF SAMPLE SIZE ($f(q)$ UNIFORM)

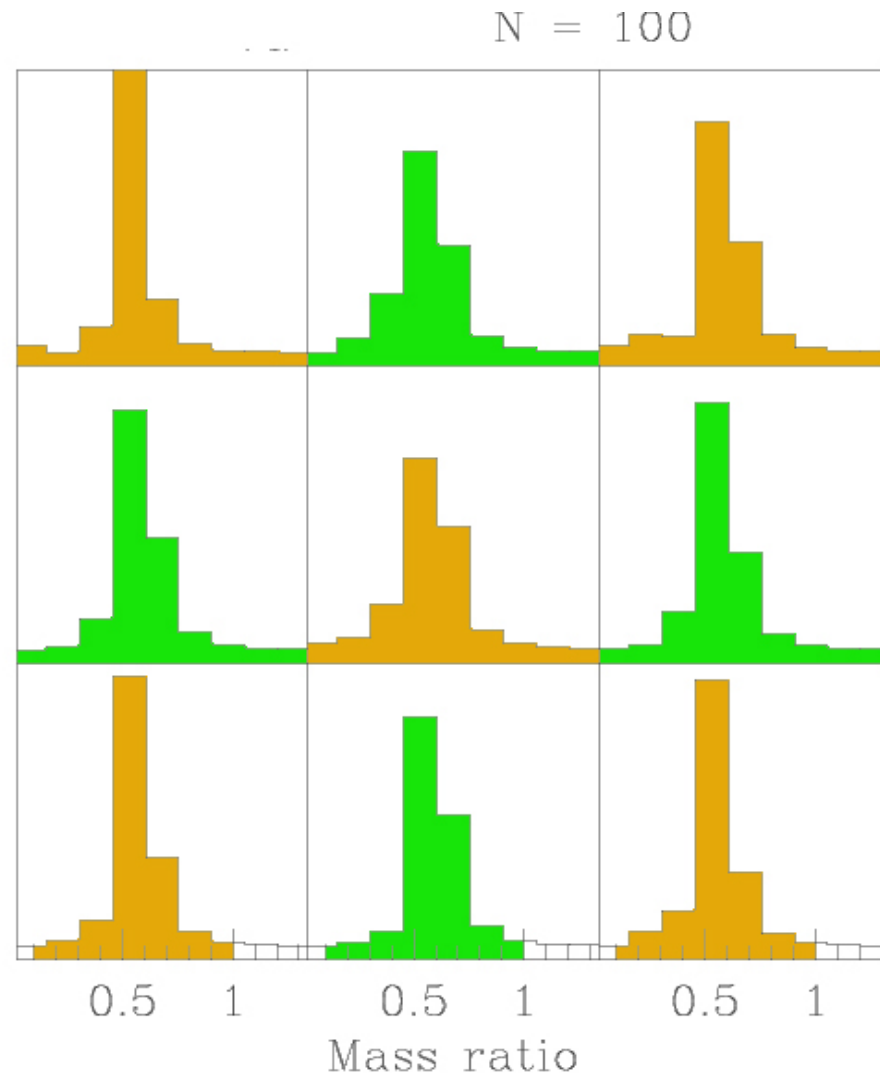
$f(q) = \text{cst}; N = 100$



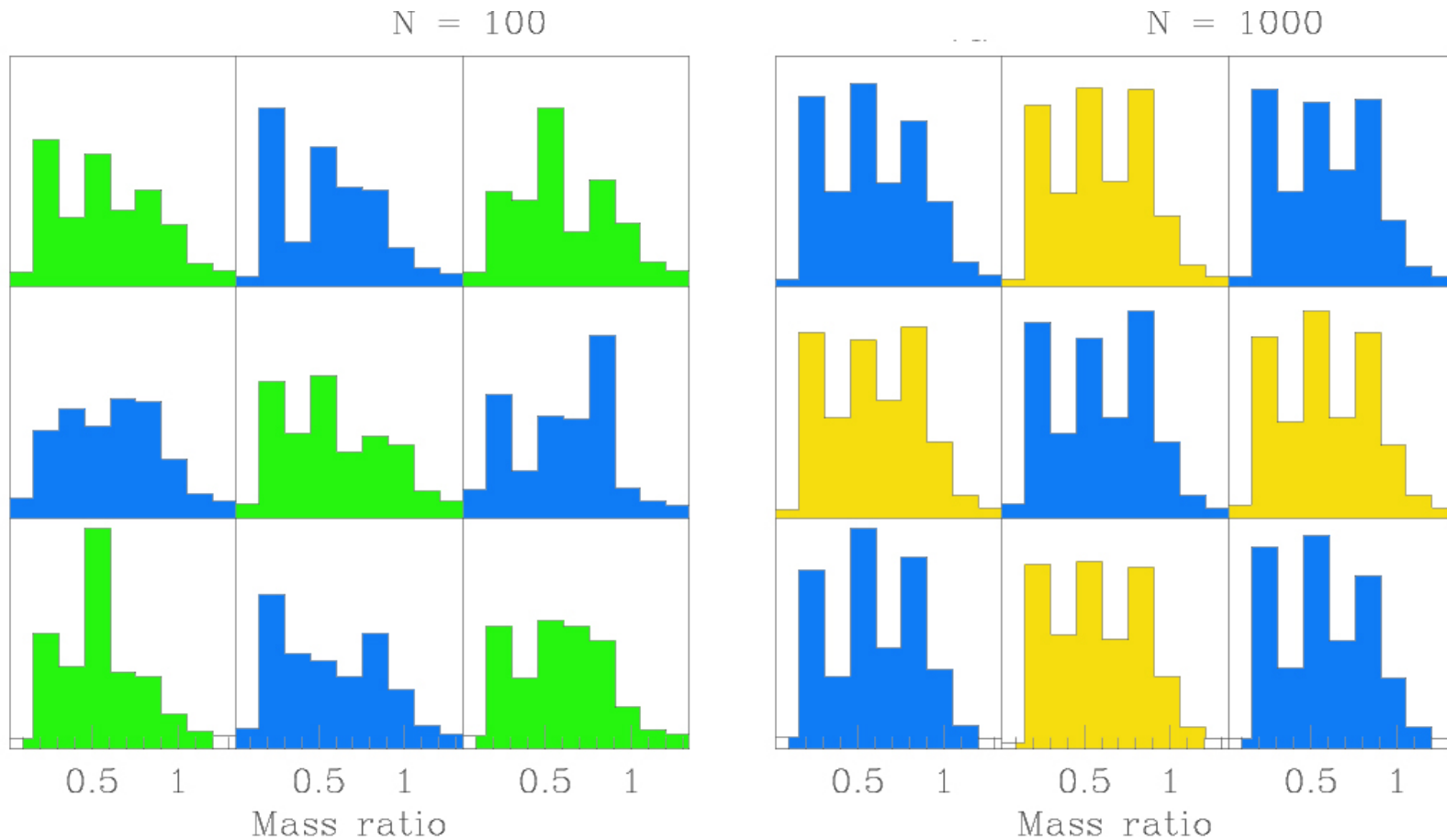
$f(q) = \text{cst}; N = 1000$



SINGLE PEAK SEEMS MORE ROBUST



