On the feasibility of a quasi-instantaneous wavelength map for the RVS.

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Abstract

We present a numerical model describing the wavelength and the AC position of photons received from a given object, as a function of the object's time of detection, its AC position in the FoV (at the time of detection), the photon's AL position within the spectrum and integration time. This model should allow quasi-instantaneous wavelength calibration which, in turn, should allow an optimal co-addition of the spectra from different CCDs.

1 Introduction.

The motivation for the present study came from several questions which were raised during the discussions of the RVS-WG and in particular during the preparation of the document RVS-ACC-001.

In the first place, of course, there is the need for a proper wavelength calibration of the observed spectra. In the very earliest stages of the project, given the expected stability of the instrument and the assumption that transverse motion would be adequately compensated by a tilt mechanism, it was stated rather loosely that the wavelength calibration would be obtained on the ground from 'first principles' using the astrometry data, the satellite's attitude and the geometry of the instrument. However, this idea was never worked out in detail, and at any rate it could not be implemented before the instrument has flown for a sufficiently long time to allow an accurate determination of how the optical components and the detector really behave on-board. E.g. we know very little about the refractive index of the optical materials at the temperature prevailing in orbit (F. Chemla, private communication). The present baseline for the wavelength calibration is that the RVS will be self-calibrated in a global iterative approach using objects with known radial velocity (late-type RV standards, asteroids etc.) (Katz et al. 2003, 2004). Application of this approach will rely heavily on linear interpolation so it will require that a sufficiently dense grid of standard objects has been observed and calibrated, and this may take a considerable time. On the other hand, if it could be supported by an interpolating function which models (if only phenomenologically) the main characteristics of the combined effect of the scanning law, diffraction, imaging and TDI, a calibration would require a less dense grid of standard objects.

Apart from the above, however, recent simulations of the spectra have indicated that some kind of on-board wavelength calibration may be required if the spectra from the several CCD's are to be summed on-board, because the so-called along-scan (AL) distortion depends on the across-scan (AC) position of the spectrum and moreover differs from one CCD to the next. If this is ignored it could degrade the wavelength resolution of the summed spectra.

Let us return to the original idea of a 'first principles' calibration: if we cannot predict with sufficient accuracy how the complex optical system will perform under GAIA's operating conditions – and since instantaneous ray-tracing may be beyond the on-board computational capacity anyway – it should be possible nevertheless in a laboratory set-up, to study the motion and distortion of a spectrum in sufficient detail to derive a phenomenological model which mimics the combined effects of the scanning law, diffraction, imaging and TDI and which depends on a number of parameters that could be adapted to the in-flight conditions relatively quickly by fitting the model to the observations of a number of RV-standards. If this succeeds it would take us as close to a 'first principles' calibration as we can get.

In the present contribution we propose such a model, based on a suggestion by F. Crifo in her contribution to the 6^{th} RVS Workshop. Essentially it is a direct relation which yields the wavelength and AC position of photons from a given source as a function of AL position and time, parametrized by the source's time of detection and its position on the sky. More specifically, for an object detected at a time T_d at an AC position Y_d, it allows to predict at which later times recording of the output signals from each CCD has to start and stop, which pixels in the readout register have received a contribution from this object and what is the distribution of the AC position and wavelength of the collected collected in these pixels during TDI. We shall refer to it hereafter as the Position-Wavelength Map (PWM).At least it should allow safe non-linear interpolation between calibrated standard spectra on a not very dense grid in T_d and Y_d, thus providing the means for an instantaneous on-board wavelength calibration which is sufficiently accurate for on-board summation of spectra from different CCD's. But it could also be useful to generate approximate simulated spectra at a greater computational speed than can be realized using full-fledged ray-tracing software.

Throughout this paper we assume the detector lay-out is as described by Katz et al. (2004), but this implies no limitation: any future changes in the base-line design could be easily accomodated.

