Approximation of the gravitational potential of a non spherical asteroid

Application to binary and triple asteroidal systems

Audrey Compère[1], Julien Frouard[2] and Benoît Carry[3]

1 NaXys, University of Namur, Belgium
2 DEMAC, Unesp - Univ. Estadual Paulista Rio Claro, SP, Brazil
3 European Space Astronomy Center

Workshop Orbital couples : "Pas de deux"

10 – 12 October 2011, Paris, France

Plan

1. Context
2. Shape models of asteroids
3. Spherical harmonics development
4. Applications to multiple systems
    1. Search for resonances in the triple system Sylvia
    2. Validation of orbits (for the system Daphne)
5. Conclusion

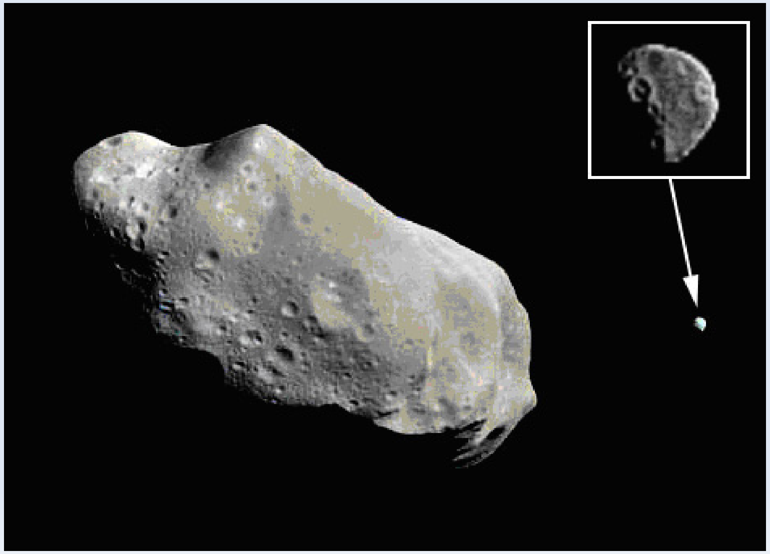
1. Context

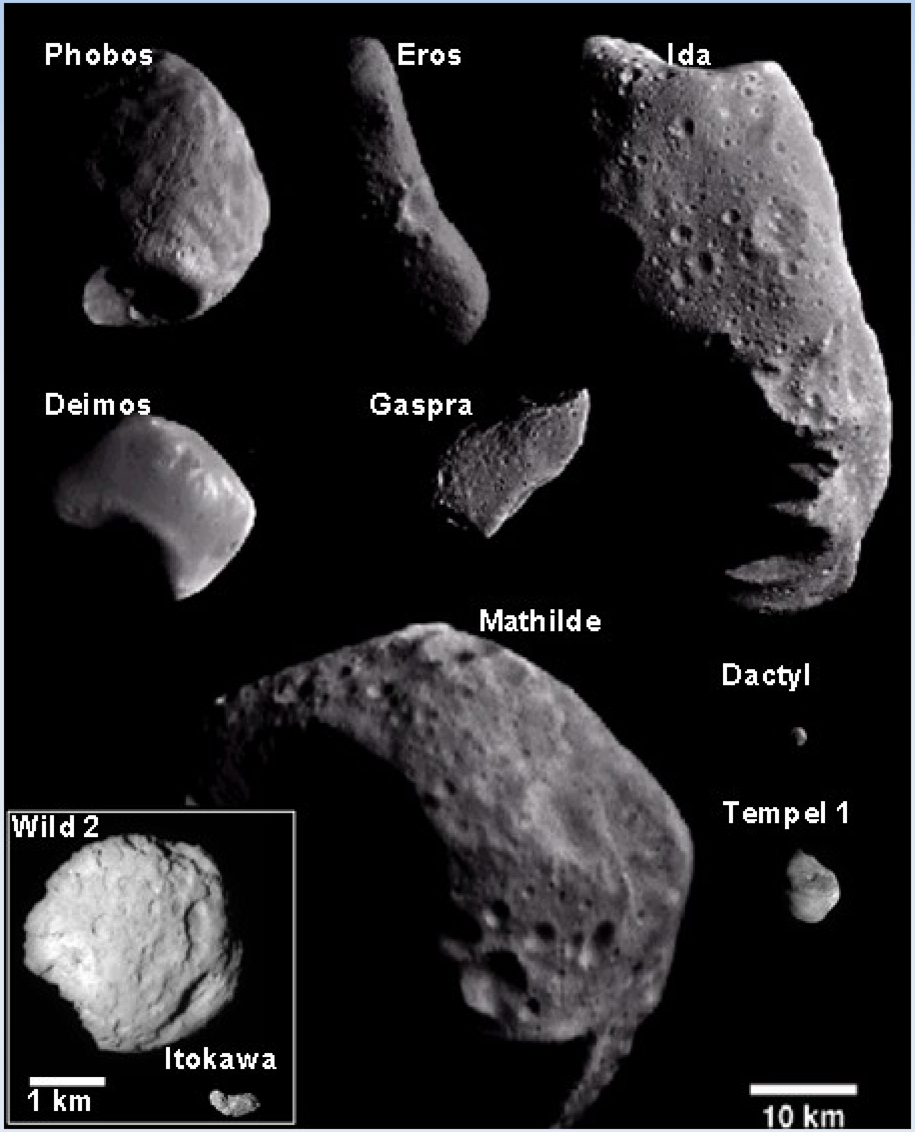
Asteroids usually have a very non-spherical shape :

	Radius of the ellipsoid (km)				
Ida	29.0	×	12.7	×	9.3
Sylvia	175	×	132	×	113.5
Daphne	119.5	×	91.5	×	76.5

Our interest : satellites of asteroids

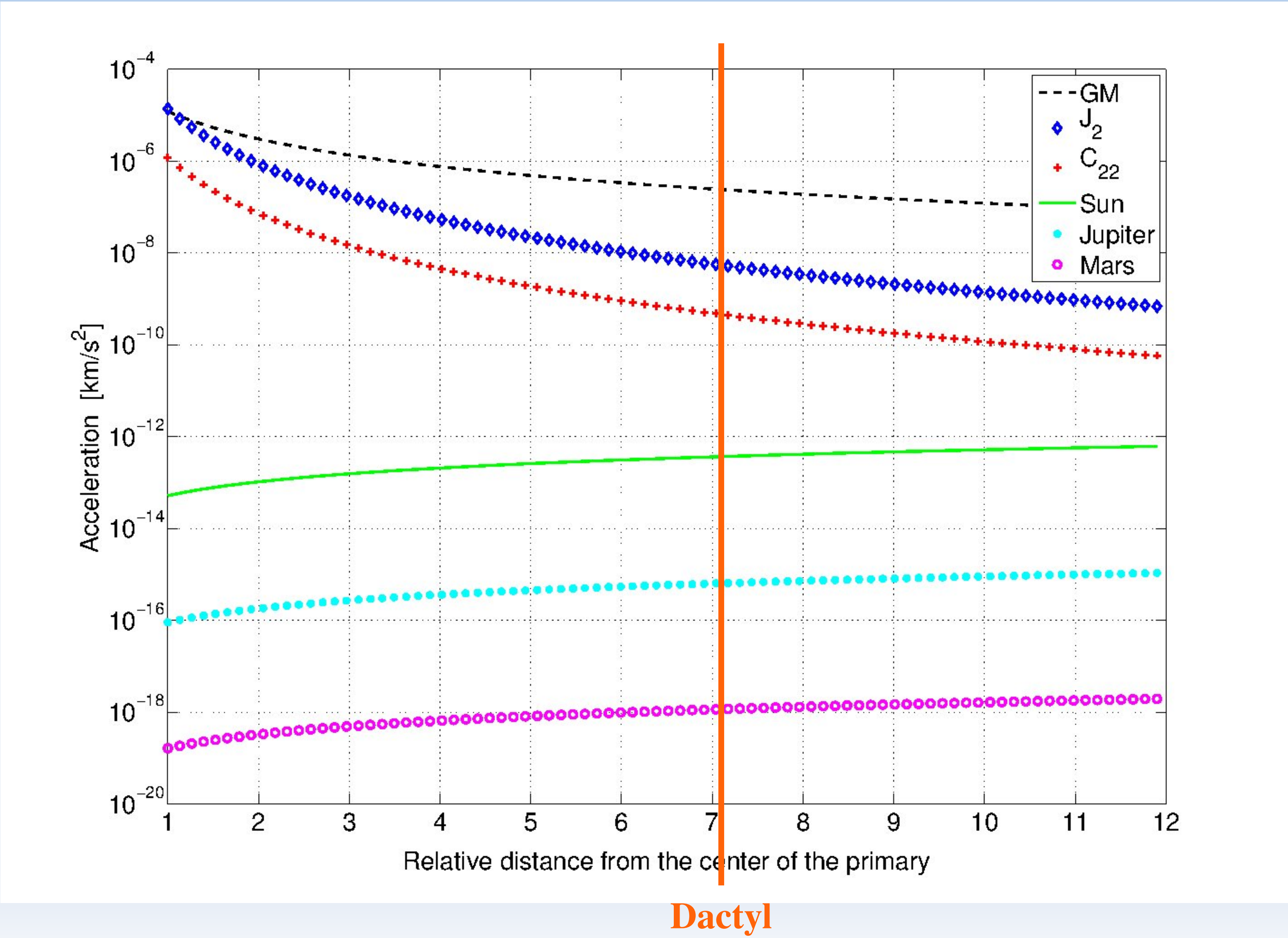
243 Ida and Dactyl





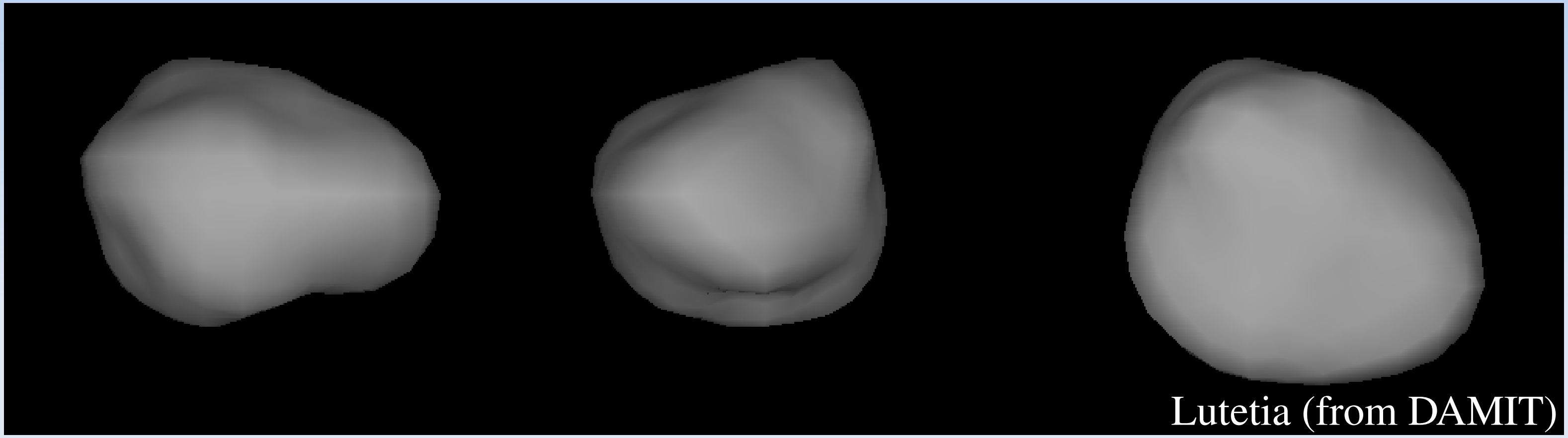
The shape can have a strong influence on the motion of satellites