3 PEAKS



See also Brown 11, for application to exoplanets

A CATALOGUE OF 213 G-K GIANTS

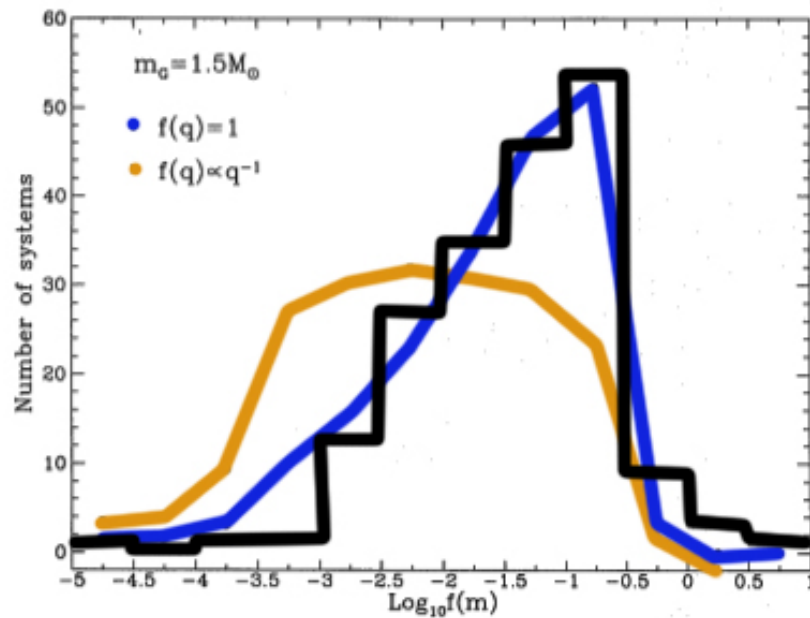
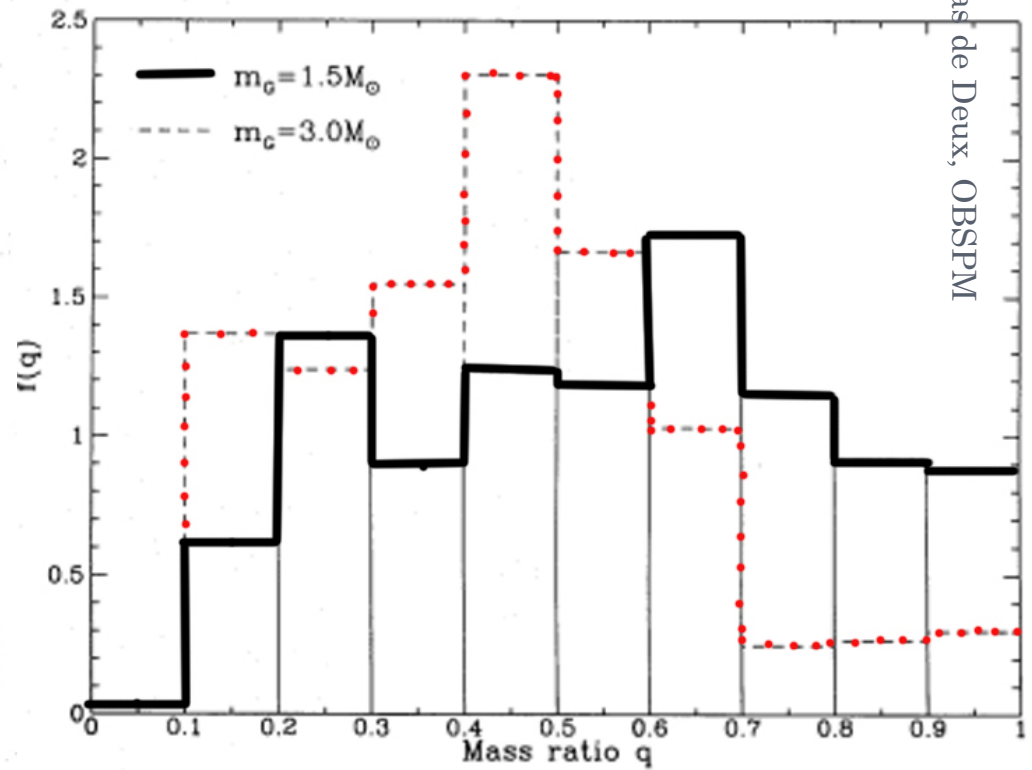


Fig. 10. Distribution of $\log f(m)$ computed by the Monte Carlo technique using a giant mass $m_G = 1.5 M_{\odot}$ and $f(q) = 1$ (●) or $f(q) \propto q^{-1}$ (■), compared to the observed one (solid line)



Boffin, Cerf & Paulus 93

WHAT ABOUT GAIA?