2 Strategy.

In order to devise a model and to test its suitability we simulated the formation of spectra in the RVS, considering not only diffraction and imaging but also the motion of the spectrum and TDI. The latter two aspects are independent of the former, so that at first we could use a simplified simplified version of the optical set-up. Afterwards, when the basic structure of the map was established, we refined it on the basis of ZEMAX simulation results provided by F. Chemla and D. Katz.

In the simplified optical design both the collimator and the camera were replaced by a single ideal lens with an 'effective' focal distance, so that the images could be calculated using simple geometrical optics; also the telescope was assumed to be ideal and as yet no attempt was made to introduce the proper PSF in any other way. For the description of the dispersion we used the characteristics of the grism and the configuration of the respective optical axes according to the present baseline design. Values for the refractive index (as a function of wavelength) were obtained by non- linear interpolation from a table for LAK9 glass (all data were kindly provided by F. Chemla).

Instead of merely yielding the position on the detector where a given wavelength is imaged, the simplified optics allowed us to solve the "inverse problem", i.e. to give the AL position on the detector (e.g. the centre of a CCD row) and to calculate the corresponding AC-position and the wavelength which is imaged there from the object's position in the focal plane at a specific time. This greatly helped in finding an appropriate functional form for the PWM.

The scanning law was implemented without the small seasonal variation (in the transverse velocity) corresponding to the excentricity of the orbit. For the description of TDI we used the specifications given by Katz et al. (2004).

The simulation for a given object starts with the specification of T_d and Y_d; taking account of the scanning law, the position of this object in the telescope's focal plane can then be calculated at any later time, to predict the motion of the spectrum over the detector. The spectrum moves over the detector at a rate which is nearly constant (the rate varies slightly due to the AL distortion). In TDI mode the charge is transferred periodically from one row to the next so that on average it moves at the mean rate of the spectrum. As a consequence the photons which are detected on any pixel at any time have nominally the same wavelengths as those which caused the charge that was transferred to that pixel.

For a more detailed description of this charge collection it is convenient to define a *TDI tick* as the midmost instant between successive charge-transfers and a *TDI bin* as the small set of pixels, on successive CCD rows at successive TDI ticks, which receive photons of about the same wavelength from a given object during the integration; at readout these few pixels form a cross-cut of the two-dimensional spectrum and the sum (weighted or otherwise) of the flux recorded on them constitutes the flux within a bin of the extracted spectrum. The object of the position-wavelength map is to associate a series of TDI bins to a given object after detection and to keep track of the wavelength of the photons collected in these bins during the transit, as well as of their AC positions: in this way one knows which pixels in the readout register will contain the spectrum of that object and what is the wavelength bin they represent.

The solution of the "inverse problem" mentioned above directly yields these wavelengths and AC positions e.g. for the AL centre of all TDI bins and at all TDI ticks during an integration. But the results depend on the AC position of the detected object and on the time of detection (i.e. on the phase of the transverse motion), so if one would store the map numerically the calculations should be done on a sufficiently dense grid of (T_d, Y_d) values. Obvously this would result in a mass of data which is far too large to be kept in memory. However, we found that for all required cases of (T_d, Y_d), the individual contributions to a TDI bin can be represented almost perfectly by a second-degree polynomial in the integration time (e.g. represented by the TDI tick number) and that the coefficients of these polynomials can be represented, in turn, by a second-degree polynomial in the TDI-bin number (or in any other suitable coordinate along the spectrum). The latter's coefficients, in their turn, can be represented in good approximation by simple functions of (T_d, Y_d).

The several functional dependencies in this model are clearly revealed in a detailed inspection of the solution of the "inverse problem", but the latter is not readily available from ray-tracing simulations, let alone from actual spectrum recordings with the instrument itself. Therefore, once the structure of the model has been established, the coefficients in it will have to be determined otherwise, e.g. from a global least squares fit.