For Ida (approched by an ellipsoid) :



2. Shape model

Shape models = polyhedrons with triangular surface facets



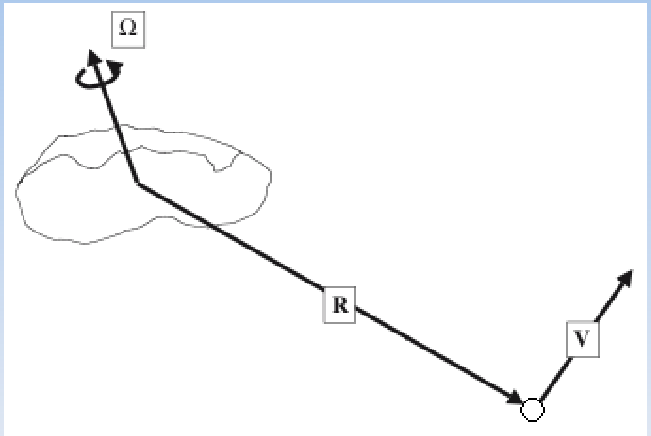
Available on DAMIT : Database of Asteroid Models from Inversion Techniques
(Ďurech et al., 2010)

Models computed by :

- Lightcurves inversion method (Kaasalainen et al., 2001)
- KOALA algorithm : Knitted Occultation, Adaptative-optics, and Lightcurve Analysis (Carry et al. 2010a, Kaasalainen 2011)

3. Spherical harmonics development

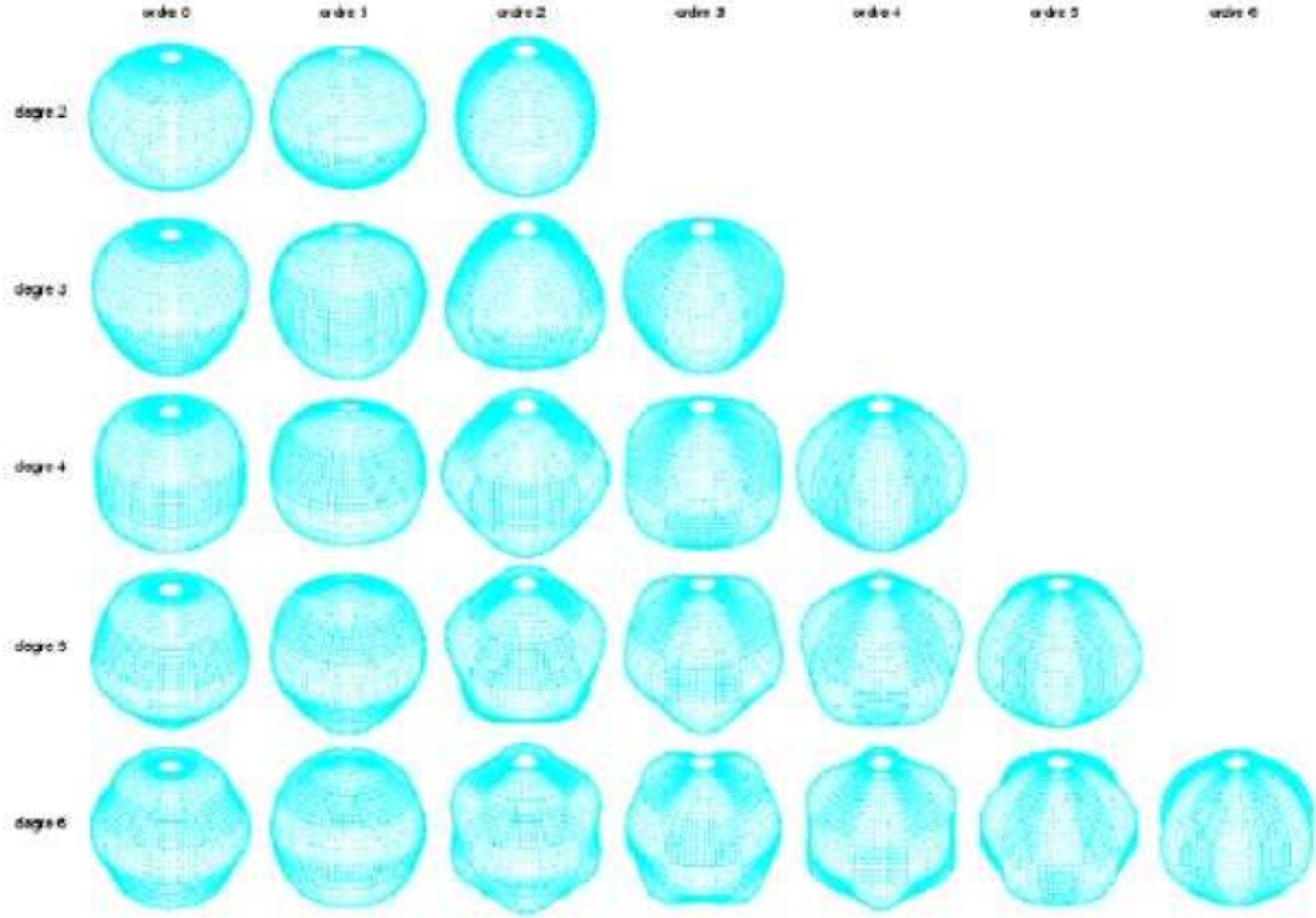
Expression of the gravitational potential due to the strange shapes of these bodies :



$$V(r,\lambda,\phi) = -\frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_e}{r}\right)^n \mathcal{P}_n^m(sin\phi)\,(C_{nm}\,cos\,m\lambda + S_{nm}\,sin\,m\lambda),$$

where

- r, λ and ϕ = spherical coordinates of the point
- $\mu = G\,M$ = the gravitational constant of the body
- R_e = mean radius of the body
- $\mathcal{P}_n^m$ = Legendre functions
- C_{nm} and S_{nm} = spherical harmonics coefficients (depending on the mass distribution of the body)


ordre 0
ordre 1
ordre 2
ordre 3
ordre 4
ordre 5
ordre 6
degre 2
degre 3
degre 4
degre 5
degre 6

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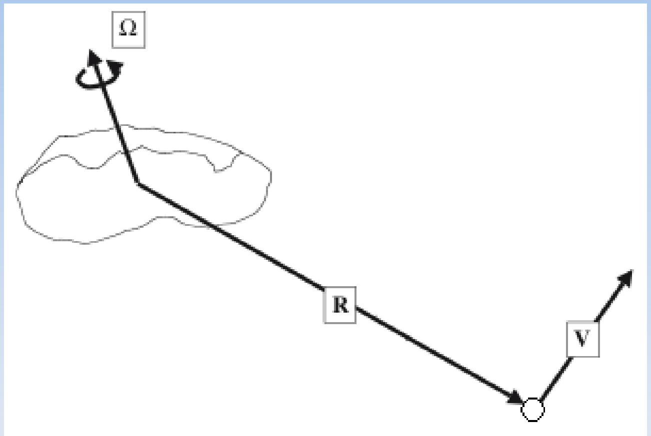

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?

Shape model of the asteroid → Software SHTOOLS (Wieczorek, 2009) → Coefficients of the spherical harmonics expansion

Shape model

↓

Spherical expansion of the **shape** of the body

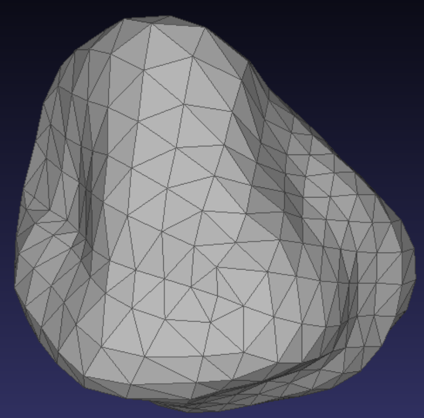
↓

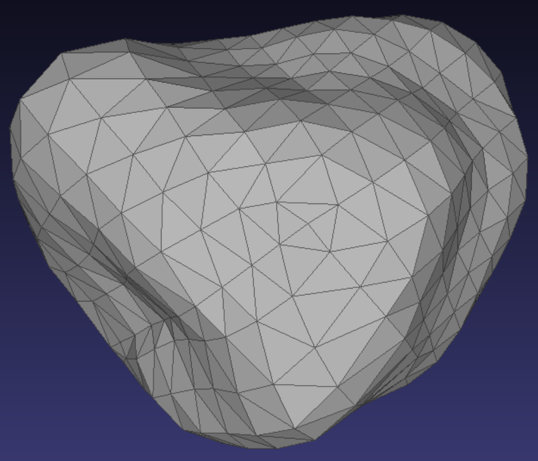
Regular shape model (with a large number of facets)

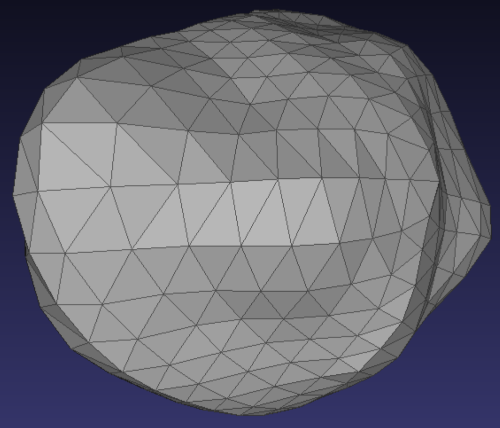
↓

Spherical expansion of the **potential** due to the body

Example : (41) Daphne







$$J_2 \sim 0.13$$

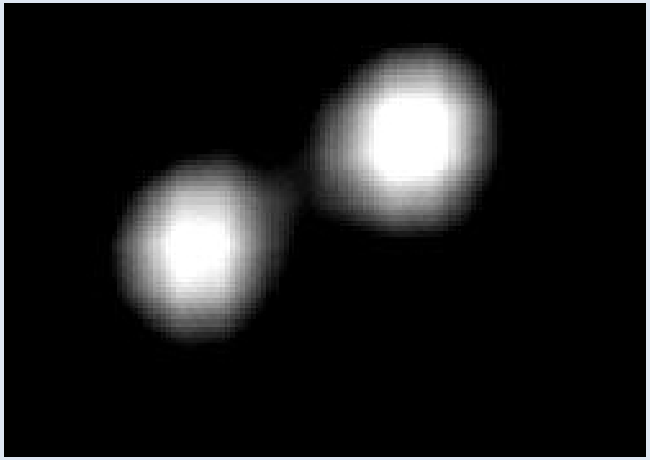
$$C_{22} \sim 0.03$$

4. Applications : multiple systems

Binary asteroid : system of two asteroids rotating around each other

Double asteroid

The two bodies have more or less the same mass



(90) Antiope

Satellite of asteroid

One of the bodies is really more massive than the other

(243) Ida et Dactyl

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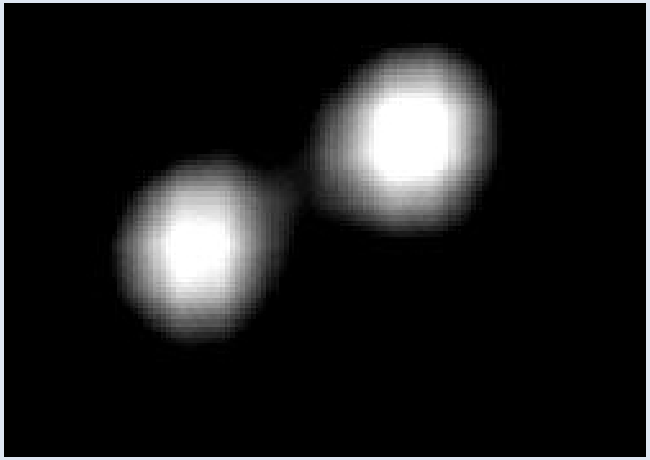

