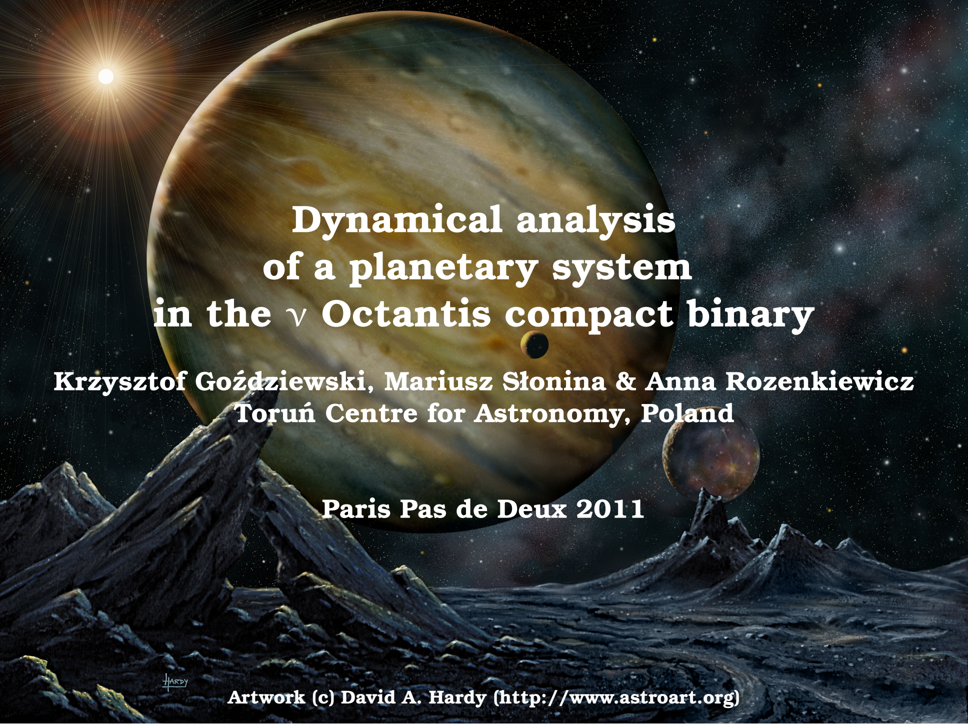


Dynamical analysis of a planetary system in the ν Octantis compact binary

Krzysztof Goździewski, Mariusz Słonina & Anna Rozenkiewicz
Toruń Centre for Astronomy, Poland

Paris Pas de Deux 2011

Agenda

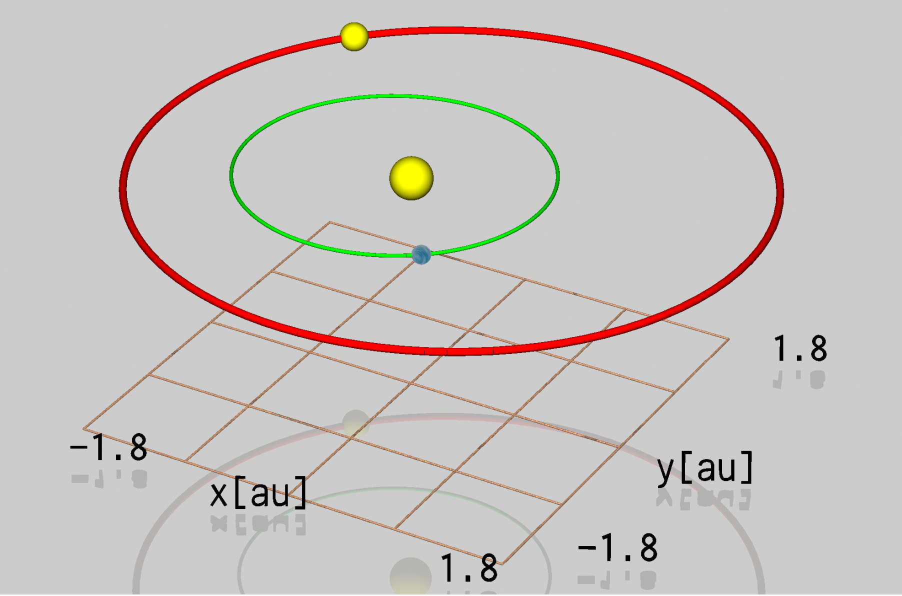
- Characterization of the ν Octantis binary
- Discovery of a Jovian planet (Ramm+, MNRAS, 2009)
- Retrograde orbit hypothesis by Eberle & Cuntz (ApJL, 2010)
- Fitting the Radial Velocities of ν Octantis A (Keplerian and N-body fits)
- Dynamical analysis of the E&B (2010) orbital setup
- A toy model and the Arnold web? (see our poster by *Słonina, Goździewski & Migaszewski*)
- Summary (work in progress . . .)

Characterization of the ν Octantis binary

- A single–line spectroscopic binary investigated for more than 80 years (11 Radial Velocities by Colacevich 1935 and astrometric orbit by Alden, 1939)
- Components: a K1 III giant primary (1.4 ± 0.3 $M_\odot$) and unseen red dwarf secondary K7-M1 V (0.5 ± 0.1 $M_\odot$) with semi-major axis $a_b = 2.55 \pm 0.13$ au, eccentricity $e_b = 0.2358 \pm 0.0003$ and orbital period $P_b = 1050.11 \pm 0.13$ days (Ramm+, 2009)
- Orbital inclination 71° with an error of less than 1° (D. Pourbaix, Hipparchos astrometry + RV)
- Precision Radial Velocity variations attributed to Jovian planet with $m_p \sin i = 2.5$ M_{Jup} in the orbit of $a_p = 1.2 \pm 0.1$ au and eccentricity $e_p = 0.123 \pm 0.037$ (Ramm+, 2009); other explanations (e.g., stellar variability, spots) basically excluded.

Unusual and interesting binary system due to stability paradox

The planetary orbit is found almost in the middle between primaries. According to stability criterion, like in Holman and Wiegert (1999), such a configuration is unstable.


1.8
-1.8
x[au]
y[au]
-1.8
1.8

Modeling observations of extrasolar planets

Keplerian model

Observed signal (e.g., Radial Velocity, Light Travel Time, Astrometry) is geometric superposition of fixed Keplerian orbits — no interactions, internal degeneracy (nodal lines and inclination undetermined)

$$V_r(t) = \sum K_i \left[\cos(\omega_i + \nu_i(t)) + e \cos \omega_i\right] + V_0$$

Newtonian model

The model of motion described in a framework of the N-body problem (Laughlin & Chambers, ApJL, 2001) — includes mutual planetary interactions. In principle, Keplerian degeneracies removed.

Newtonian model with direct or indirect stability constraints

A generalized Newtonian model for systems with strongly interacting companions. Because the phase space of the N-body problem has non-continuous and complex structure — dynamical stability is an implicit observable (many references here).