3 The model.

We assume that there is a single TDI clock for all CCD's (including the SSM) so that time can be reckoned in TDI ticks during the entire transit. The CCD's will be numbered by $I_{\mathrm{c}} = 1, \ldots, N_{\mathrm{c}}$. The calibraton will be defined for a set of TDI bins labeled $I_{\mathrm{s}} = 1, 2, \ldots, N_{\mathrm{s}}$, where N_{s} is slightly larger than the nominal spectrum length so that their read-out times (specified as $I_{\mathrm{br}}(I_{\mathrm{c}})$ to $I_{\mathrm{br}}(I_{\mathrm{c}}) + N_{\mathrm{s}} - 1$ TDI ticks after T_d) can be fixed independently of Y_d without the risk of losing any of the targeted wavelengths because the AL distortion may vary with Y_d. Let us label the CCD-rows by $I_{\mathrm{r}} = 1, 2, \ldots, N_{\mathrm{r}}$ according to the order in which a spectrum moves over them. Then it is easily seen that a bin labeled I_{s} coincides with the row labeled I_{r} of a CCD labeled I_{c} at a TDI tick given by

$$T_{\mathrm{bin}}(I_{\mathrm{c}}, I_{\mathrm{s}}, I_{\mathrm{r}}) = T_d + I_{\mathrm{br}}(I_{\mathrm{c}}) - N_{\mathrm{r}} + I_{\mathrm{s}} + I_{\mathrm{r}} - 2 \tag{1}$$

In particular it follows that a bin is in the readout register at

$$T_{\mathrm{ro}}(I_{\mathrm{c}}, I_{\mathrm{s}}) = T_{\mathrm{bin}}(I_{\mathrm{c}}, I_{\mathrm{s}}, N_{\mathrm{r}} + 1) \tag{2}$$

In our model a TDI bin is represented by an array of data characterizing the photons collected during TDI; naturally this data structure may also be referred to as a *spectrum cross-cut*. Its elements are labeled by the row-index I_{r}. Each element of a spectrum cross-cut consists of a couple (λ, y) where λ is the wavelength received at the centre of the row I_{r} at the instant (1), from the position of the source in the focal plane at that moment, and y is the AC position where it is received. Thus one can visualize a TDI bin or spectrum cross-cut as a window moving over a CCD according to the TDI clock; at each TDI tick it corresponds to a CCD row and acquires the "contribution" $(\Lambda(I_{\mathrm{c}}, T_{\mathrm{d}}, Y_{\mathrm{d}}, I_{\mathrm{s}}, I_{\mathrm{r}}), Y(I_{\mathrm{c}}, T_{\mathrm{d}}, Y_{\mathrm{d}}, I_{\mathrm{s}}, I_{\mathrm{r}}))$.

According to the remarks given in the previous section, we now build the PWM in three stages:

- (1) the "sub-pixel model" which (for wavelengths and AC positions alike) is a quadratic function of integration time for a given TDI bin, this time being represented by e.g. the sequence number I_{r} of the contributions in a spectrum cross-cut:

$$\Lambda(I_{\mathrm{c}}, T_{\mathrm{d}}, Y_{\mathrm{d}}, I_{\mathrm{s}}, I_{\mathrm{r}}) = \sum_{i=0}^{2} A_i^{\Lambda, I_{\mathrm{c}}}(T_{\mathrm{d}}, Y_{\mathrm{d}}, I_{\mathrm{s}})(I_{\mathrm{r}})^i \tag{3}$$

$$Y(I_{\mathrm{c}}, T_{\mathrm{d}}, Y_{\mathrm{d}}, I_{\mathrm{s}}, I_{\mathrm{r}}) = \sum_{i=0}^{2} A_i^{\mathrm{Y}, I_{\mathrm{c}}}(T_{\mathrm{d}}, Y_{\mathrm{d}}, I_{\mathrm{s}})(I_{\mathrm{r}})^i \tag{4}$$

- (2) the "bin model" which is a quadratic function of the TDI (or spectrum-) bin number and which yields the coefficients for the sub-pixel model in each TDI bin:

$$A_i^{\Lambda,I_c}(T_\mathrm{d},Y_\mathrm{d},I_\mathrm{s}) \;=\; \sum_{j=0}^{2} B_{ij}^{\Lambda,I_c}(T_\mathrm{d},Y_\mathrm{d})(I_\mathrm{s})^j \qquad (5)$$

$$A_i^{\mathrm{Y},I_c}(T_\mathrm{d},Y_\mathrm{d},I_\mathrm{s}) \;=\; \sum_{j=0}^{2} B_{ij}^{\mathrm{Y},I_c}(T_\mathrm{d},Y_\mathrm{d})(I_\mathrm{s})^j \qquad (6)$$