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massive than the other

(243) Ida et Dactyl

Application 1 : Sylvia

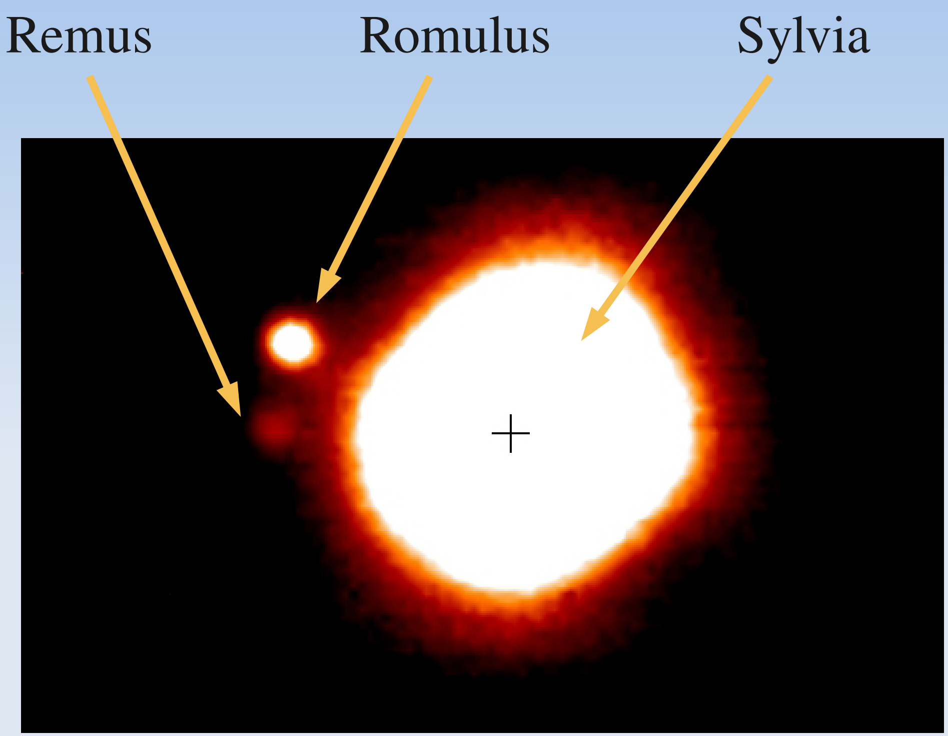
(in collaboration with Julien Frouard)

- Characteristics of the asteroid :
  - first triple asteroid discovered
  - large asteroid of the Main Belt

($R = 130.47\,km$ and $M = 1.4780 \pm 0.006 \times 10^{19} kg$)

- Characteristics of the satellites :
  - much smaller than Sylvia (ratio 10^4 and 10^5)
  - nearly circular and planar orbits

	a (km)	e	i (deg)
Romulus	1356 ± 5	0.001 ± 0.001	1.7 ± 1.0
Remus	706 ± 5	0.016 ± 0.011	2.0 ± 1.0



(Marchis et al., 2005)

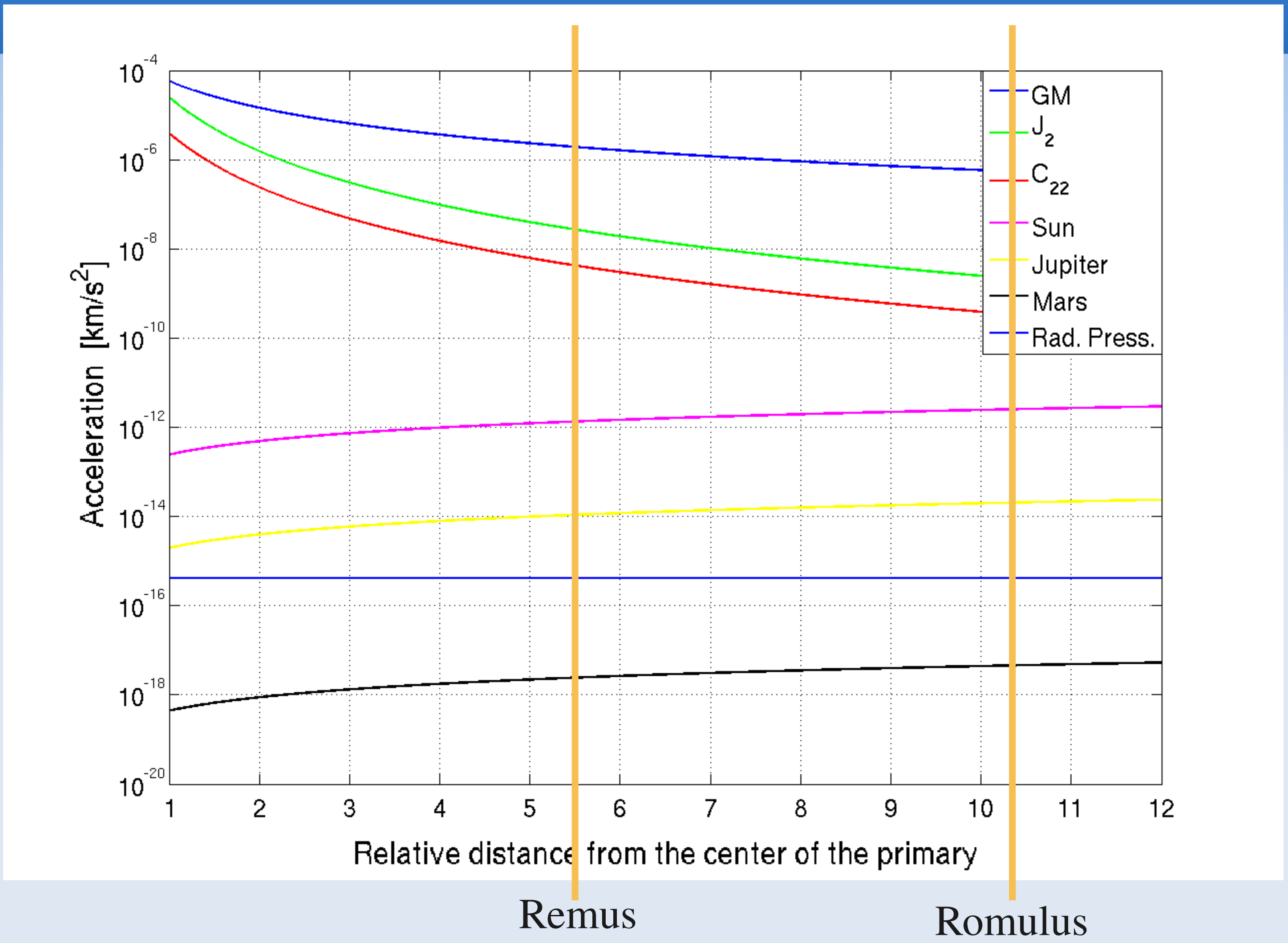
First study : (Winter et al., 2009)

Sylvia = sphere → Instability due to a secular resonance

Sylvia = sphere + 2nd degree gravity field → Secular resonance disappears

Objective of our work : Showing that other resonances are present in the non-spherical case

Satellites small and distant from Sylvia → Point mass satellites



We look for two kinds of resonances :

Mean-motion resonances and secular resonances

Tools

- Chaos indicator : **MEGNO** (Mean Exponential Growth factor of Nearby Orbits)

Tangent vector : $\delta_\phi(t)$

The MEGNO is $$Y_\phi(\mathbf{t}) = \frac{2}{\mathbf{t}} \int_0^{\mathbf{t}} \frac{\dot{\delta}_\phi \cdot \delta_\phi}{\delta_\phi \cdot \delta_\phi} \, s \, ds$$ (Cincotta & Simó, 2000)

= characterization of the rate of divergence of two nearby orbits

Mean MEGNO $\overline{Y}_\phi$
- tends to 0 for orbits close to a periodic one
- tends to 2 for quasi-periodic orbits
- increases linearly with time for chaotic orbits

- Frequency map analysis (Laskar, 1999)

= numerical search for a quasiperiodic approximation of the solutions of the system over a finite time span

Mean-motion resonances

- Model : - three body problem (Sylvia-Romulus-Remus) + influence of the Sun
  - spherical harmonics expansion
  - complete equations of motion
  - constant rotation of Sylvia (axis of rotation = principal axis of inertia)

- Program : NIMASTEP (Delsate and Compère)

Mean-motion resonance = resonance between the orbital periods of two objects orbiting the same primary.

$$P_{Romulus} = 3.6496 \pm 0.0007\, days$$
$$P_{Remus} = 1.3788 \pm 0.0007\, days$$

(Marchis et al., 2005)

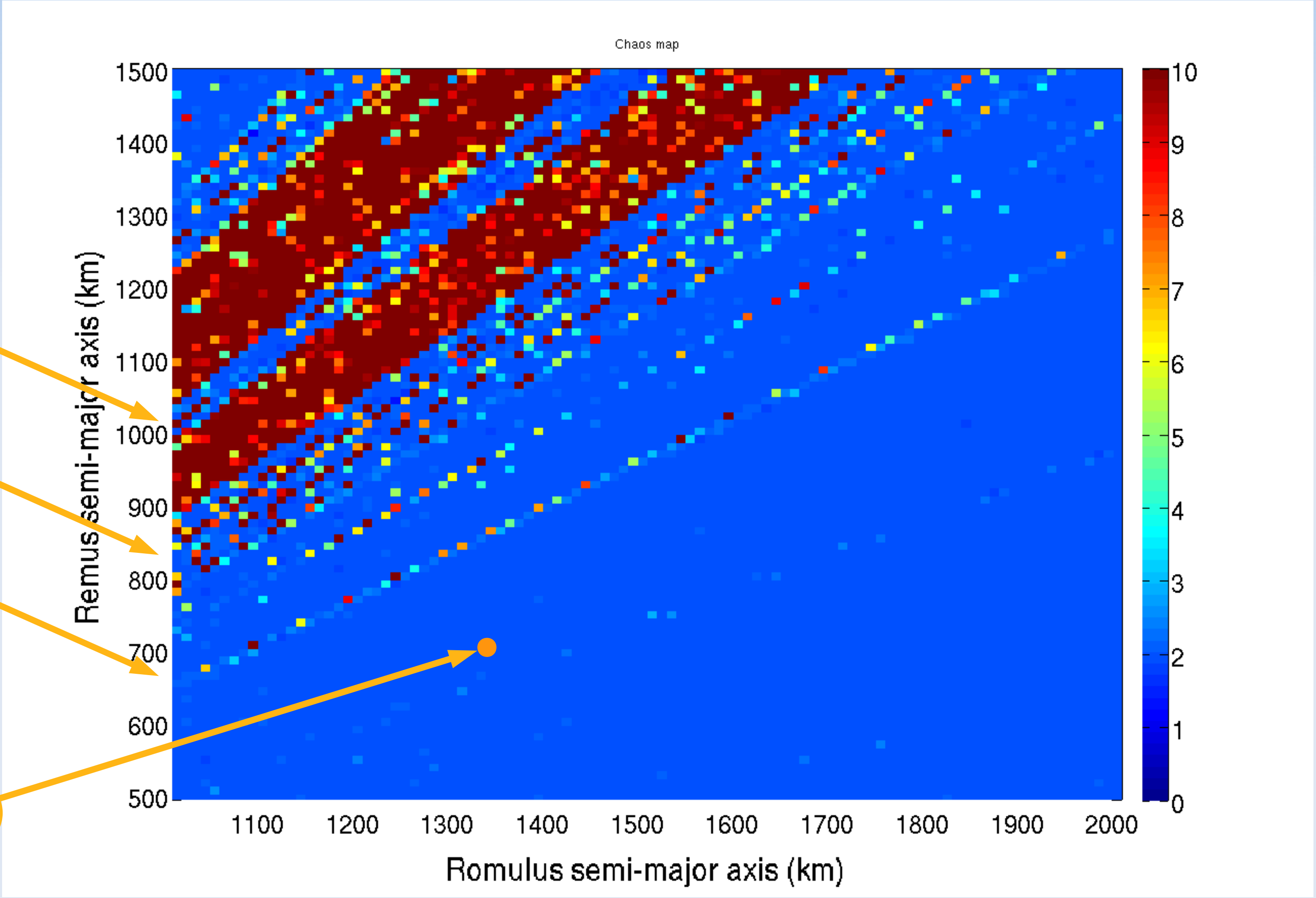
$$\frac{P_{Romulus}}{P_{Remus}} = 2.6469$$ → between 2:1 and 3:1 resonance