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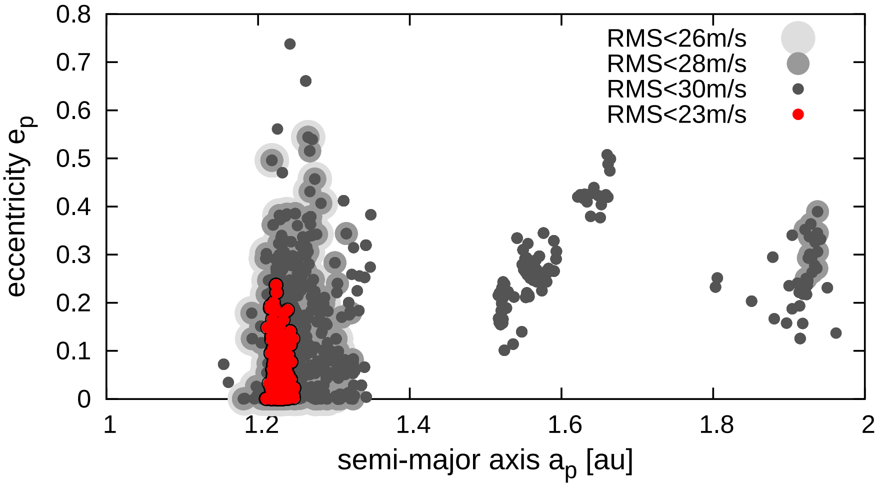
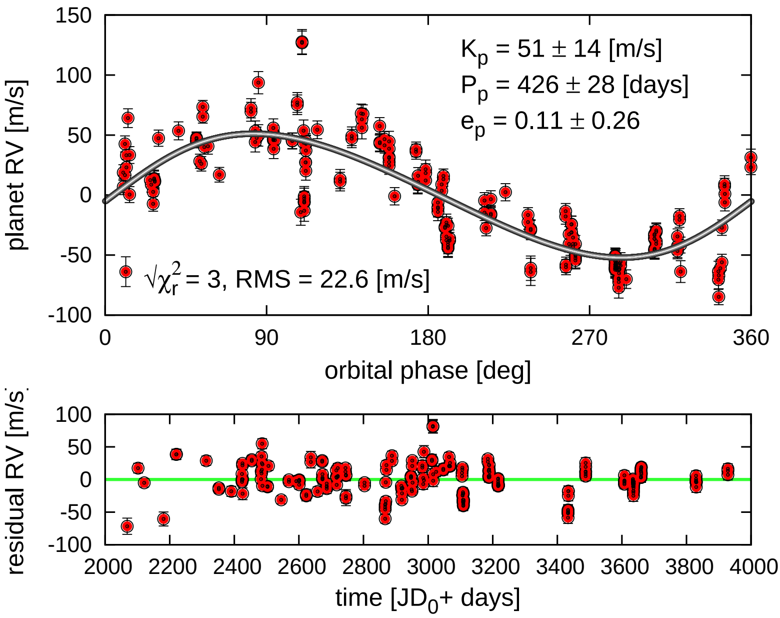
Keplerian fits by the hybrid algorithm (GA+simplex)



Keplerian best fit RV signal



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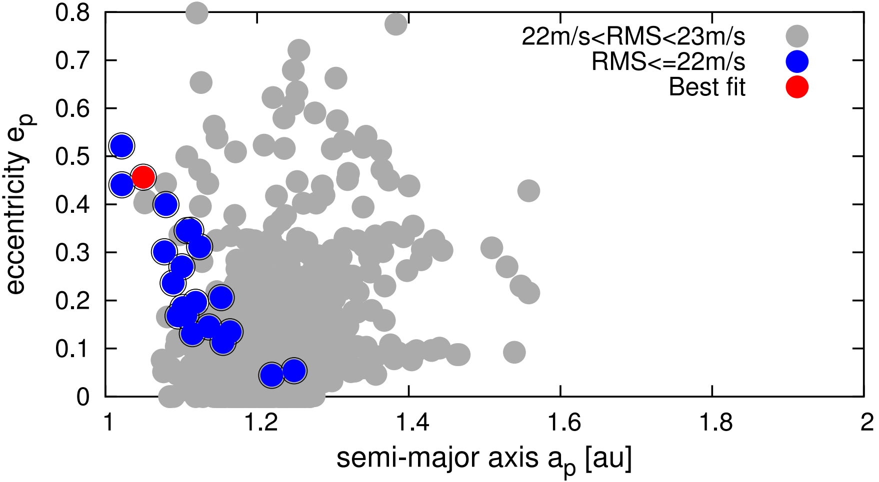

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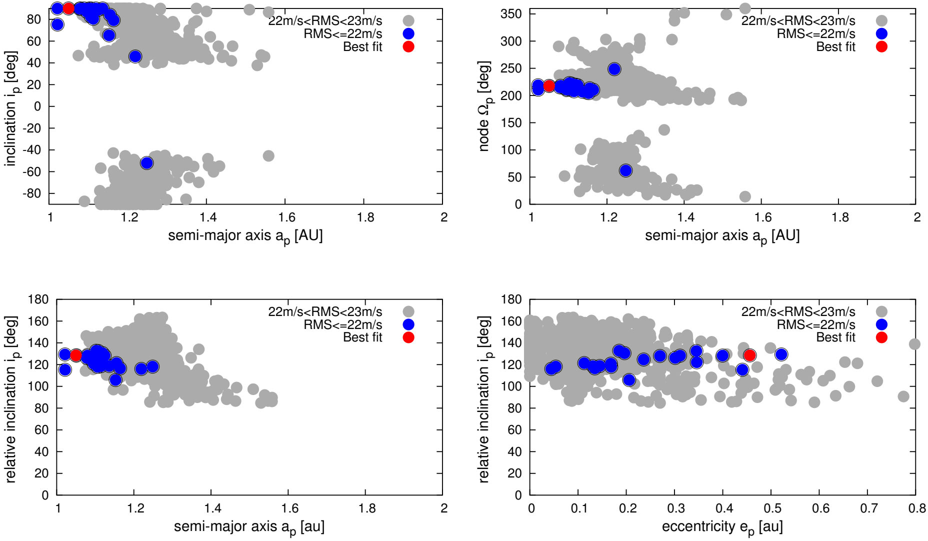
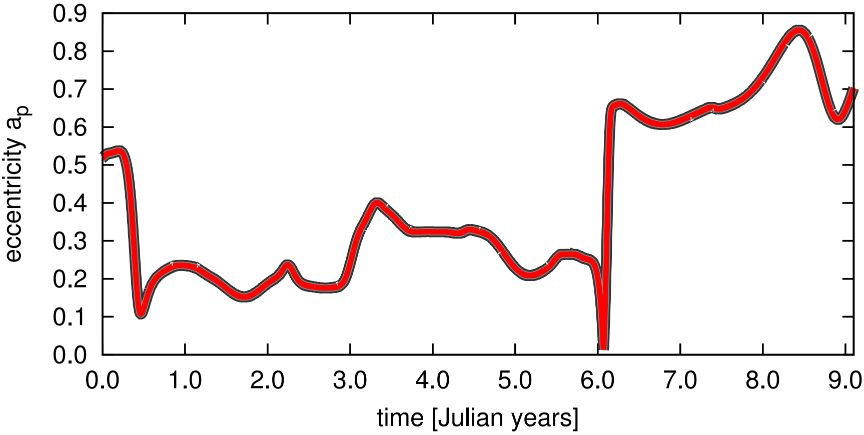


Strong instability of the best fit solution



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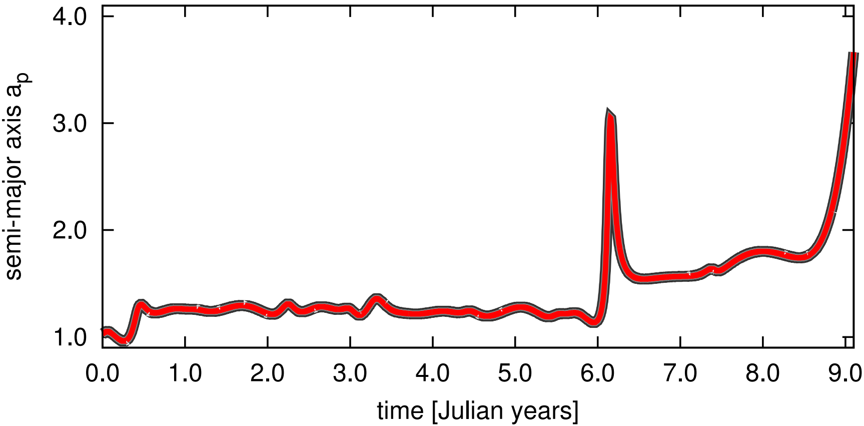
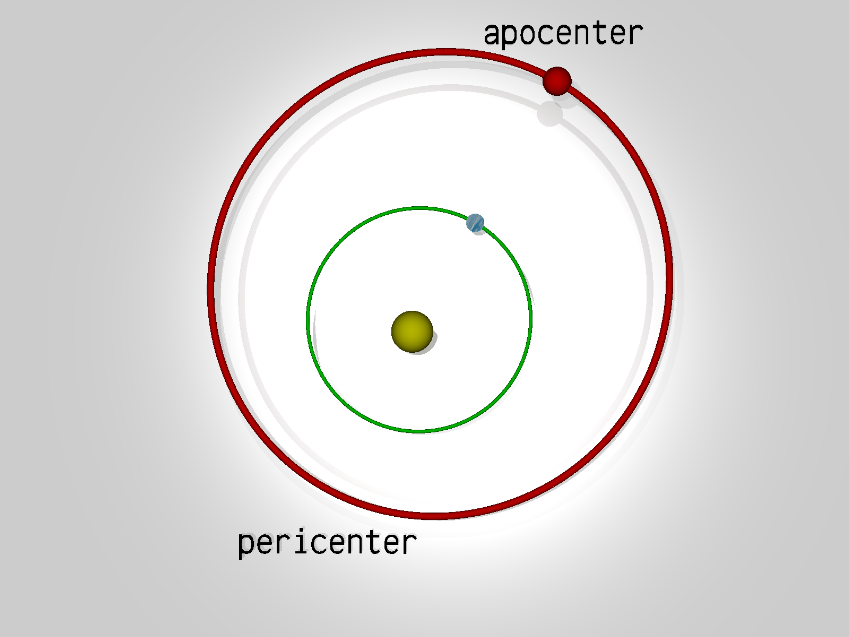


