The astrometric solution of Gaia: A hard problem

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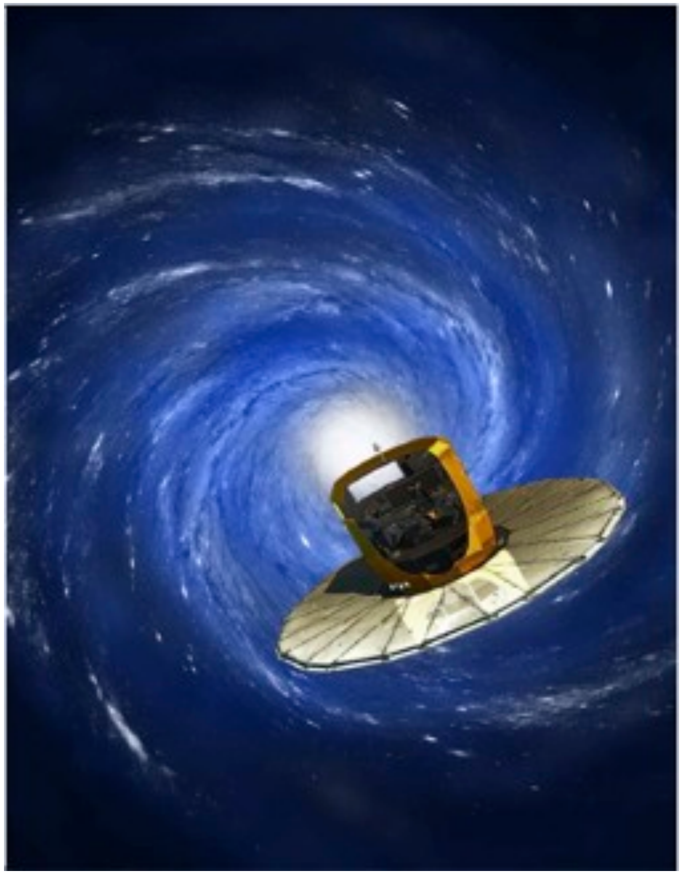
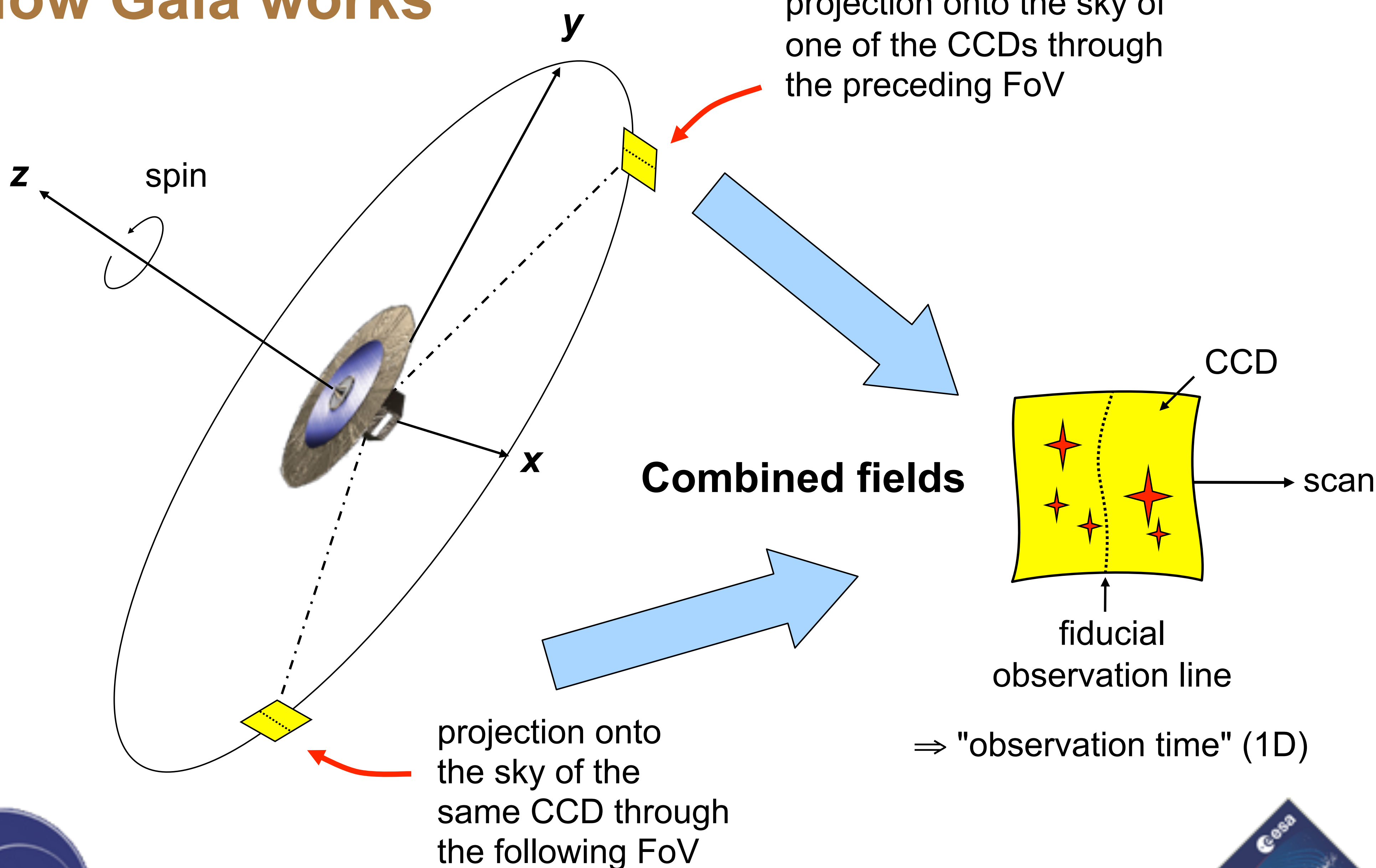
Basic principles of scanning space astrometry