MEGNO map (on 20 years) :

1:1 resonance

3:2 resonance

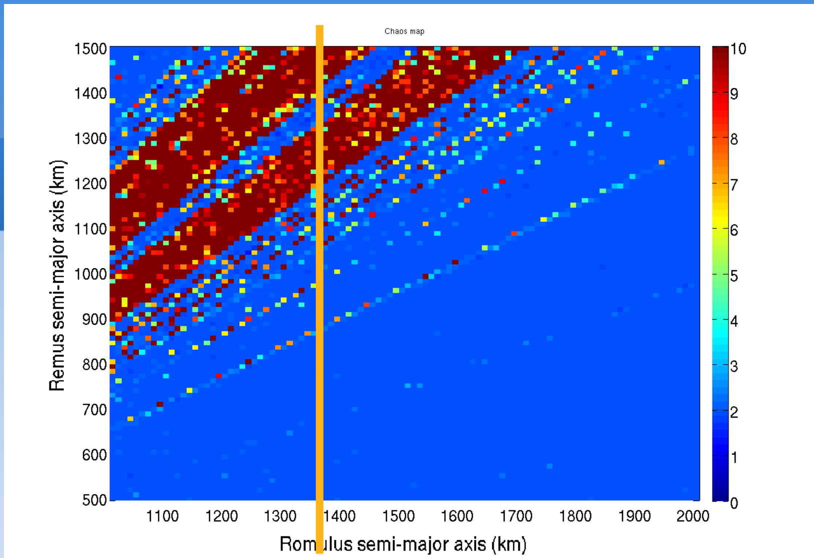
2:1 resonance



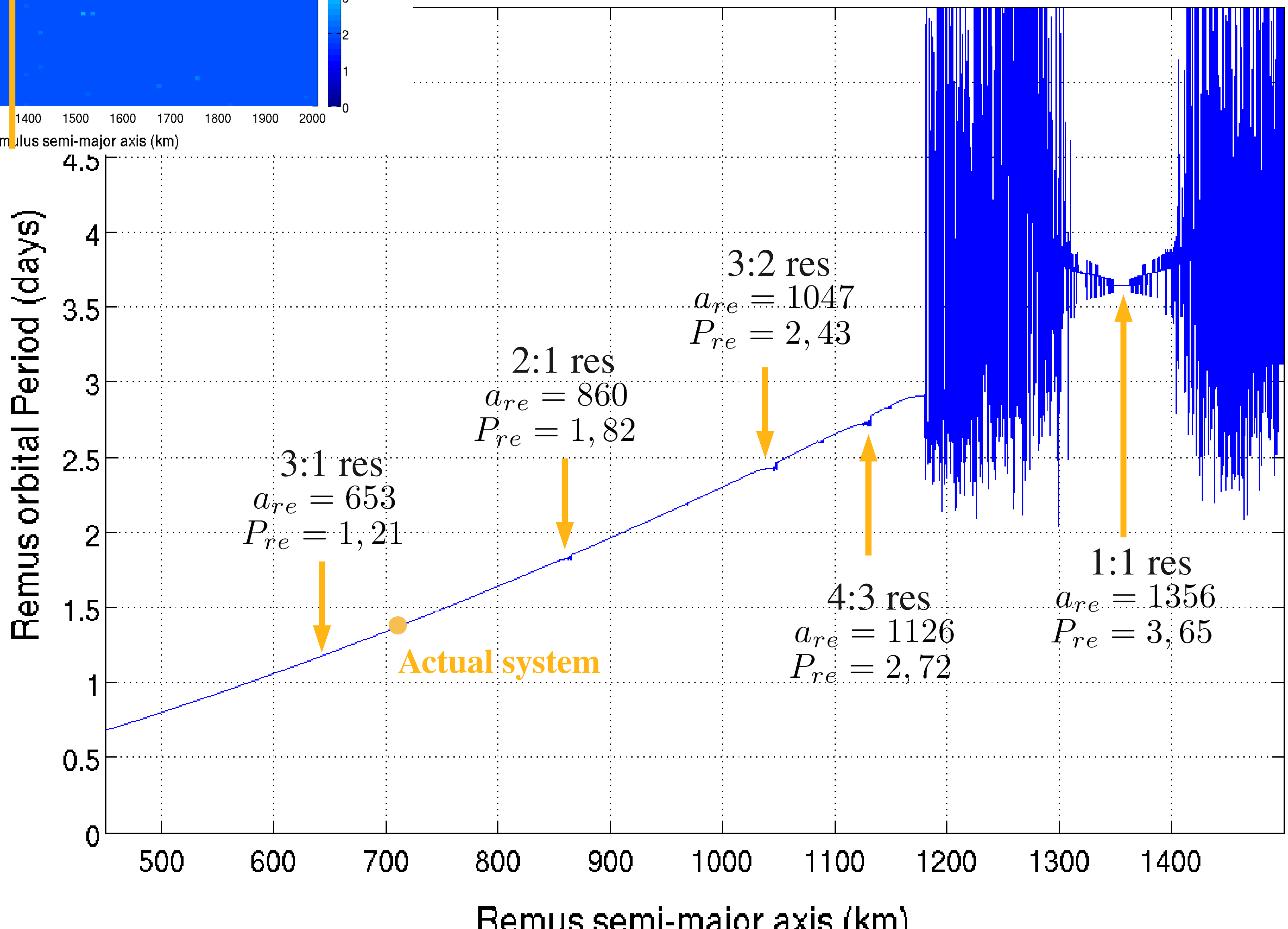
System Sylvia-Romulus-Remus

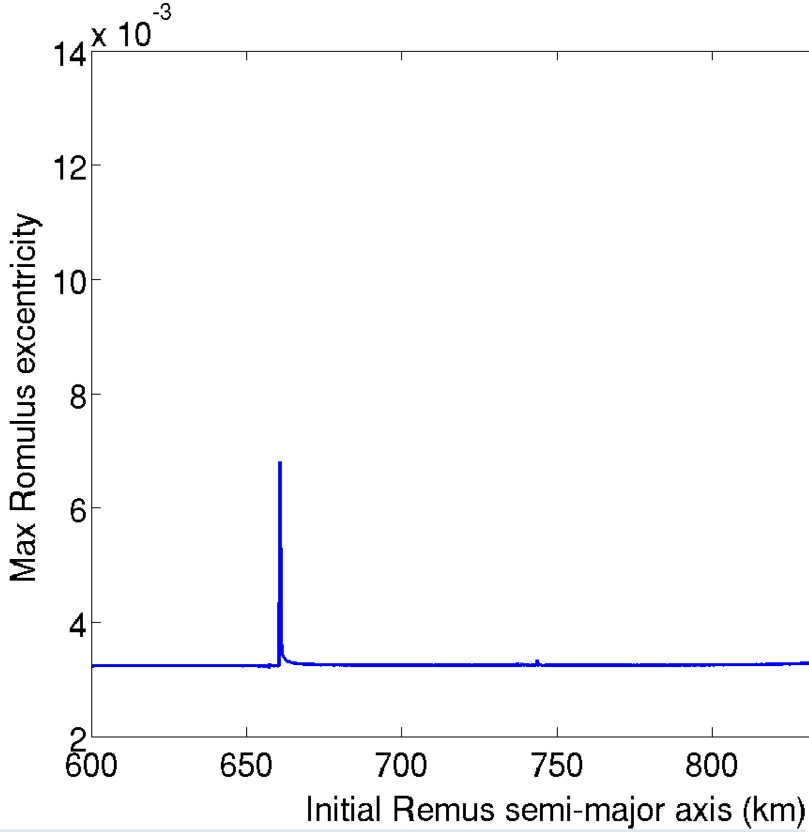
Romulus initial semi-major axis = 1356 km

→ Romulus orbital period = 3.6496 days



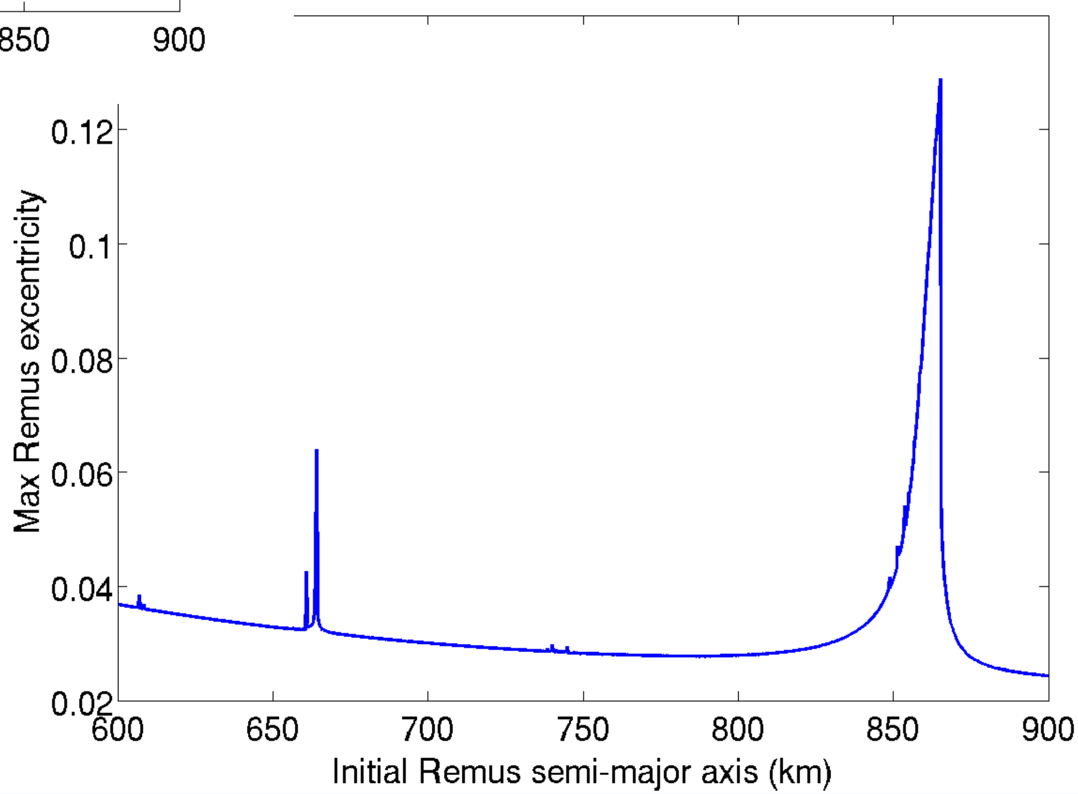
Frequency map analysis of the signal $(a_{re}\cos(\lambda_{re}), a_{re}\sin(\lambda_{re}))$