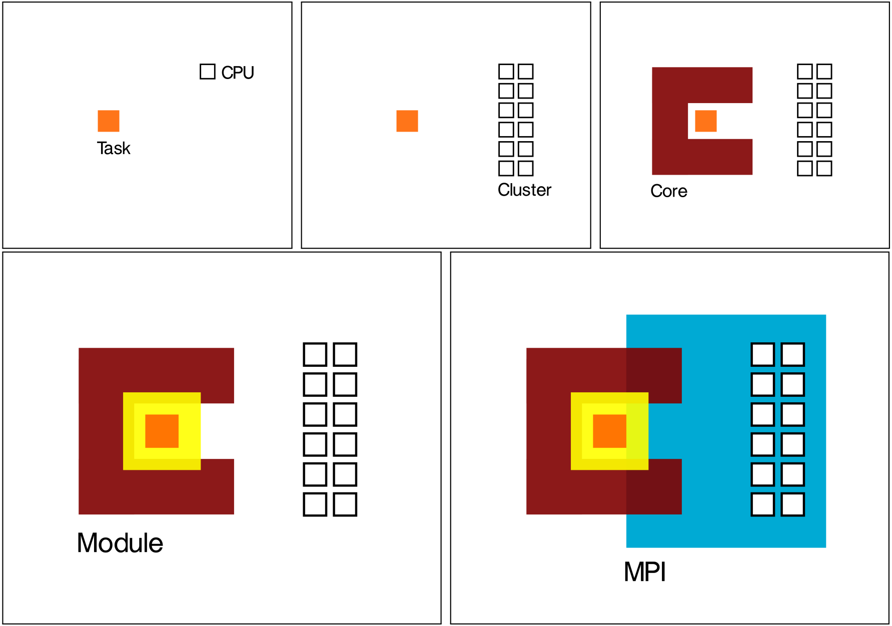
Stability study setup by Eberle & Cuntz (2010)



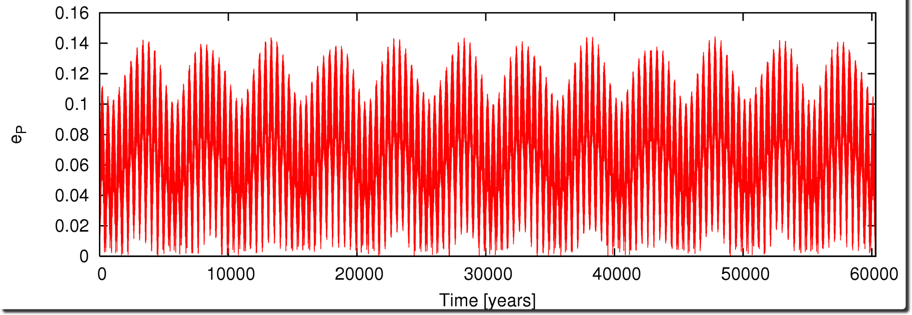
Numerical setup

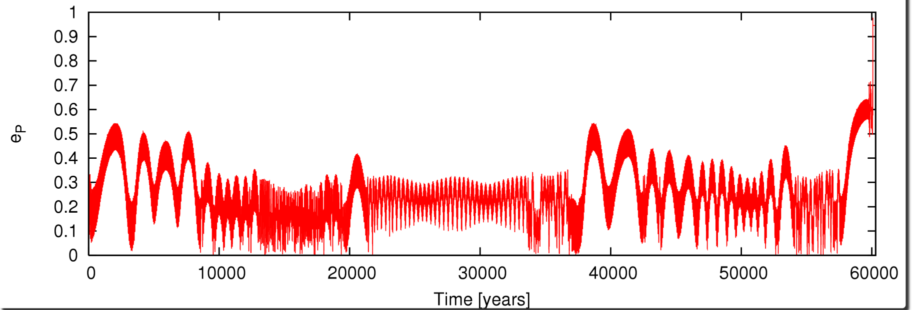
- Fast Indicator instead of the direct numerical integration $\equiv$ CPU efficient resolution of stable/unstable configurations
- Mean Motion Resonances time scale $\equiv 10^3$–10^5 binary periods $\equiv$ detection of strongest instabilities
- High resolution dynamical maps (1440×900 initial conditions $\equiv$ 1.3 Mpixels) $\equiv$ detection of the fine structure of the phase space
- MECHANIC MPI framework (see our poster) $\equiv$ intensive CPU-cluster computations