The same applies to Hipparcos, Gaia, nano-Jasmine, ...

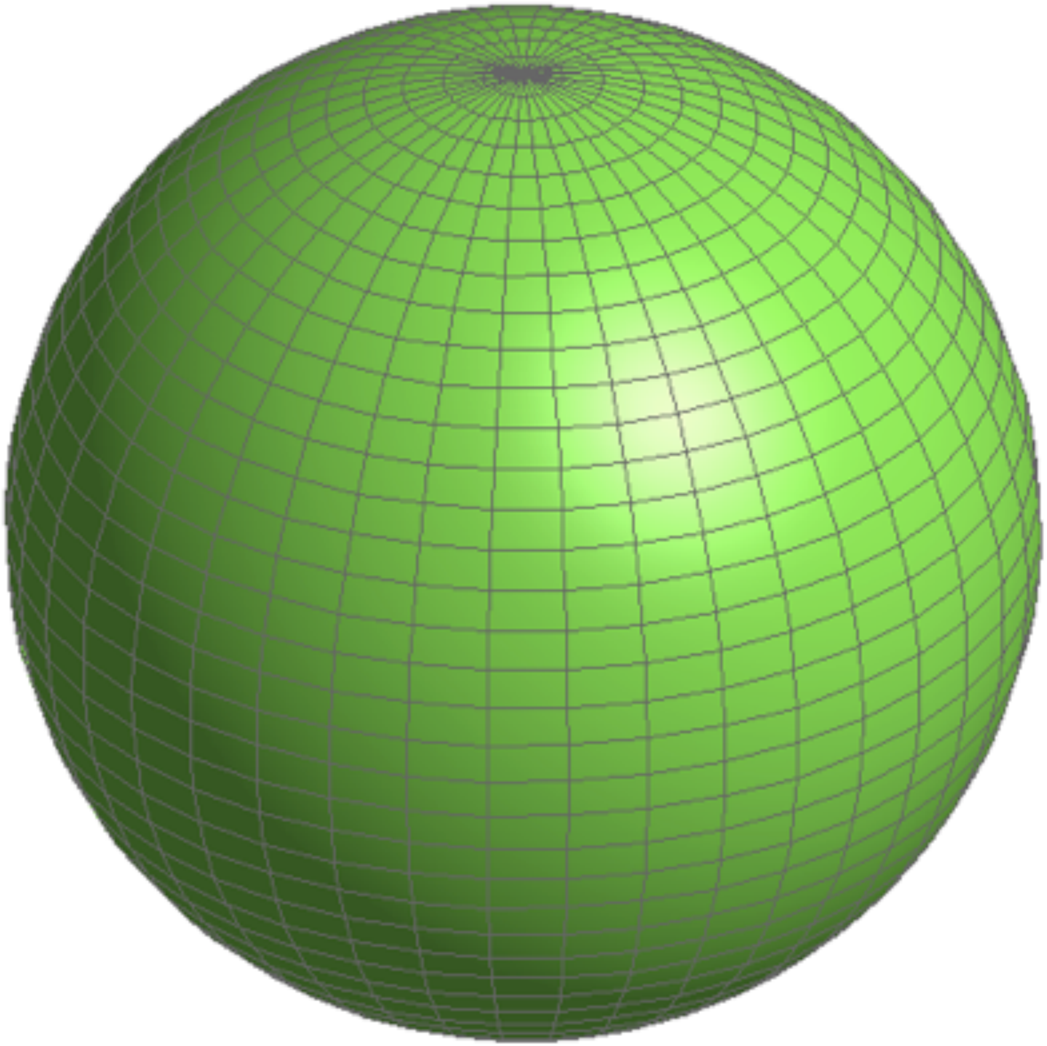
- Why measurements are 1D
- How to get absolute parallaxes
- Scanning law
- Choice of basic angle
- Self-calibration principle
- The resulting numerical problem