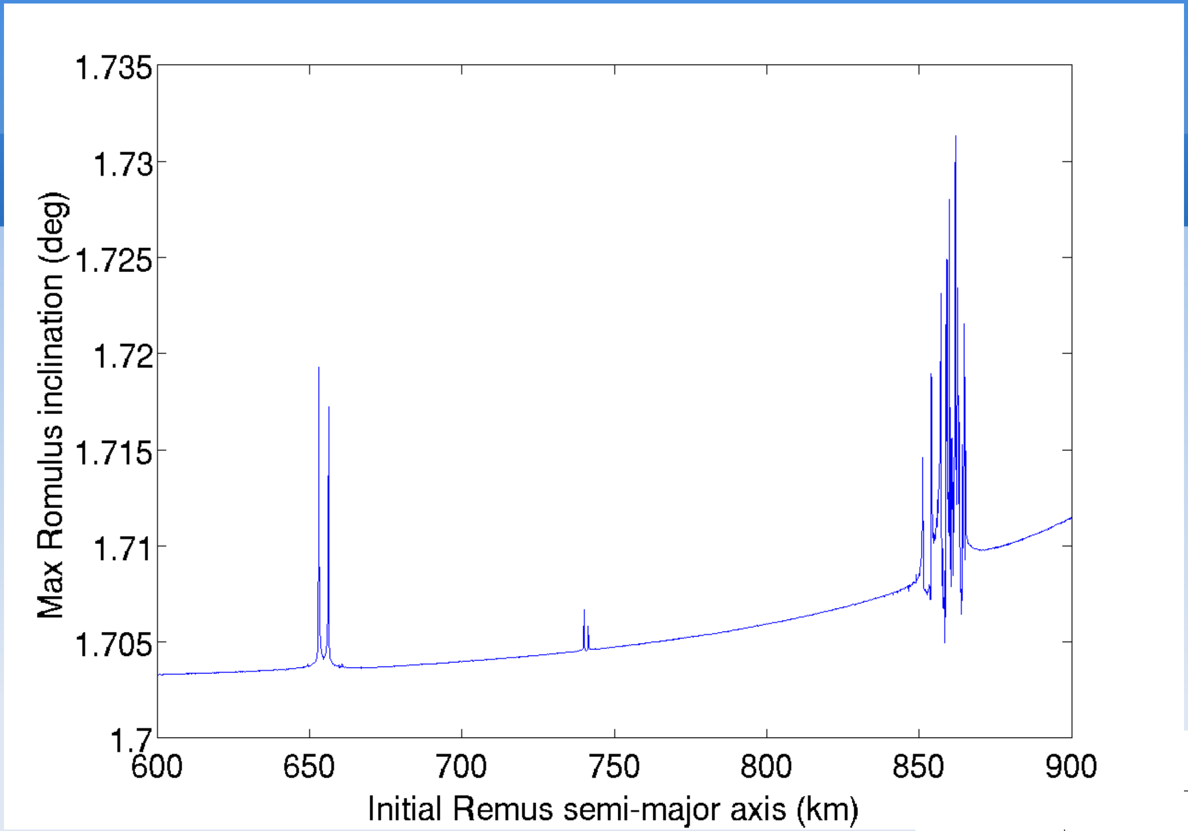




← Maximum eccentricity of Romulus

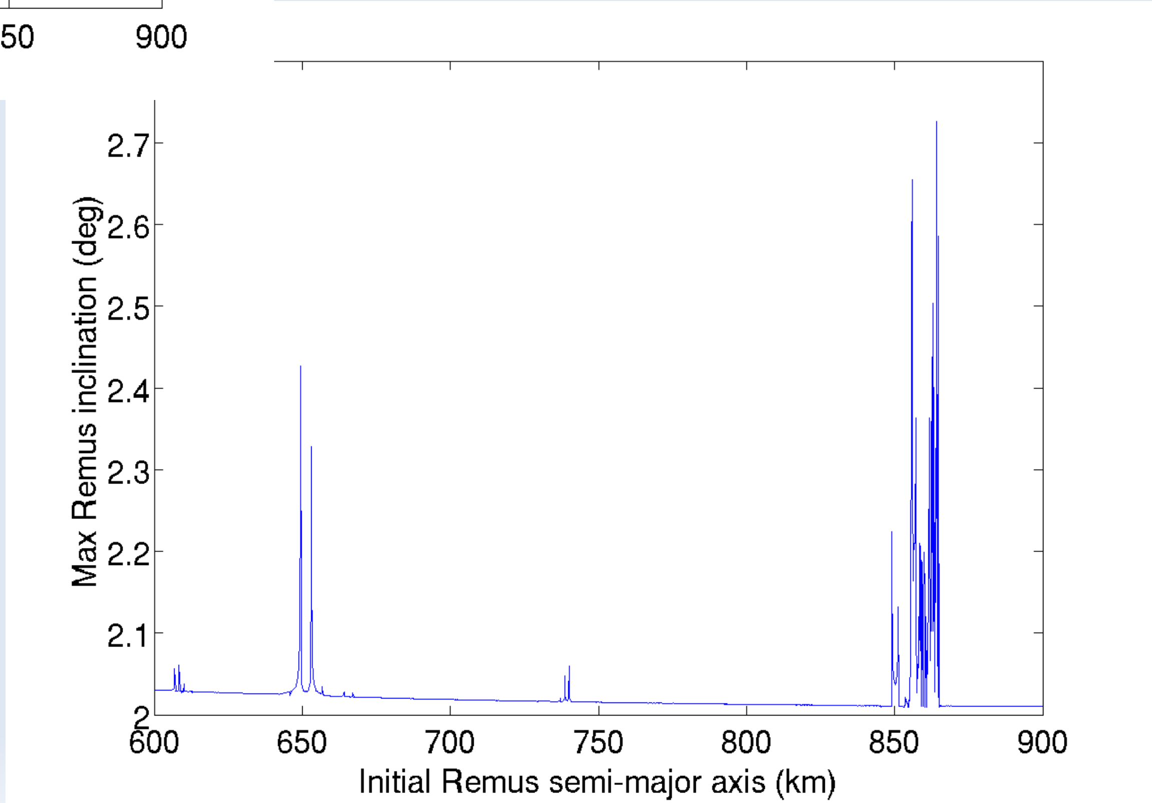
Maximum eccentricity of Remus →





Maximum inclination of Romulus

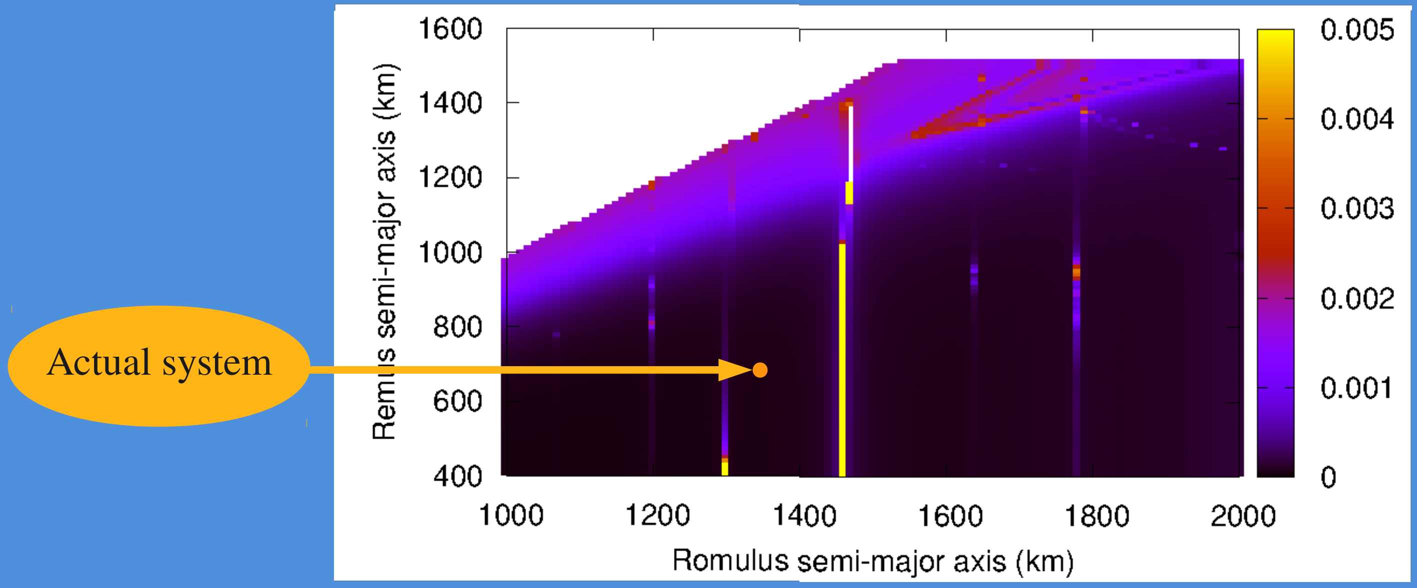
Maximum inclination of Remus



Secular resonances

• Model : - numerical integrations of the equations of motion, averaged over the mean longitudes of the satellites and the spin angle of Sylvia.
- oblateness of Sylvia $(J_2,\ J_2^2\ and\ J_4)$
- non-averaged solar perturbation
- secular interactions of the satellites

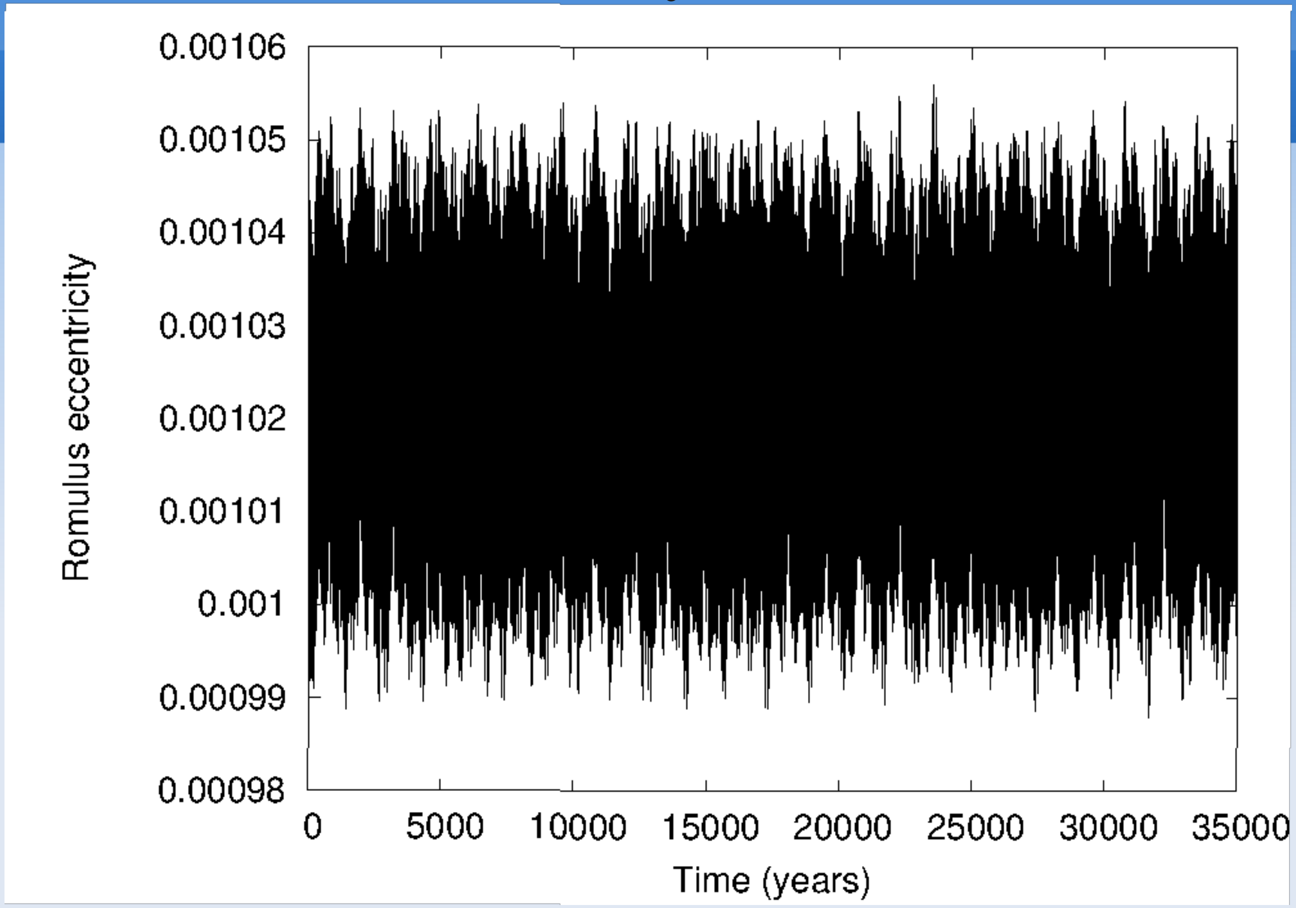
Secular map : Maximum eccentricity range reached by Romulus on 10^4 yr.