- (3) a "supermodel" which is a polynomial in Y_d and periodic in T_d (because of the periodicity of the transverse motion) and which yields the coefficients for the bin model:

$$\begin{aligned} B_{ij}^{\Lambda,I_c}(T_\mathrm{d},Y_\mathrm{d}) &= C_{ij0}^{\Lambda,I_c} + C_{ij1}^{\Lambda,I_c} Y_\mathrm{d} + C_{ij2}^{\Lambda,I_c} \sin\omega_\mathrm{s} T_\mathrm{d} \\ &= +C_{ij3}^{\Lambda,I_c} {Y_\mathrm{d}}^2 + C_{ij4}^{\Lambda,I_c} Y_\mathrm{d} \sin\omega_\mathrm{s} T_\mathrm{d} + C_{ij5}^{\Lambda,I_c} Y_\mathrm{d} \cos\omega_\mathrm{s} T_\mathrm{d} \qquad (7) \\ B_{ij}^{\mathrm{Y},I_c}(T_\mathrm{d},Y_\mathrm{d}) &= C_{ij0}^{\mathrm{Y},I_c} + C_{ij1}^{\mathrm{Y},I_c} Y_\mathrm{d} + C_{ij2}^{\mathrm{Y},I_c} \sin\omega_\mathrm{s} T_\mathrm{d} \\ &= +C_{ij3}^{\mathrm{Y},I_c} \cos\omega_\mathrm{s} T_\mathrm{d} + C_{ij4}^{\mathrm{Y},I_c} Y_\mathrm{d} \sin\omega_\mathrm{s} T_\mathrm{d} \\ &= +C_{ij5}^{\mathrm{Y},I_c} Y_\mathrm{d}^2 + C_{ij6}^{\mathrm{Y},I_c} Y_\mathrm{d}^3 + C_{ij7}^{\mathrm{Y},I_c} Y_\mathrm{d}^5 \qquad (8) \end{aligned}$$

  since the angular frequency of the transverse motion of the spectrum equals the scan rate ω_s. Contrary to the case of the lower-level models, the functional form of the supermodel describing the AC positions is different from the one describing the wavelengths.

Using this construction one can effectively simulate how (according to geometrical optics) the signal on each TDI pixel is built up during the scan; the average of the wavelengths collected on a pixel provides the basic wavelength calibration while their extreme values describe how the wavelength bins are blurred due to the AL distortion.

Probably the sub-pixel model can be combined with the PSF to generate a realistic distribution of wavelengths and AC positions for each TDI bin, but this has not yet been worked out. The distribution of wavelengths would allow an accurate wavelength calibration. Likewise the distribution of the AC positions determines which TDI pixels from the CCD's output register must be added to extract the spectrum, as well as (at least in principle) their weights if an "optimal" extraction is attempted. Finally, these distributions can be combined with known stellar spectra (synthetic or observed) to simulate how these spectra would be observed by the RVS.

The model contains 126 parameters per CCD. These can be determined from three components of three spectrum cross-cuts calculated on each CCD for fourteen detection-configurations (T_d, Y_d). Of course, as the model is not exact, one had better use a larger number of data in order to reduce the systematic errors caused by the limitations of the model.

Naturally the question arises whether such a large number of parameters is really required. Of course one can argue that this number is not surprising since they serve two functions of four variables each, so there is no obvious reason to suspect over-fitting. Nevertheless, if these parameters have to be determined from noisy data, their large number will make them the less robust. As a matter of fact it is quite possible that *some* of the coefficients C_{ijk}^{Λ,I_c} and C_{ijk}^{Y,I_c} could be omitted without compromising the accuracy of the model, but it will take a dedicated study to find out which ones.