MECHANIC numerical environment

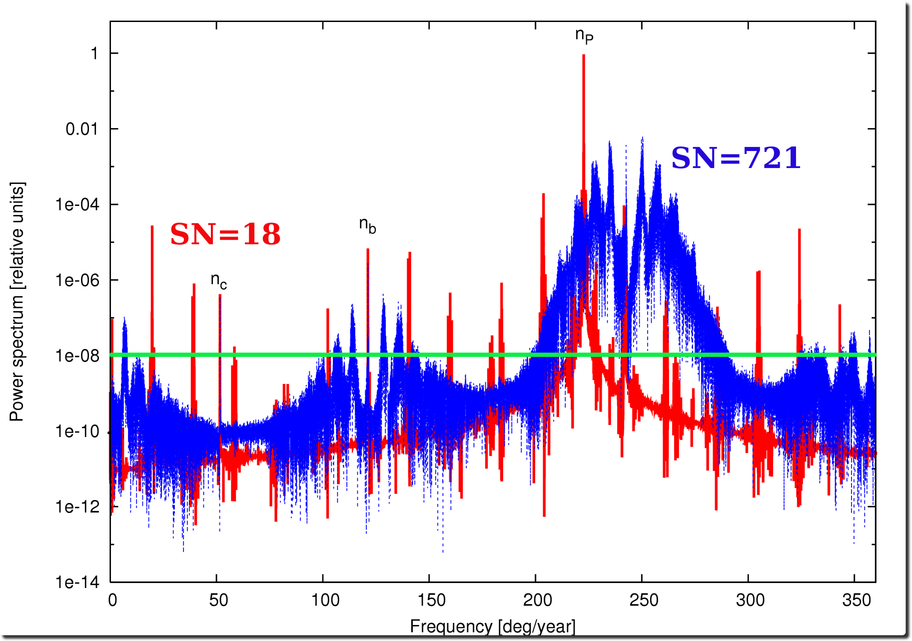


Regular vs regular motion

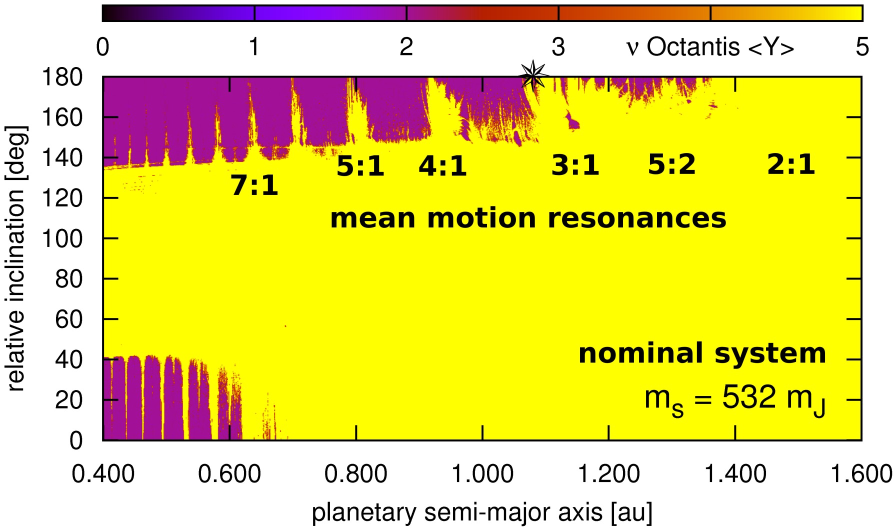




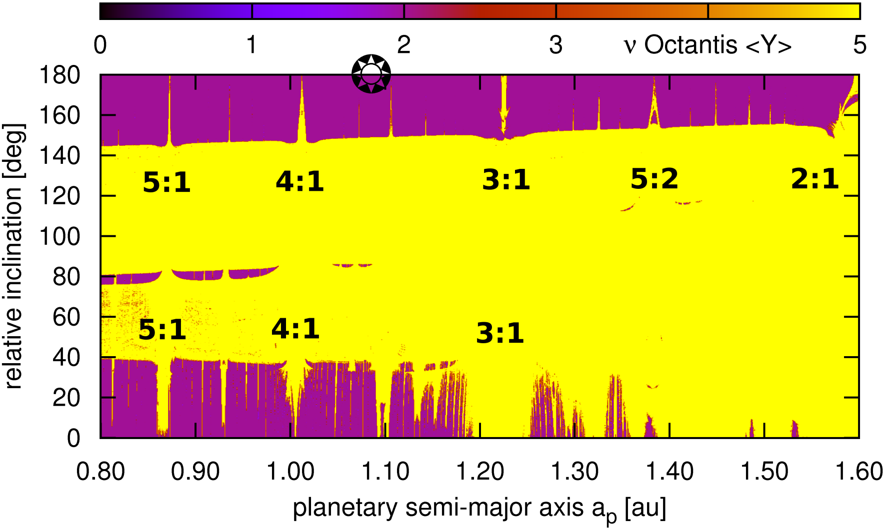
Regular vs regular motion – FFT signal



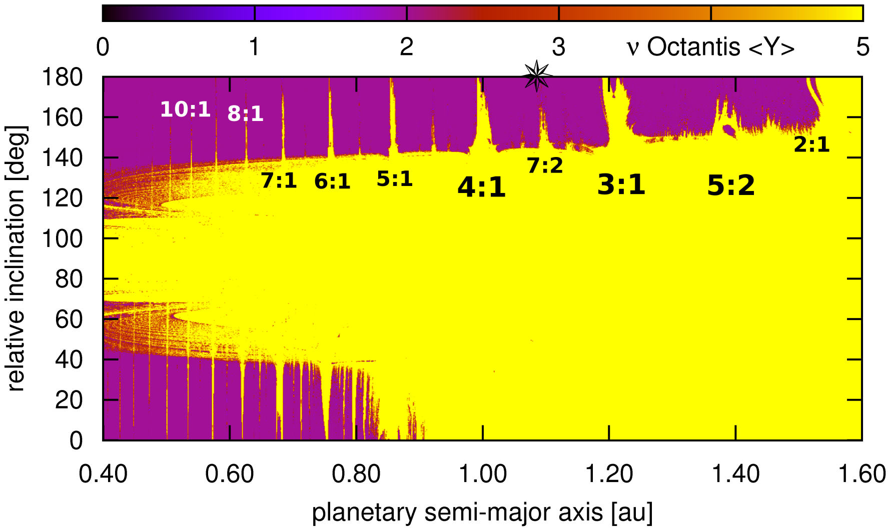
Global dynamical map of ν Octantis (MEGNO indicator)



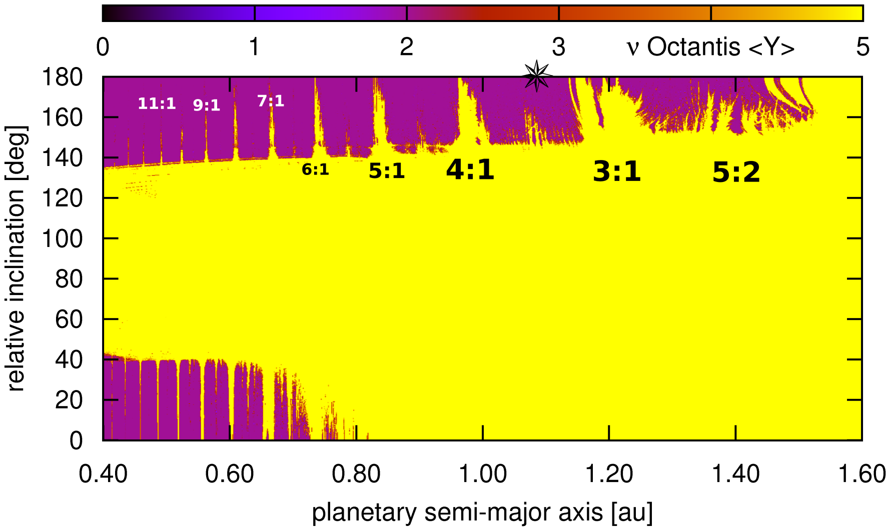
Dynamical map of ν Oct ($m \sim 5.32 M_{Jup} \equiv \epsilon \sim 0.003$)



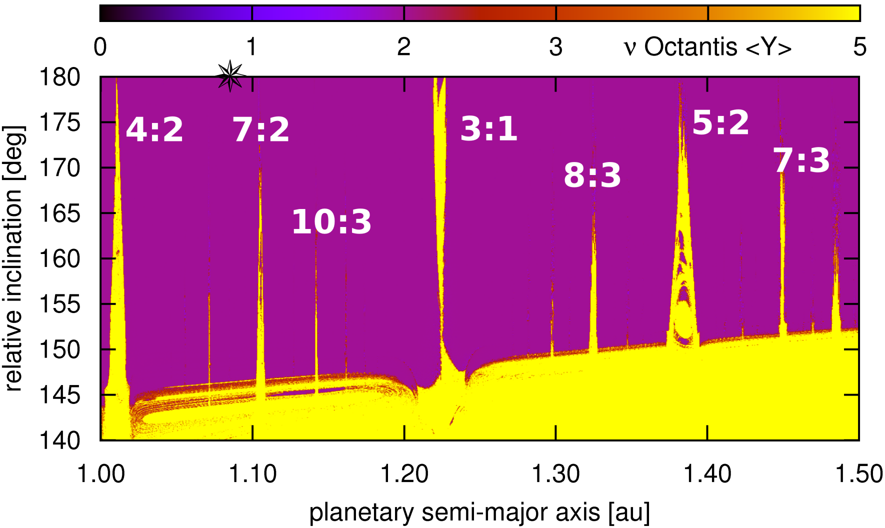
Dynamical map of ν Oct ($m \sim 100 M_{Jup} \equiv \epsilon \sim 0.070$)