• Zones of secular instabilities : evection-related resonances (Touma & Wisdom 1998) between the longitude of pericenter of the satellites and the longitude of the Sun.

$$\varpi_{Ro} - k\lambda_{\odot}$$

Actual system :



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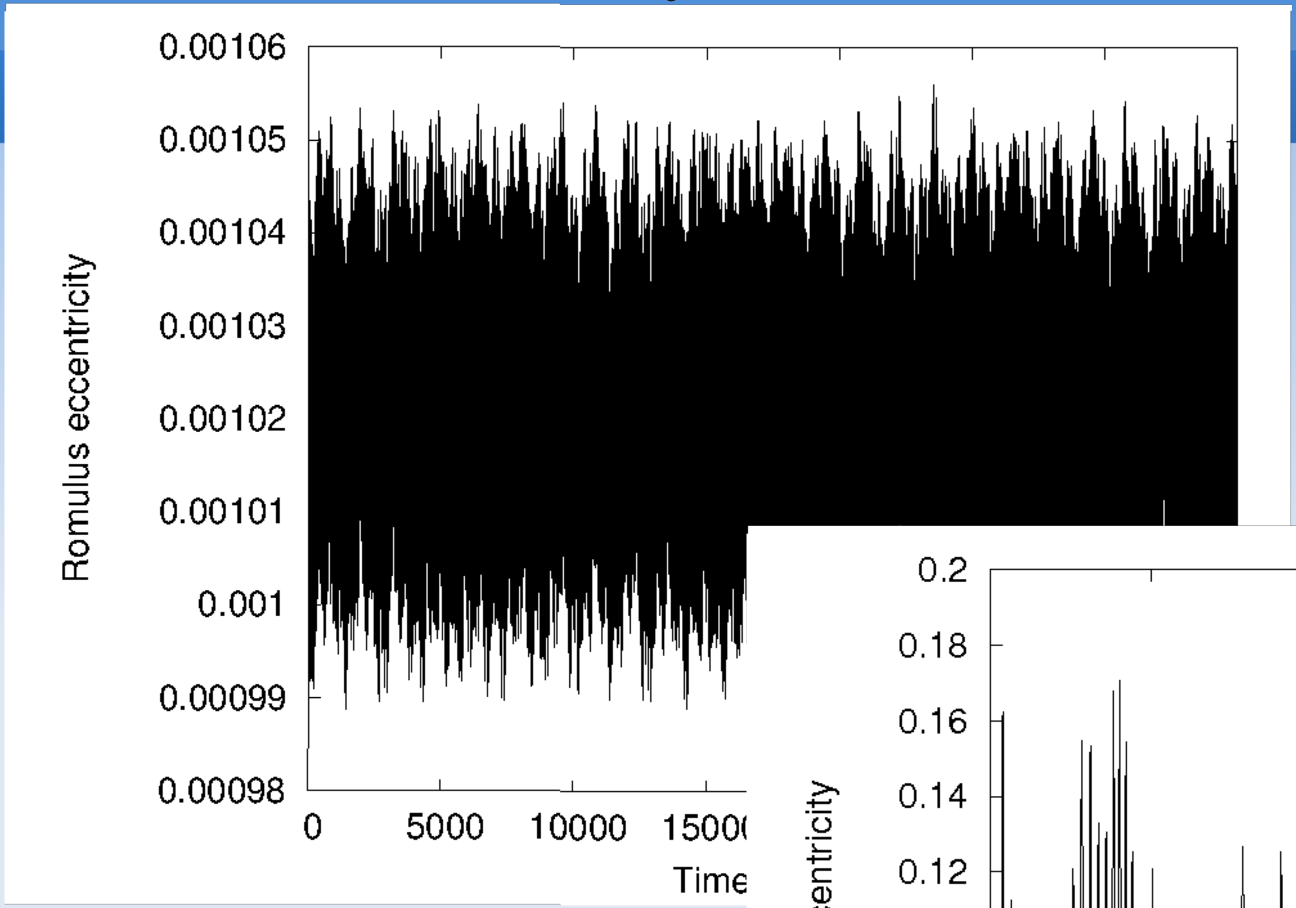
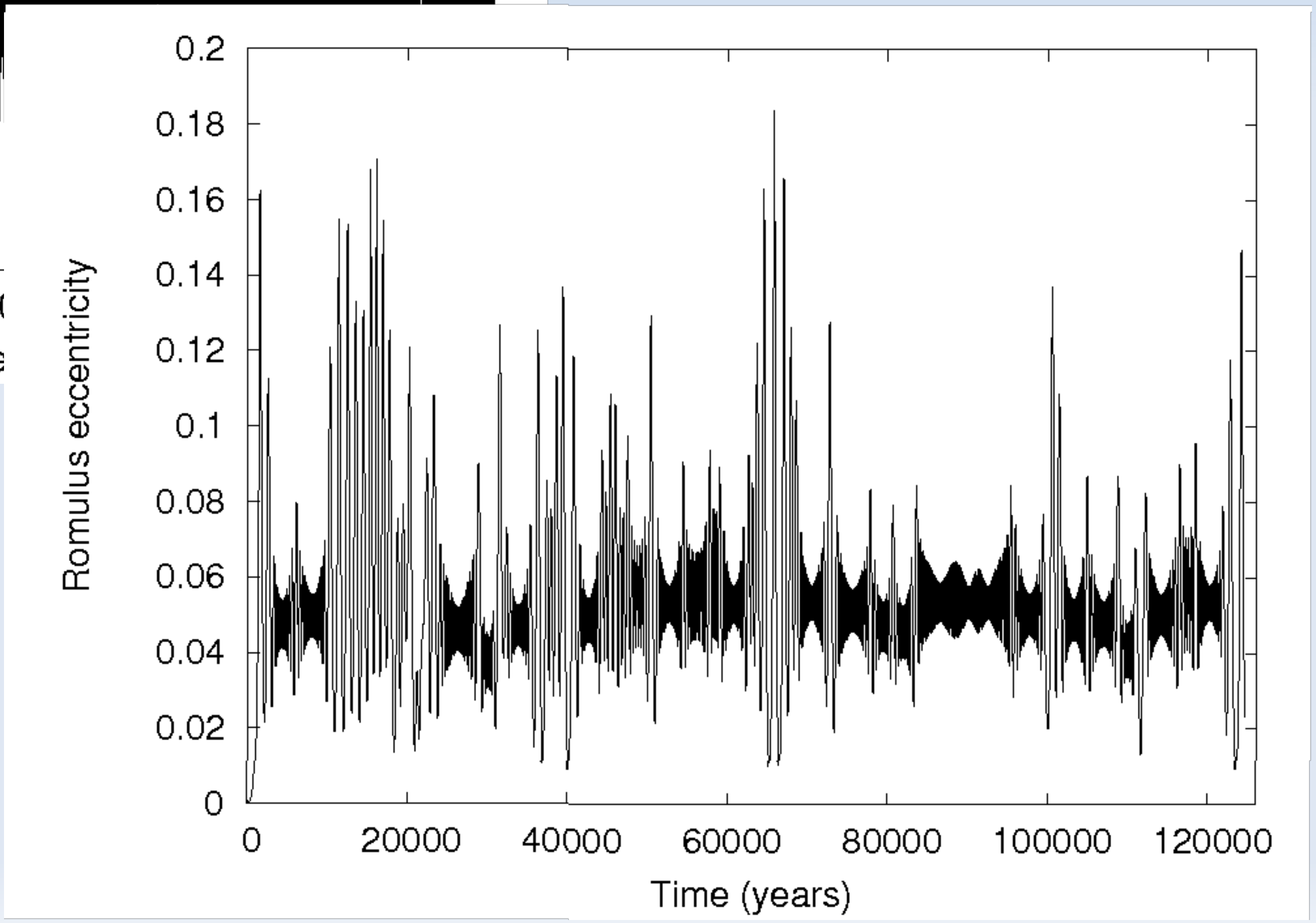


In the evection resonance :



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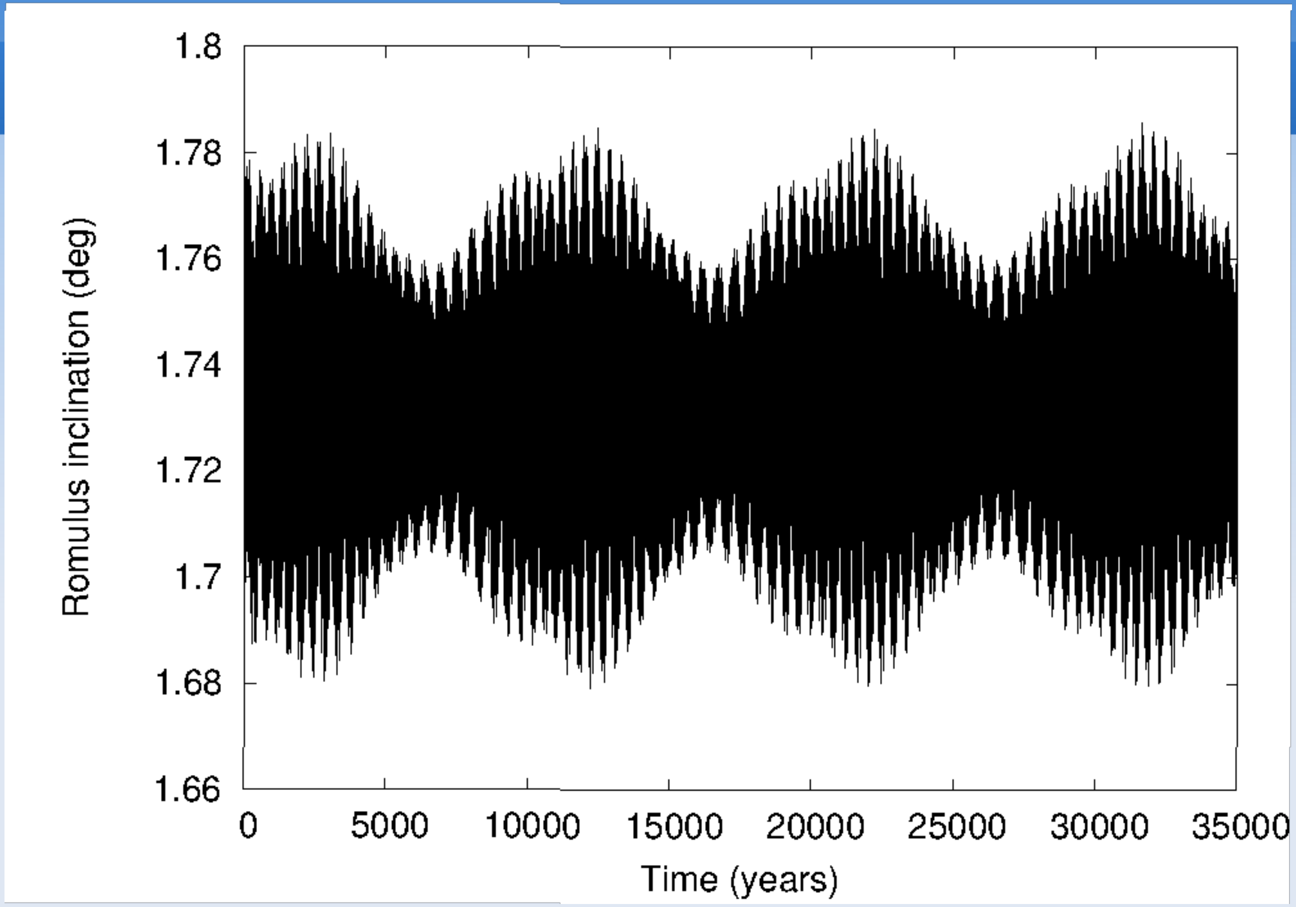