In practice there may be a more fundamental problem with the determination of the parameters, notably the fact that the individual elements of a spectrum cross-cut can be obtained only from a numerical solution of the diffraction and imaging problems. Indeed, if the model must be tuned to observed spectra on-board, one can associate the mean wavelength of each cross-cut to the (known) wavelength of the corresponding spectrum bin, and its mean AC position to the centroid of the acros-scan intensity profile.

But to determine the individual coefficients A_0^{Λ,I_c} etc. we would need to know in addition e.g. the extreme wavelengths and AC positions contributing to a spectrum bin, or two higher moments of the distribution of these quantities, but these cannot be derived from an observed spectrum. In the case of the AC positions one might attempt to model the acros-scan intensity profile and so estimate the higher moments, but that would imply a deconvolution to eliminate the PSF and this would not yield robust results. And for the wavelengths nothing of this kind is possible anyway.

So under those circumstances a reduced form of the wavelength map must be used. This is easily derived since the map is linear in the coefficients of the sub-pixel model. The first stage is now the "bin model":

$$\bar{\Lambda}(I_{\rm c}, T_{\rm d}, Y_{\rm d}, I_{\rm s}) = \sum_{j=0}^{2} \bar{B}_j^{\Lambda, I_{\rm c}}(T_{\rm d}, Y_{\rm d})(I_{\rm s})^j \tag{9}$$

$$\bar{Y}(I_{\rm c}, T_{\rm d}, Y_{\rm d}, I_{\rm s}) = \sum_{j=0}^{2} \bar{B}_j^{{\rm Y}, I_{\rm c}}(T_{\rm d}, Y_{\rm d})(I_{\rm s})^j \tag{10}$$

while the "supermodel" is written as:

$$\begin{aligned} \bar{B}_j^{\Lambda, I_{\rm c}}(T_{\rm d}, Y_{\rm d}) &= \bar{C}_{j0}^{\Lambda, I_{\rm c}} + \bar{C}_{j1}^{\Lambda, I_{\rm c}} Y_{\rm d} + \bar{C}_{j2}^{\Lambda, I_{\rm c}} \sin\omega_{\rm s} T_{\rm d} \\ &= +\bar{C}_{j3}^{\Lambda, I_{\rm c}} {Y_{\rm d}}^2 + \bar{C}_{j4}^{\Lambda, I_{\rm c}} Y_{\rm d} \sin\omega_{\rm s} T_{\rm d} + \bar{C}_{j5}^{\Lambda, I_{\rm c}} Y_{\rm d} \cos\omega_{\rm s} T_{\rm d} \end{aligned} \tag{11}$$

$$\begin{aligned} \bar{B}_j^{{\rm Y}, I_{\rm c}}(T_{\rm d}, Y_{\rm d}) &= \bar{C}_{j0}^{{\rm Y}, I_{\rm c}} + \bar{C}_{j1}^{{\rm Y}, I_{\rm c}} Y_{\rm d} + \bar{C}_{j2}^{{\rm Y}, I_{\rm c}} \sin\omega_{\rm s} T_{\rm d} \\ &= +\bar{C}_{j3}^{{\rm Y}, I_{\rm c}} \cos\omega_{\rm s} T_{\rm d} + \bar{C}_{j4}^{{\rm Y}, I_{\rm c}} Y_{\rm d} \sin\omega_{\rm s} T_{\rm d} \\ &= +\bar{C}_{j5}^{{\rm Y}, I_{\rm c}} Y_{\rm d}^2 + \bar{C}_{j6}^{{\rm Y}, I_{\rm c}} Y_{\rm d}^3 + \bar{C}_{j7}^{{\rm Y}, I_{\rm c}} Y_{\rm d}^5 \end{aligned} \tag{12}$$

The relation between the coefficients of the full model and those of the reduced one is simple:

$$\bar{C}_{jk}^{*, I_{\rm c}} = C_{0jk}^{*, I_{\rm c}} + C_{1jk}^{*, I_{\rm c}} \frac{(N_{\rm r}+1)}{2} + C_{2jk}^{*, I_{\rm c}} \frac{(N_{\rm r}+1)(2N_{\rm r}+1)}{6} \tag{13}$$

where the '*' stands for Λ or Y. The reduced form of the wavelength map can still be used for calibration and for locating the spectrum in the AC direction, but it contains no information about the distribution of the wavelengths within a TDI bin, nor about the intensity profile in the AC direction. However, it is not inconceivable that a complete wavelength map, tuned to e.g. ZEMAX simulations, could be used to provide that additional information, albeit with lower accuracy depending on how well the in-flight performance of the instrument will be known.