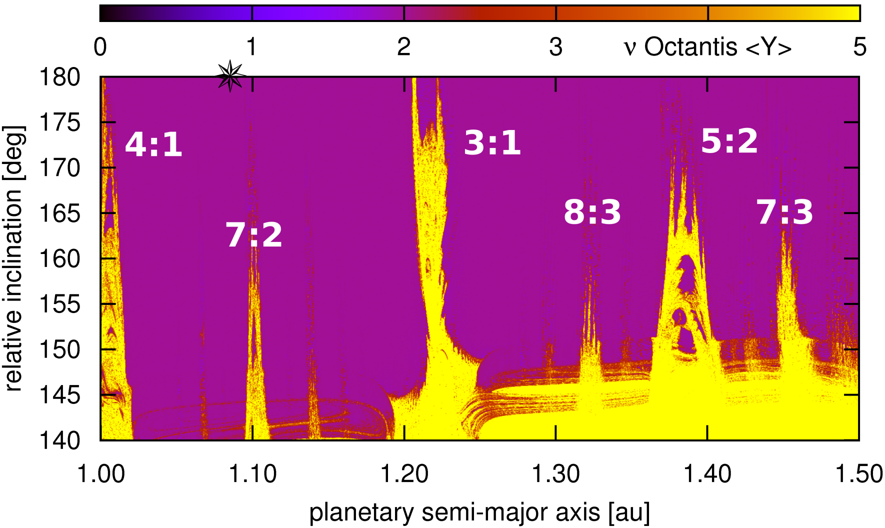
Dynamical map of ν Oct ($m \sim 250 M_{Jup} \equiv \epsilon \sim 0.178$)



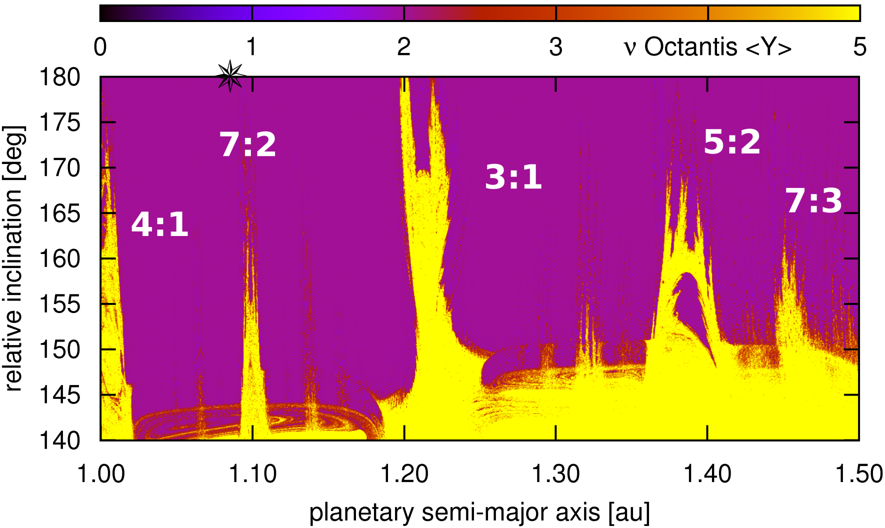
Dynamical map of ν Oct ($m \sim 14 M_{Jup} \equiv \epsilon \sim 0.010$)



Dynamical map of ν Oct ($m \sim 53 M_{Jup} \equiv \epsilon \sim 0.038$)



Dynamical map of ν Oct ($m \sim 75 M_{Jup} \equiv \epsilon \sim 0.054$)



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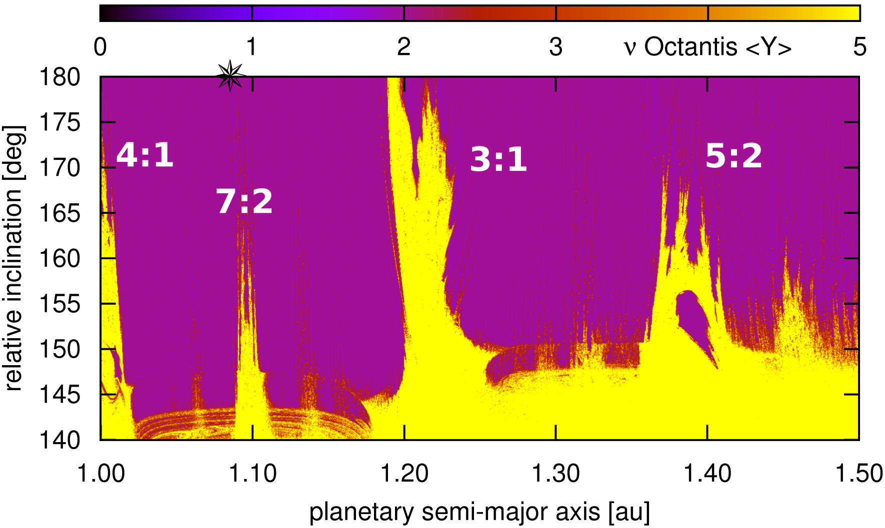
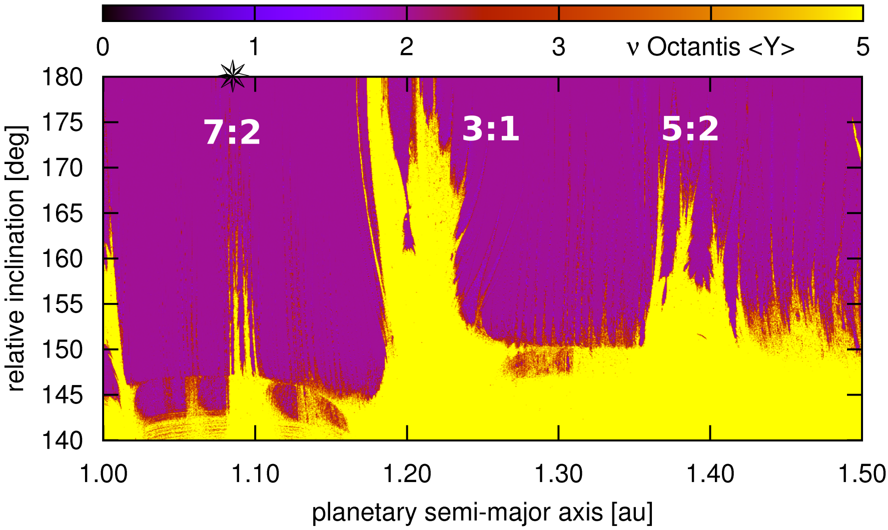
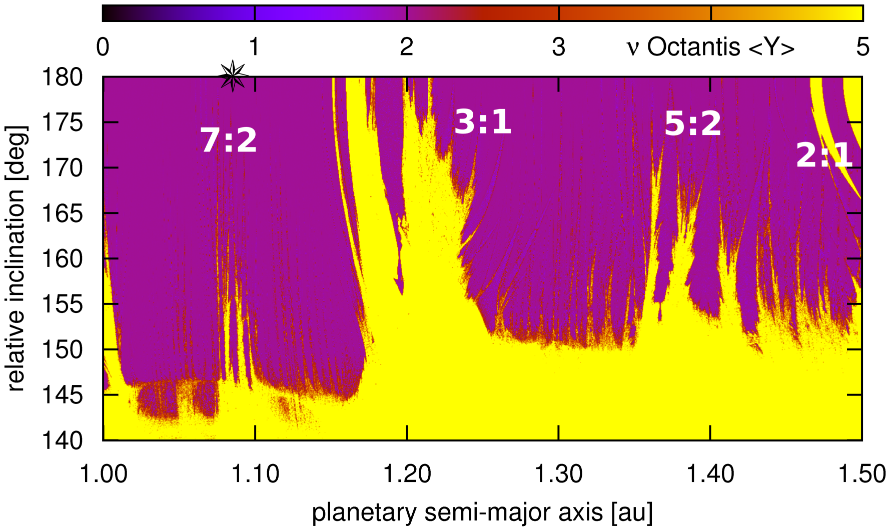