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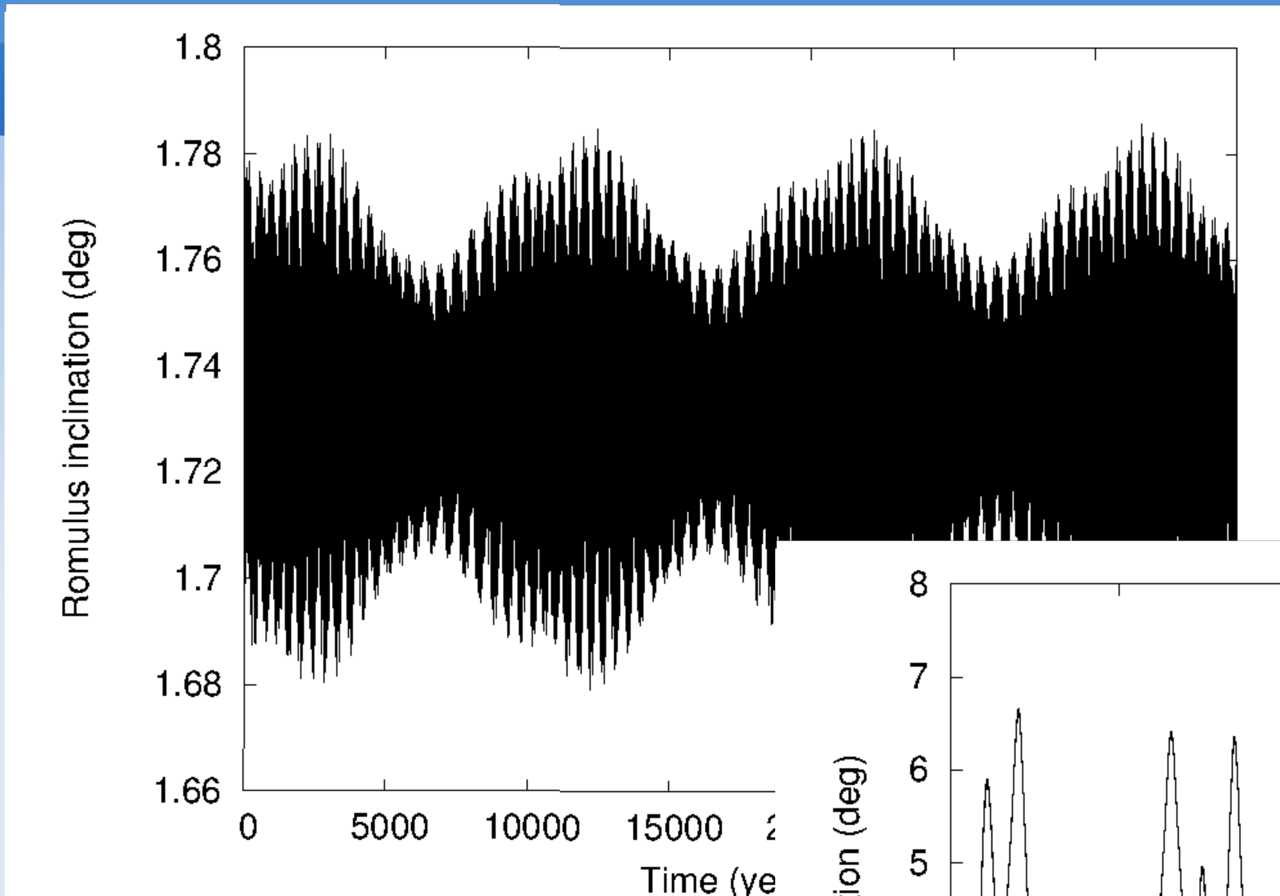

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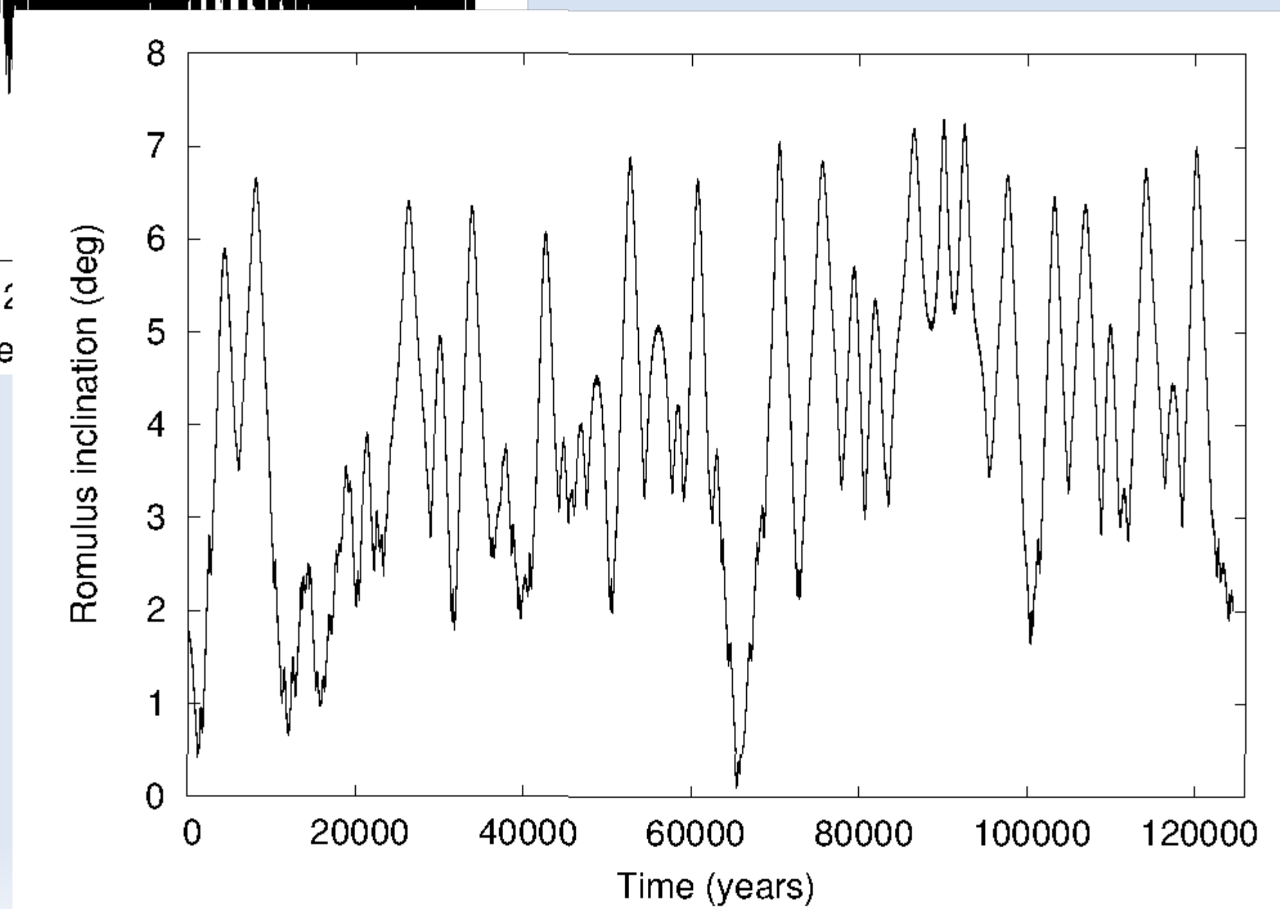
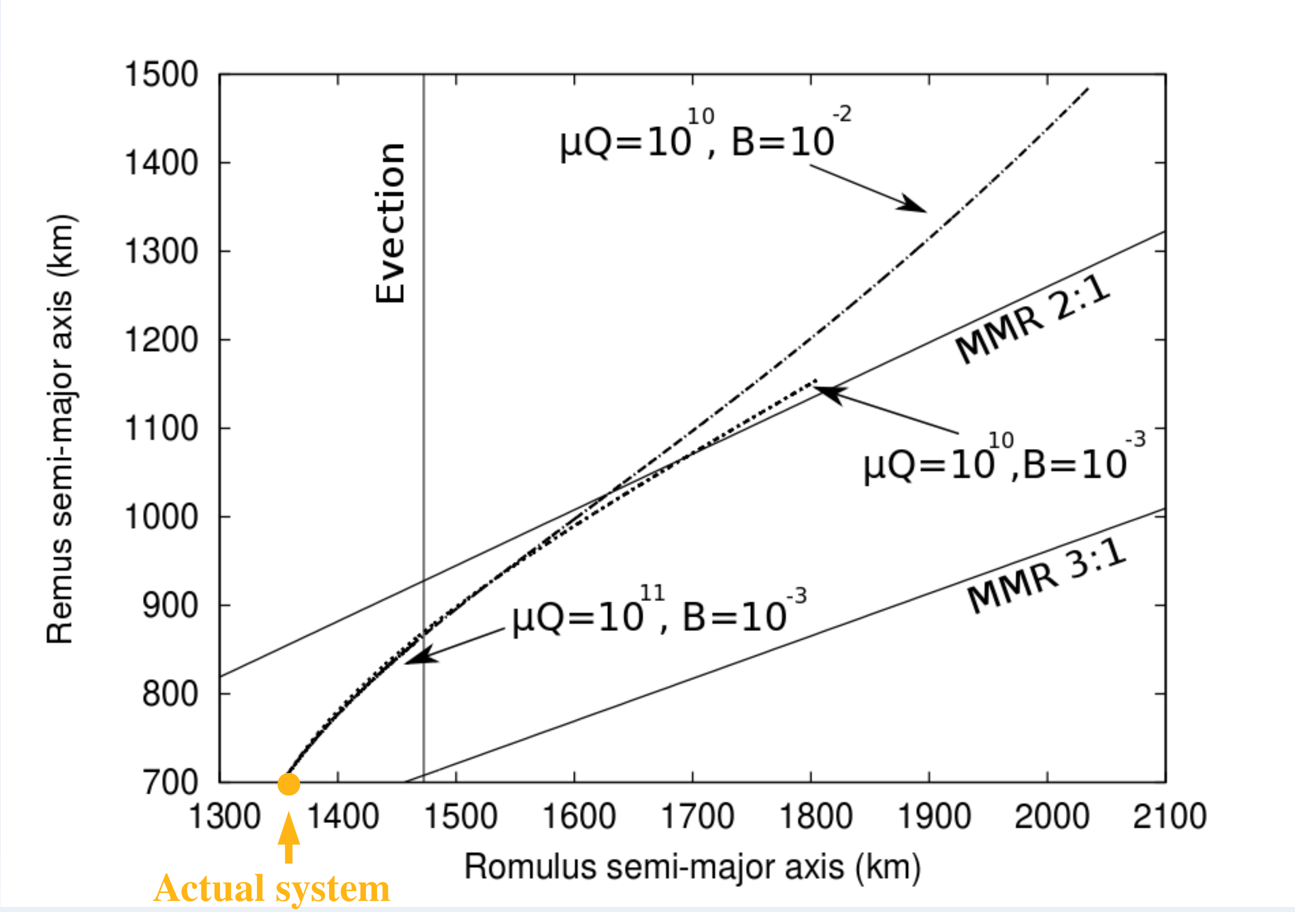