4 Interpolation.

According to the definitions above, each value of the index $I_{\rm r}$ corresponds to an instant of time mid-between two successive charge transfers of a TDI bin, while each value of the index $I_{\rm s}$ corresponds to the centre (AL) of a CCD-row in the sense that the contributions to the bin are assumed to be calculated at such an instant and at the centre of the row which contains the bin. However, in the mathematical expressions constituting the map, both $I_{\rm r}$ and $I_{\rm s}$ may be replaced by continuous variables $T_{\rm int}$ and $X_{\rm b}$ describing, respectively, the integration time for a given bin and the AL position within this bin relative to the centre of the first bin. They are related to absolute time T (in units of TDI ticks) and AL position X (in pixels) as follows:

$$T_{\rm int}(I_{\rm c}, I_{\rm s}) = T - T_d - (I_{\rm br}(I_{\rm c}) - N_{\rm r}) - (I_{\rm s} - 1) \tag{14}$$

$$X_{\rm b}(I_{\rm c}, I_{\rm s}) = X_{I_{\rm c}} + {\rm round}(T_{\rm int}(I_{\rm c}, 1)) - X \tag{15}$$

where $X_{I_{\rm c}}$ is the AL coordinate of the centre of the first row on the CCD. In practice, $T_{\rm int}$ will take values in the range $[-0.5, N_{\rm r} - 0.5]$ and $X_{\rm b}$ in the range $[-0.5, N_{\rm s} - 0.5]$. Thus it is possible to determine the parameters in the PWM from data which do not correspond precisely to the centre of a row or to a TDI tick, as well as to apply the map to abitrary times and positions on the detector, i.e. to interpolate between the data which have been used to fix the parameters.

5 Tuning and tests.

The ZEMAX results mentioned above describe the optical performance of the current RVS baseline which is based on the Offner Relay concept. They comprise 175 directions of incidence corresponding to the following grid in the FoV:

- 35 points AL: from $-1.36°$ to $+1.36°$ by steps of $0.08°$;

- 5 points AC: from $-0.80°$ to $+0.80°$ by steps of $0.40°$;

In each direction 3 wavelengths were considered, notably 8480, 8610 and 8740 $\AA$. For each point in the field of view and each wavelength, 4 quantities were derived:

- the AL and AC detector-plane coordinates of the barycenter of the PSF (expressed in micron);
- the AL and AC widths (expressed in micron) containing 76% of the energy respectively of the AL and AC line spread function

In the present study we used only the coordinates of the barycenter of the PSF. In order to have data which are suited to our model we inverted the scanning law to link these results to 5 different detection instants T_d (spread over an entire scan period) and the corresponding AC positions Y_d in the telescope's focal plane.

In the PWM we used the following environmental parameters: $N_\mathrm{c} = 6$, $N_\mathrm{r} = 1010$, $N_\mathrm{s} = 720$, $\omega_\mathrm{s} = 60\,\mathrm{arcsec/sec}$, $I_\mathrm{br}(I_\mathrm{c}) = (1210 I_\mathrm{c} + 10)$ TDI-ticks, which takes account of a 2000 μm gap between the CCDs; the origin of the detector plane was chosen in the centre so that e.g. the centre of the first row of CCD #4 had $X_{4,1} = 1000\,\mu\mathrm{m}$ as AL coordinate. With this configuration the ZEMAX data provide points on three AL positions and 5 AC positions for each of the three wavelengths; with the multiplication which results from associating these data with 5 different instants of detection, we thus get, per CCD, 225 data to constrain the 126 parameters.