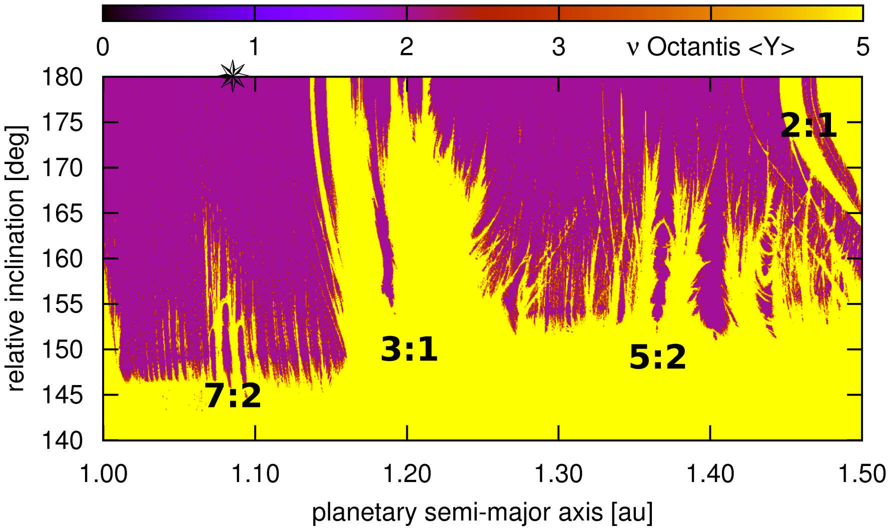
Dynamical map of ν Oct ($m \sim 150 M_{Jup} \equiv \epsilon \sim 0.110$)



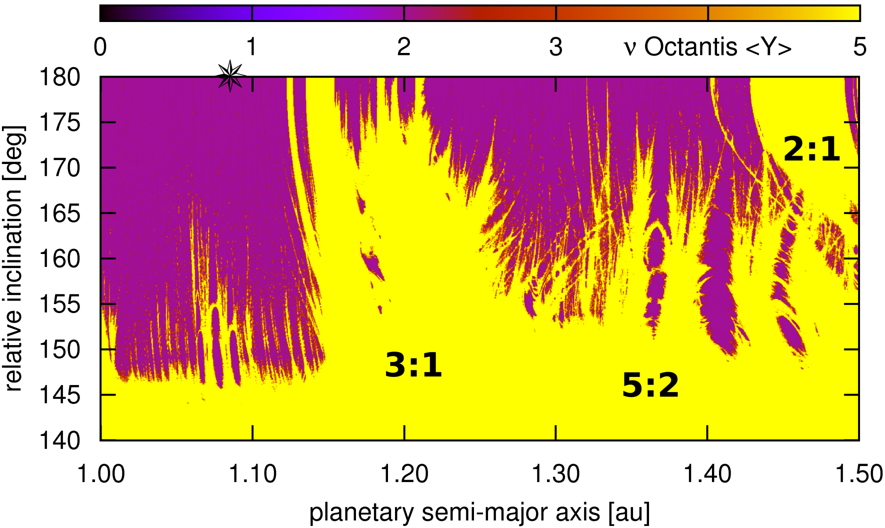
Dynamical map of ν Oct ($m \sim 200 M_{Jup} \equiv \epsilon \sim 0.140$)



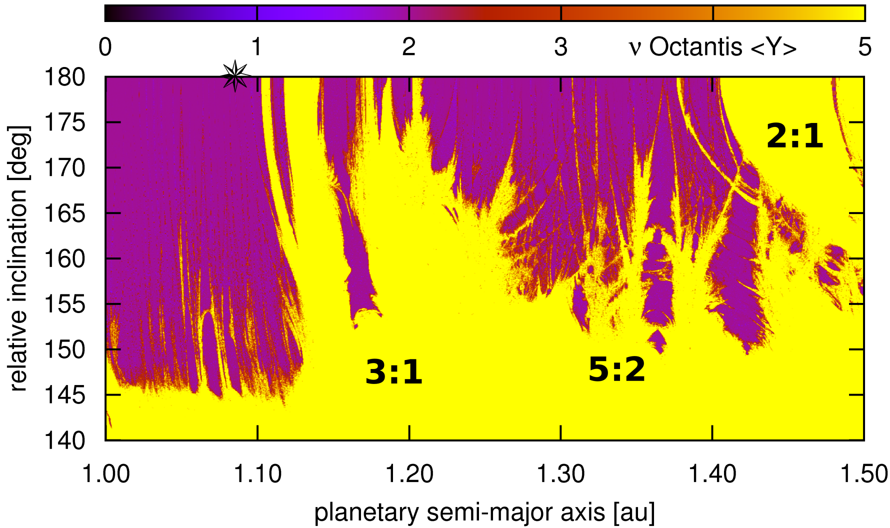
Dynamical map of ν Oct ($m \sim 250 M_{Jup} \equiv \epsilon \sim 0.179$)



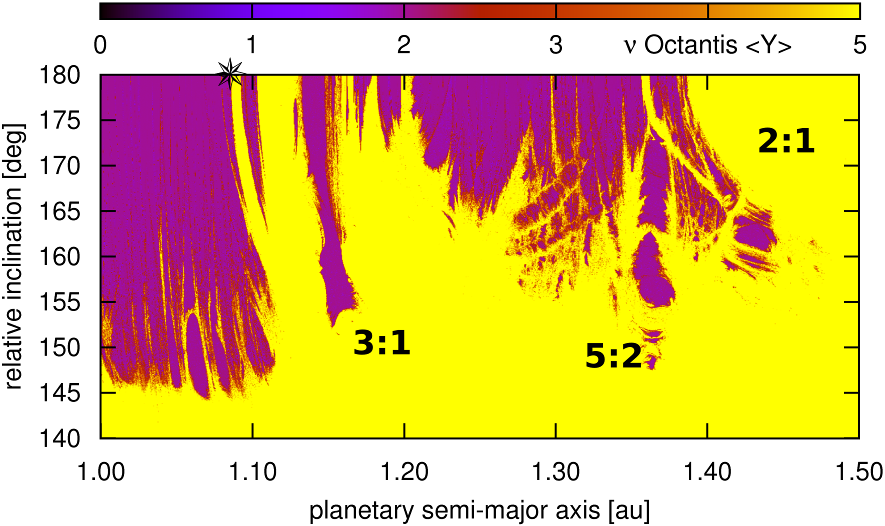
Dynamical map of ν Oct ($m \sim 300 M_{Jup} \equiv \epsilon \sim 0.214$)



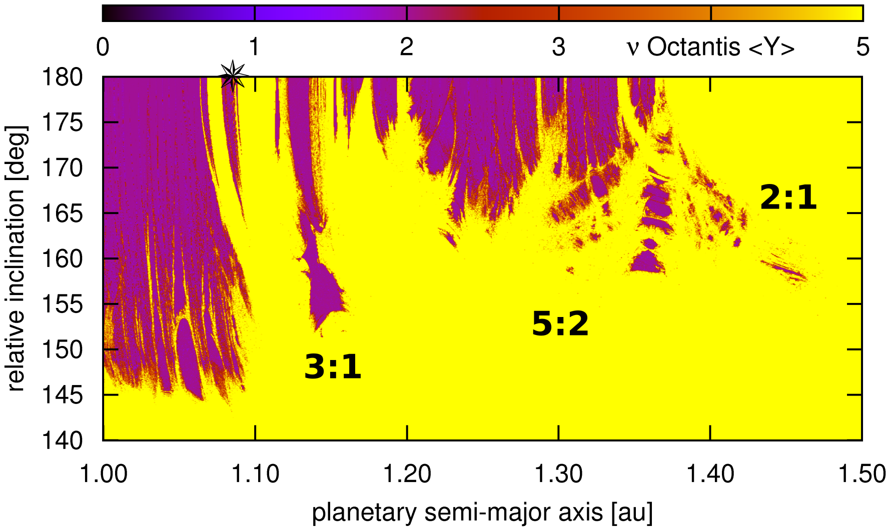
Dynamical map of ν Oct ($m \sim 380 M_{Jup} \equiv \epsilon \sim 0.271$)