Tidal and BYORP effects

- Long-term evolution of the satellites driven by tidal and BYORP effects (assuming synchronous satellites and a positive $\dot{a}$ for BYORP)
- Dependance on μQ (rigidity of the material and dissipation coefficient in $N\,m^{-2}$) and BYORP coefficient B (shape of the secondary) (Jacobson & Scheeres 2011)

Evolution of the semi-major axis of the satellites over 1 Gyr :



Conclusion for the application 1

System bounded between :

- Mean-motion resonances
- Evection resonance

Effect of Tides and BYORP : crossing of the evection resonance

➡ Important growth of the eccentricity/inclination of Romulus (and very limited for Remus)

Perspectives :

- Determination of generic instability zones for satellites of asteroids
- Position of the actual multiple systems (Eugenia, Kleopatra)
- Study of the evolutions of eccentricity/inclination, and time-scale of the satellites in the instability zones

Application 2 : validation of orbits

(in collaboration with Benoît Carry)

Principle :

Data :
- shape model → Spherical harmonics development
- rotation period and spin axis directions
- approximation of the orbit of the satellite
- approximation of the mass of both bodies

Numerical integrations of the equations of motion :

- 2 body problem (asteroid + satellite)
- spherical harmonics development
- constant rotation of the asteroid
- gravitationnal influence of other bodies (Sun, planets,...)

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→ Keplerian orbit ?
Stable system ?
The orbit fit the observations ?

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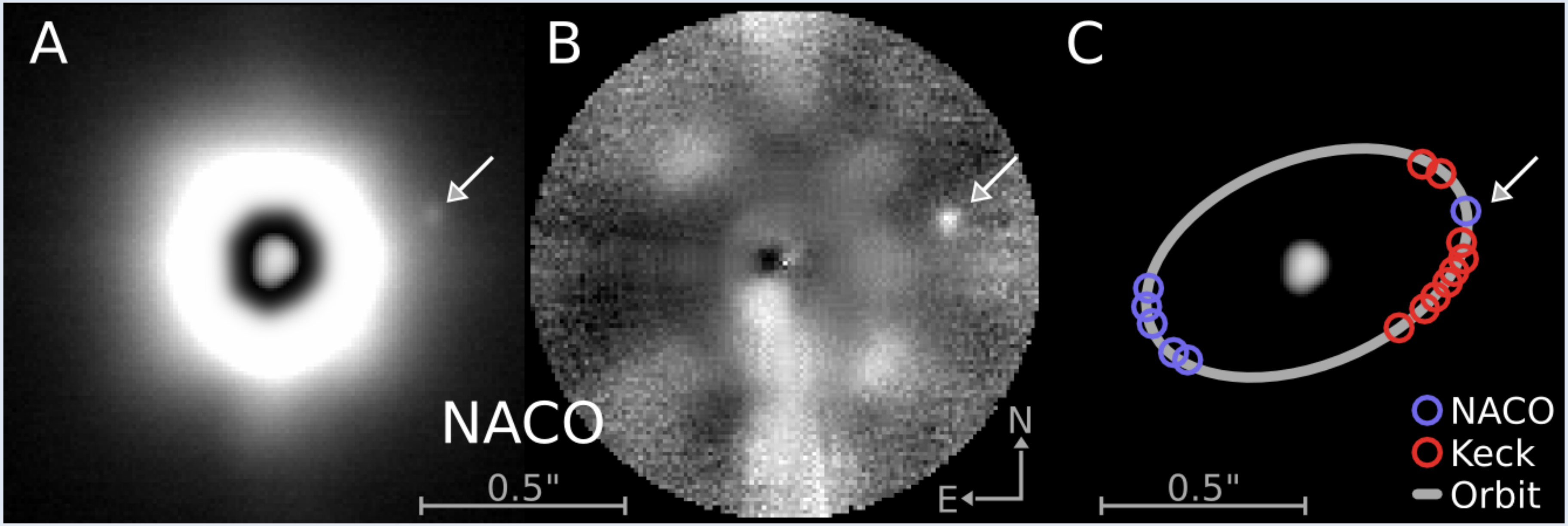
The system (41) Daphne

Daphne :
- large main-belt asteroid
- highly irregular shape : 239 x 183 x 153 km (Conrad et al. 2008)

In March 2008, discovery of a small satellite : S/2008 (41) 1

- estimated diameter of the satellite < 2 km (IAUC 8930)
- short orbital period : 1.1 days (Conrad et al. 2008)
- extreme mass ratio about 10^6 (Merline et al. 2008)



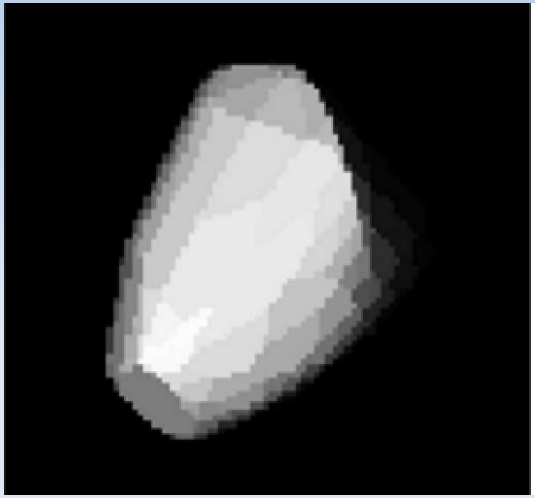
(Carry et al. In preparation)

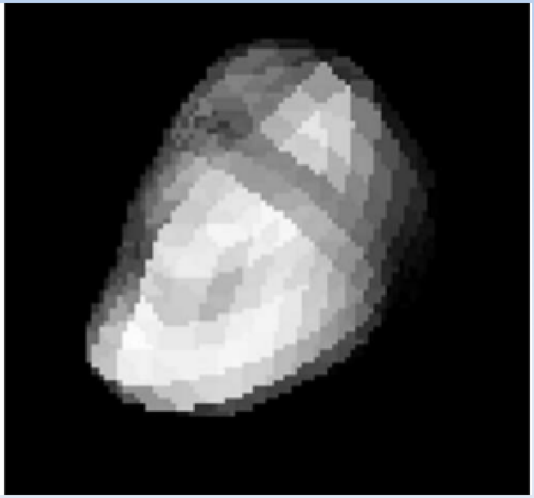
Two shape models :

Convex model
(Ďurech et al., 2011)

Non-convex model
(B. Carry with KOALA)

Unknown size





Calculated size

(Matter et al. 2011)

Pole solution :

$\lambda_0 = 198°$ $\beta_0 = -32°$

$P = 5.98798\ h$ $\phi_0 = 0°$

at $t_0 = 2444771.79382\ (JD)$

Volume equivalent diameter :
(Matter et al. 2011)
between 192 and 210 km

Pole solution :

$\lambda_0 = 198°$ $\beta_0 = -31°$

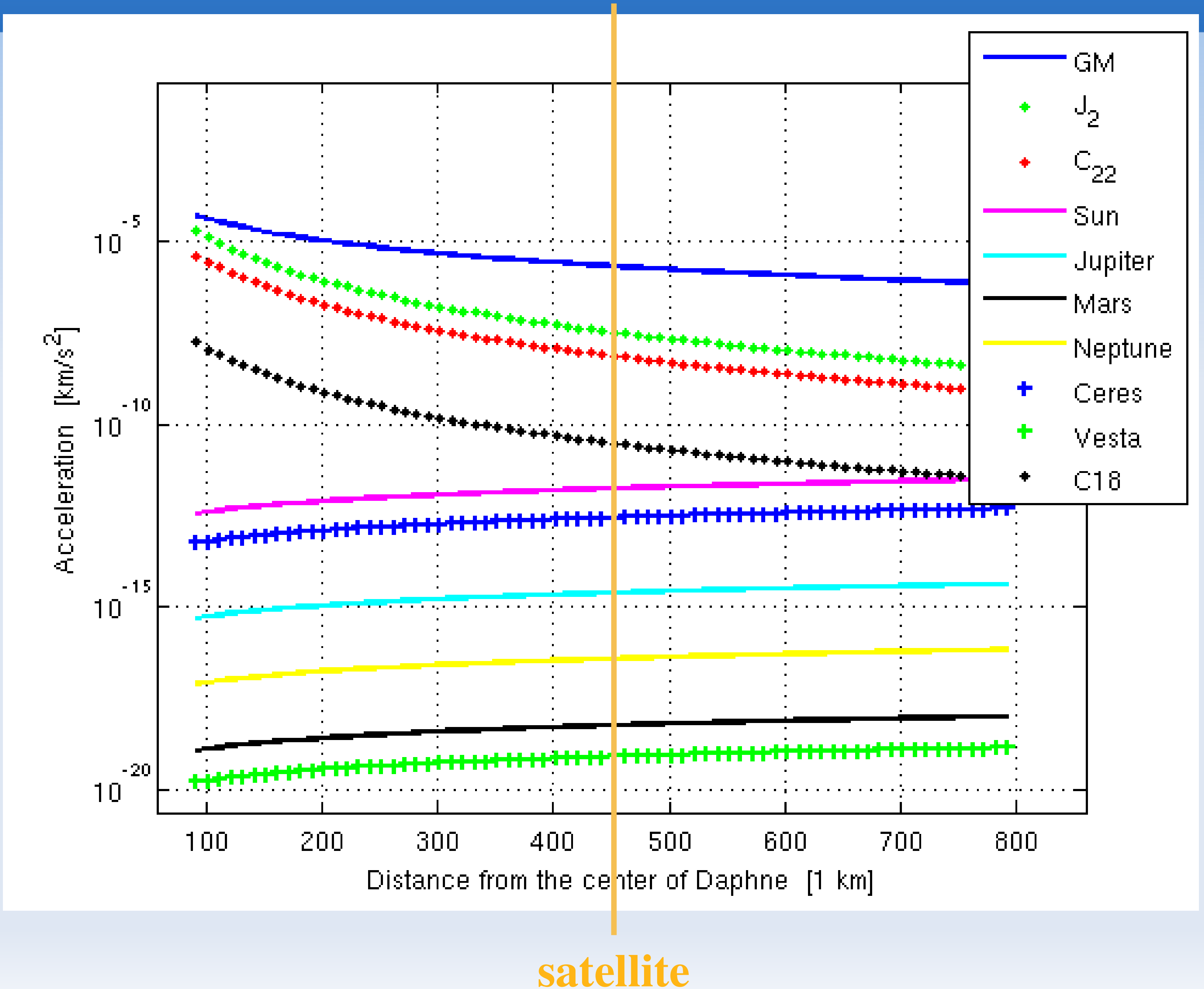
$P = 5.987980\ h$ $\phi_0 = 0°$

at $t_0 = 2444771.79382\ (JD)$

Volume equivalent diameter :

$D_V = 185 \pm 5\ km$

Main pertubations ?



To do :

- **approximation of the orbit** of the satellite and of the **mass** of the asteroid

- numerical integrations **in the future** to :
  - see if the orbit is keplerian
  - study the stability

- numerical integrations **in the past** to verify that the orbit is consistent with the observations.

Work in progress...

Conclusion

Shape models → Approximation of the gravitational potential

With it we can :

- study the dynamics of multiple systems :
  - Stability
  - Resonances
- validate approximated orbits

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Still a lot of work !


Thank you for your attention
eso