With the FoV spanning about 2° and the AL directions ranging from $-1.36°$ to $+1.36°$, one would expect that 8 or 10 of the ZEMAX images are outside the detector. However, it appears that the linear-to angular scale of the spectrograph varies considerably over the FoV so that this estimate is not valid; in fact the total AL size of the detector subtends an angle of about 1.7° instead of 2°. Furthermore we found that two or three of the ZEMAX AL directions are imaged in one of the gaps between the CCDs. As a result only 18 or 19 of the 35 ZEMAX entries are available for the fit. Fortunately the three AL positions we do get on each detector are just sufficient to constrain the sub-pixel model.

To verify the goodness of fit we calculated the residuals of the model, both for the input data which were used to determine the parameters and for a few of the ZEMAX entries which were not used because they correspond to points outside any of the CCDs. It appears that wavelengths and AC positions on the CCDs are reproduced with an error less than 0.0015 $\AA$ and 0.0015 pixels respectively. For the ZEMAX data corresponding to points which are less than 200 AL pixels away from a CCD we find that wavelengths are still reproduced with the same accuracy while the residuals on the AC position remain below 0.0025 pixels. Only further away from the detector the residuals become markedly larger. These results strongly indicate that the PWM is valid as a numerical model for the dispersion and (geometrical) imaging in the RVS.

If the large number of coefficients in the PWM proves to be a problem when one has to fit it to noisy data, one could contemplate a reduction of the supermodel (eliminating multiples of nine coefficients with each term that is omitted in (7) or (8)) provided that a somewhat lower accuracy is acceptable. After some trial and error we came up with a few possibilities listed in Table 1. The accuracy of the PWM is described here by an upper bound for the magnitude of the residuals, taken over the whole 4-dimensional domain covered by the variables $(T_d, Y_d, I_\mathrm{r}, I_\mathrm{s})$.

6 Future work.

We have demonstrated that the PWM is able to represent dispersion and image formation with the present baseline design for the RVS, with an accuracy of 0.002 $\AA$ in wavelength and 0.03 μm in the AC position, so that a quasi-instantaneous wavelength calibration is in fact feasible. However, before we can proceed to its implementation in practice the following further problems will have to be addressed:

- how can the PSF be conbined with the PWM to make a realistic prediction of the distributions of wavelength and position within each spectrum pixel in the readout register;
- assuming that the previous problem has been answered satisfactorily, test if the this prediction is sufficiently accurate in practice to derive the weights for an 'optimal extraction', and find out whether the required computations could be done on-board;

Table 1: Reduction of the 'supermodel' and its effect on the accuracy.

Parameters omitted	Wavelength residuals ($\AA$)	AC position residuals (pixels)
none	0.0015	0.0015
$C_{ij2}^{\Lambda,I_c}, C_{ij4}^{\Upsilon,I_c}$	0.007	0.01
$C_{ij2}^{\Lambda,I_c}, C_{ij4}^{\Upsilon,I_c}, C_{ij6}^{\Upsilon,I_c}, C_{ij7}^{\Upsilon,I_c}$	0.007	0.07

- is it necessary to reduce the number of parameters in the PWM, and if so, can it be done without compromising the accuracy (if at the expense of simplicity) or is a lower accuracy acceptable so that we can simply omit a few terms from the "supermodel";
- the determination of the parameters using observed standard-star spectra still needs to be worked out in detail and tested;
- in principle the PWM allows for a coaddition (with fractional shifts in the wavelength coordinate) of the spectra from different CCDs, but we have to assess whether the on-board computing capabilities are capable of performing the extraction and wavelength calibration required for such a coaddition: if this is not the case, on-board use of the PWM is not necessary.

Furthermore we can use the PWM to study in detail (and without a large amount of additional ZEMAX calculations) the blurring of the wavelength due to the variation of the AL distortion over the FOV, and the additional blurring that could result from an 'on the fly' coaddition without instantaneous wavelength calibration, as proposed by Hancock and Cropper (2003).

References

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