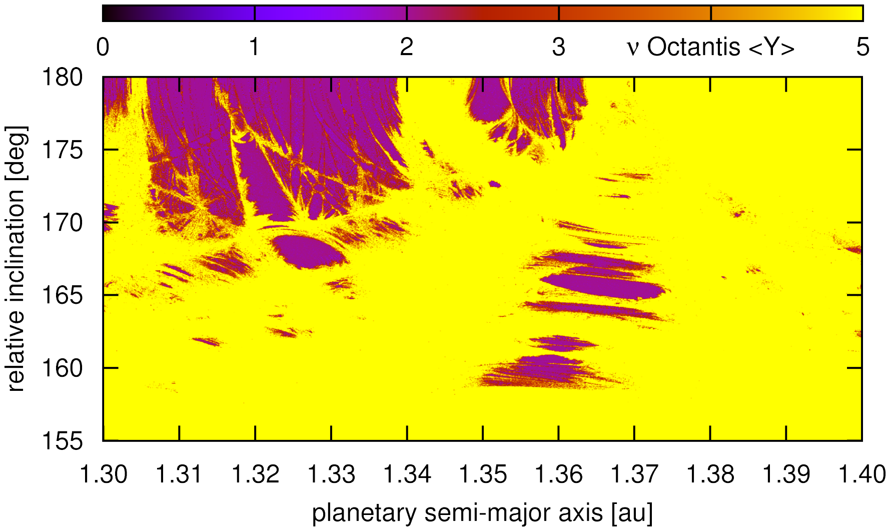
Dynamical map of ν Oct ($m \sim 462 M_{Jup} \equiv \epsilon \sim 0.330$)



Dynamical map of ν Oct ($m \sim 532 M_{Jup} \equiv \epsilon \sim 0.380$)



Arnold web in ν Oct ($m \sim 532 M_{Jup} \equiv \epsilon \sim 0.380$) – zoom



A long digression: toy model of the Arnold web

- We consider Hamiltonian of the form (Froeschlé+, 2001):

$$\mathcal{H} = \mathcal{H}_0(I_{1,2,3}) + \epsilon V(\phi_{1,2,3}) \equiv \frac{1}{2}I_1^2 + \frac{1}{2}I_2^2 + I_3 + \frac{\epsilon}{\cos\phi_1 + \cos\phi_2 + \cos\phi_3 + 4}$$

where actions $I_1, I_2, I_3 \in \mathbb{R}$ and angles $\phi_1, \phi_2, \phi_3 \in \mathbb{T}$ are canonically conjugate variables, and ϵ is the perturbation parameter

- For $\epsilon = 0$, the system of $\mathcal{H}_0$ is trivially integrable: actions I_1, I_2, I_3 are constant; $\phi_i = \omega_i t + \phi_0$ and the motions are confined to invariant tori filled up with quasi-periodic solutions with fundamental frequencies $\omega_1 = I_1$, $\omega_2 = I_2$, $\omega_3 = 1$.
- With the perturbation, the dynamics are non-integrable, but the KAM theorem predicts persistence of quasi-periodic motions, if the unperturbed tori are sufficiently non-resonant:

$$k_1\omega_1 + k_2\omega_2 + k_3\omega_3 \neq 0, \quad (k_1, k_2, k_3) \in \mathbb{Z}.$$

- Close to the resonances (lines here) up to the distance $\sim \sqrt{\epsilon}/\parallel \mathbf{k} \parallel^\alpha$ (the Arnold web), the dynamics are very complex.
- Following Froeschleé+ (2001), we determine the structure of the Arnold web with the dynamical maps.

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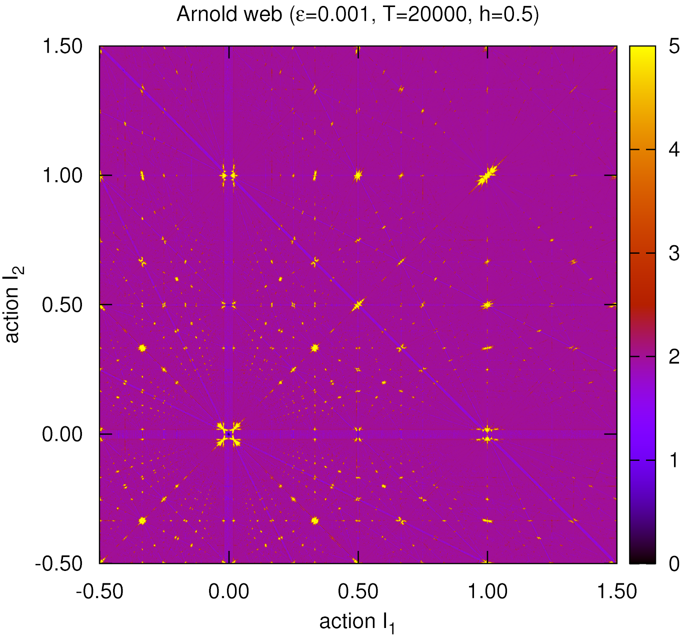
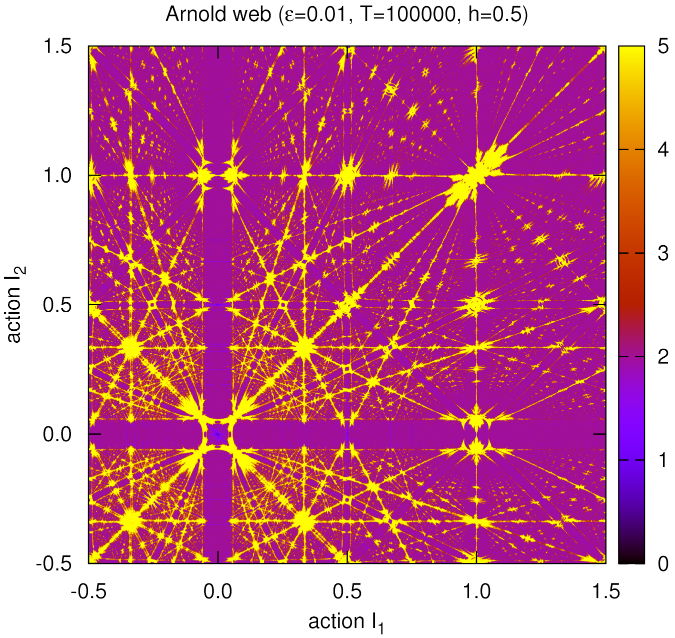
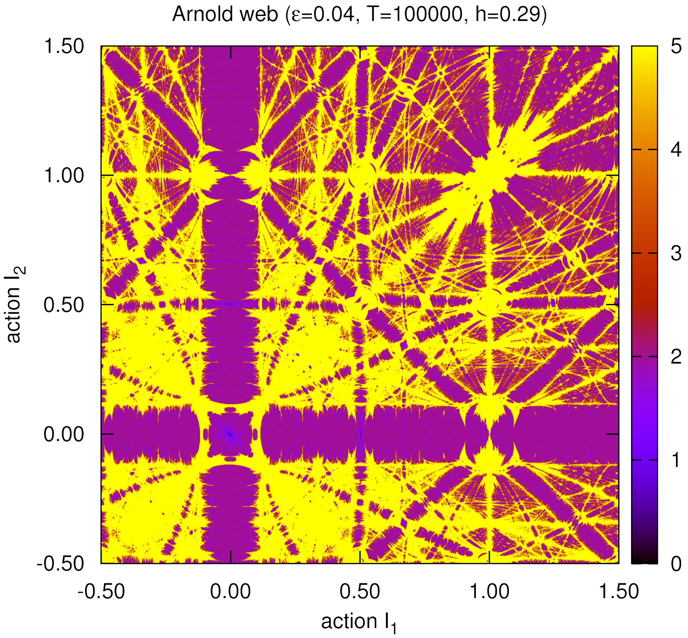
Arnold web of the test Hamiltonian ($\epsilon = 0.001$)