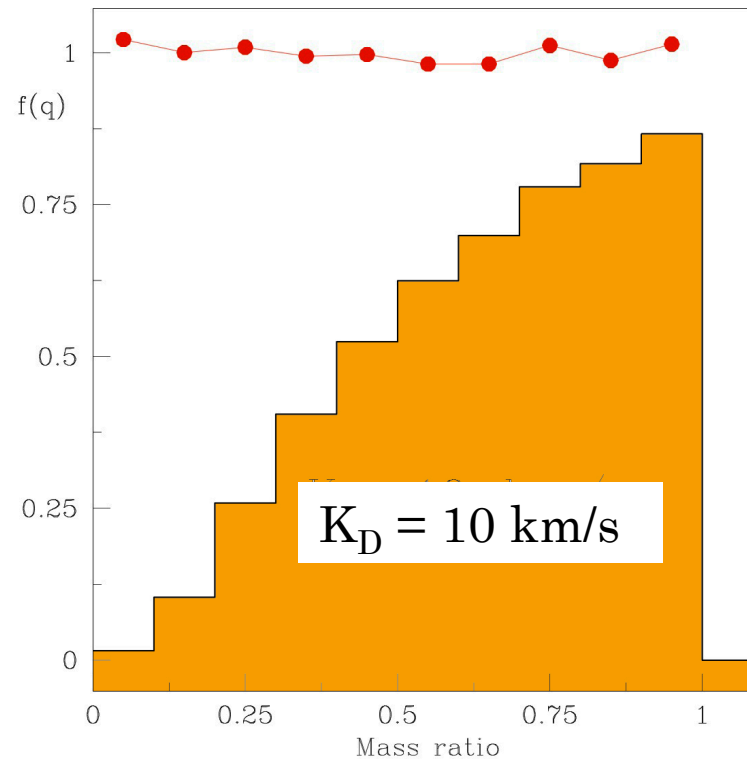
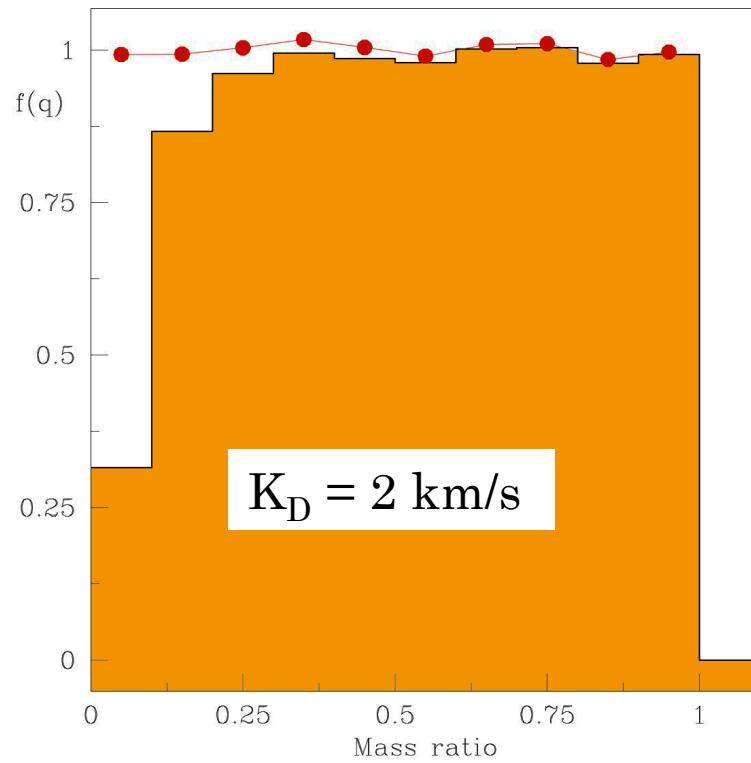
- GAIA will provide us with a flurry of new SBs
- Observe for 5 years
- $\sim 10^6$ orbits could be derived, finally making it possible to have huge samples for statistical analysis
- The survey will be homogeneous, so the bias should be quantifiable
- Simplest: look only at eclipsing binaries ($i \sim 90$)
- But why limit ourselves?

GAIA (II)

○ Problem:

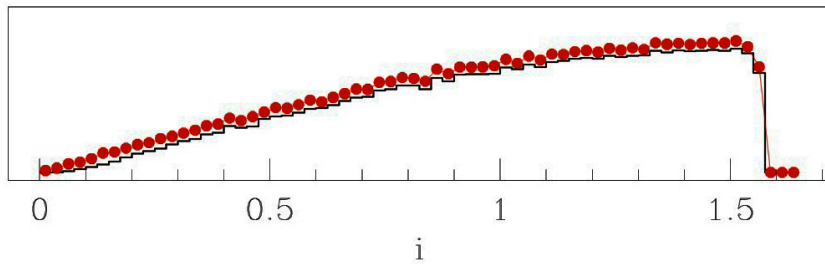
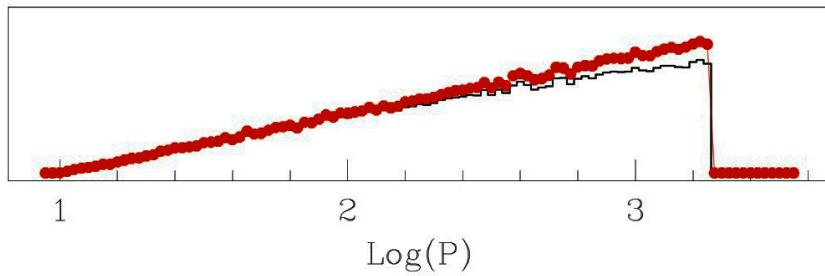
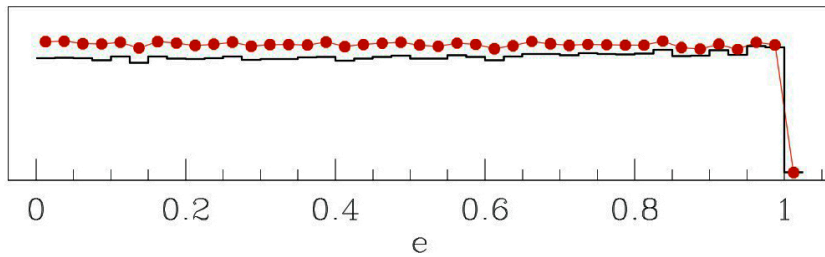
- RV accuracy degrades quickly with G and with spectral type
- $s_{RV} \sim$ a few km/s for relatively bright G-K star (single measurement)
- $s_{RV} \sim$ 10–20 km/s for A-F stars (single measurement)

EFFECT OF K ON DERIVED $f(q)$

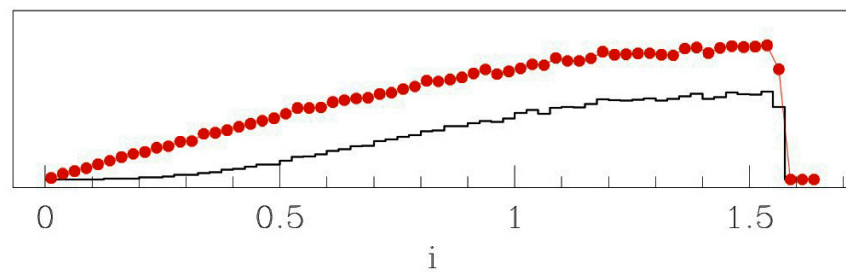
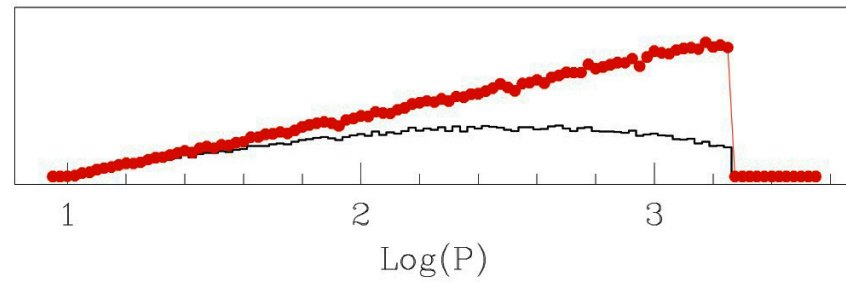
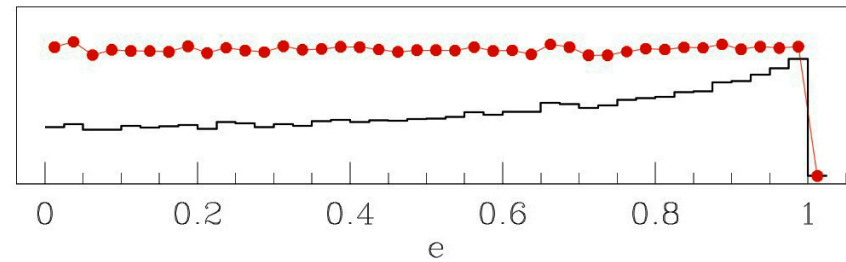


EFFECT OF K

$K < 2 \text{ km/s}$



$K < 10 \text{ km/s}$



CONCLUSION

- $f(q)$ important
- There are some ways to retrieve it by statistical methods
- But be aware of the limitations and the rules of the game

- GAIA will revolutionise this – even though it will also be limited in the binaries it can sample