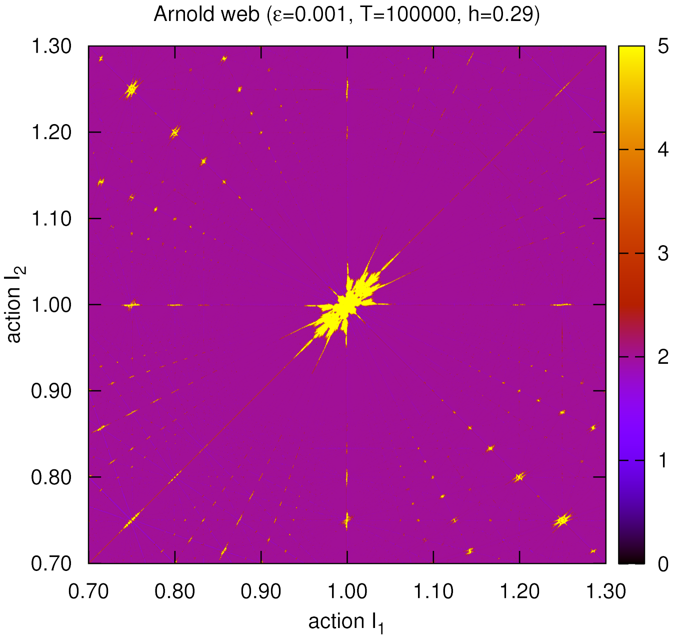
Arnold web of the test Hamiltonian ($\epsilon = 0.01$)



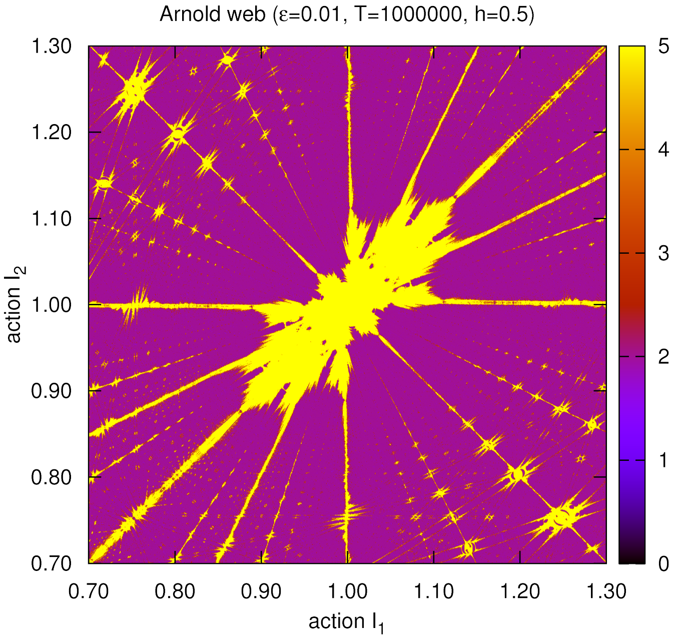
Arnold web of the test Hamiltonian ($\epsilon = 0.04$)



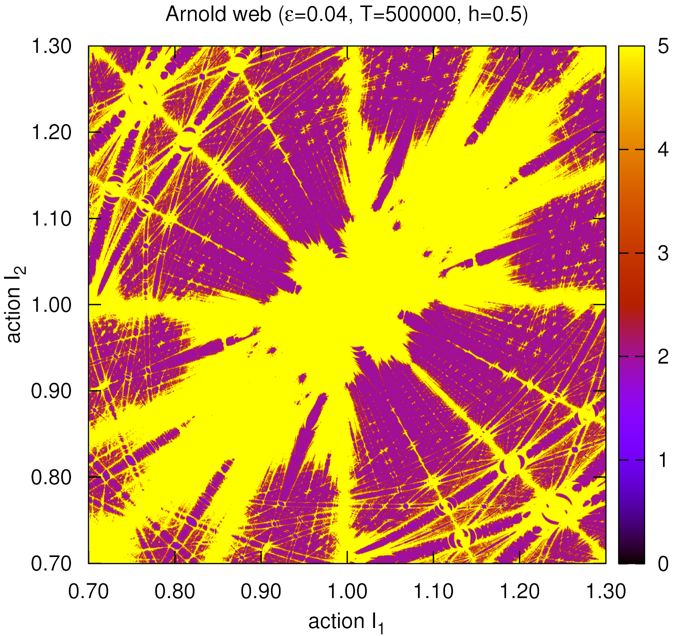
Arnold web of the test Hamiltonian ($\epsilon = 0.001$, zoom)



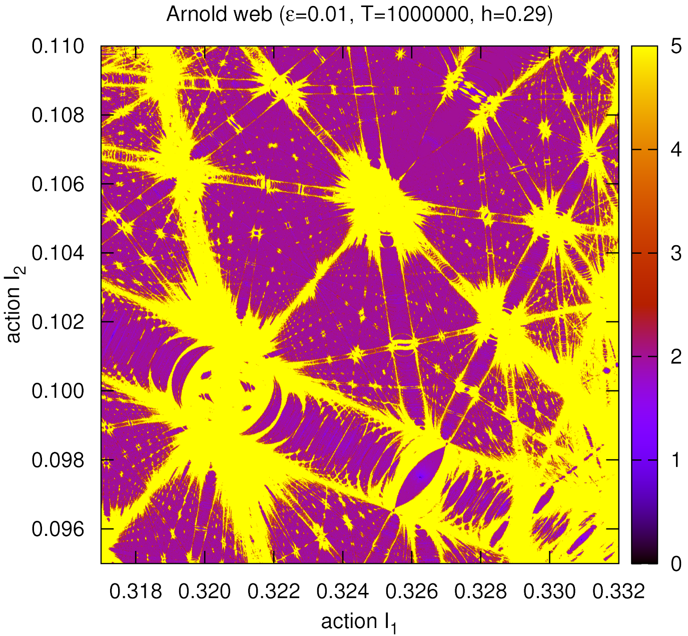
Arnold web of the test Hamiltonian ($\epsilon = 0.01$, zoom)



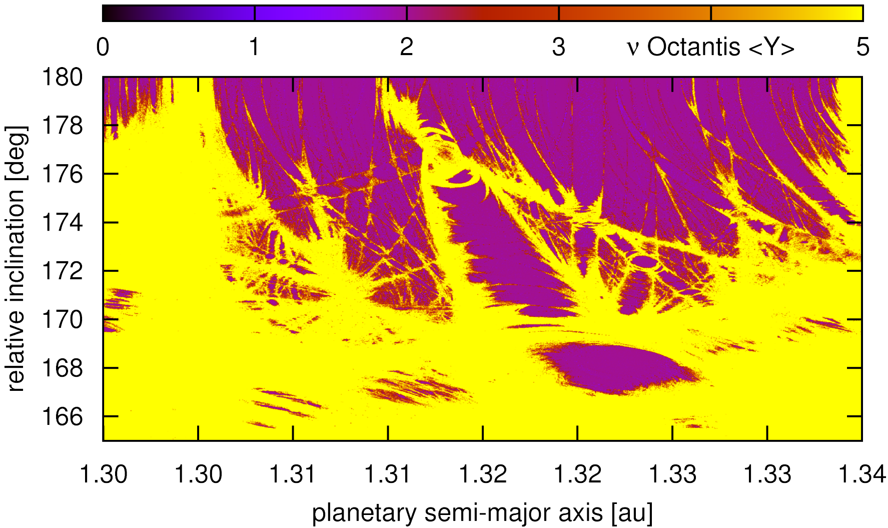
Arnold web of the test Hamiltonian ($\epsilon = 0.04$, zoom)



Arnold web of the test Hamiltonian ($\epsilon = 0.04$, zoom)



Arnold web in ν Oct ($m \sim 532 M_{Jup} \equiv \epsilon \sim 0.380$)?



Summary

- We confirmed the best fit solution to the RV measurements of ν Octantis by Ramm+ (2009)
- The N-body model reveals apparently well constrained mutual inclination but other orbital elements of a putative planet are not known (semi-major axis, eccentricity)
- The N-body fits seem formally favour retrograde orbit of the planet, in accord with hypothesis by Eberle & Cuntz (2010)
- BUT the best-fit initial conditions lead to strongly unstable configuration in time scale $\equiv$ of 1 orbital period of the binary
- We did not found yet any long-term stable system consistent with the observations