How Gaia works



⇒ "observation time" (1D)

Why are measurements one-dimensional?

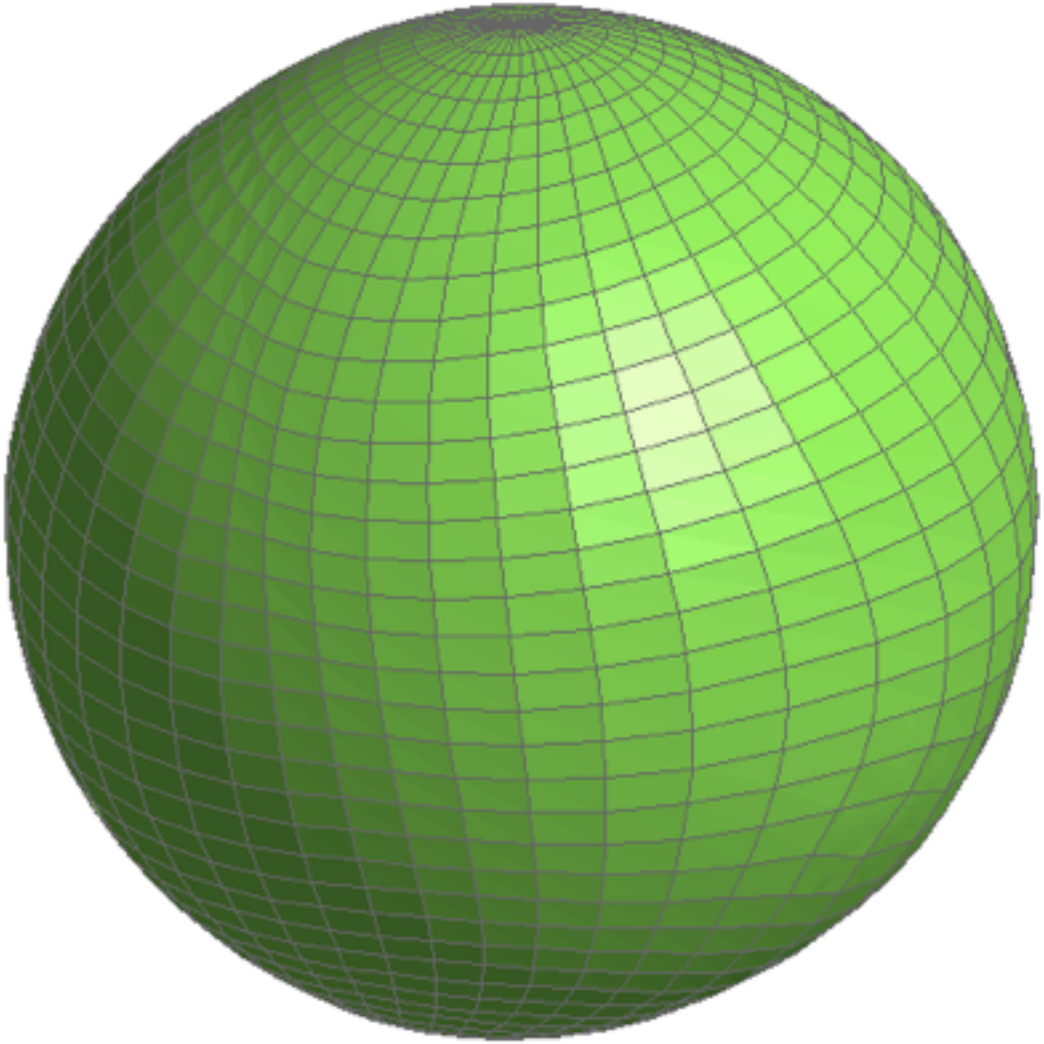


The purpose is to build an **accurate** catalogue

Accuracy implies **absence of** (significant) **systematic errors**

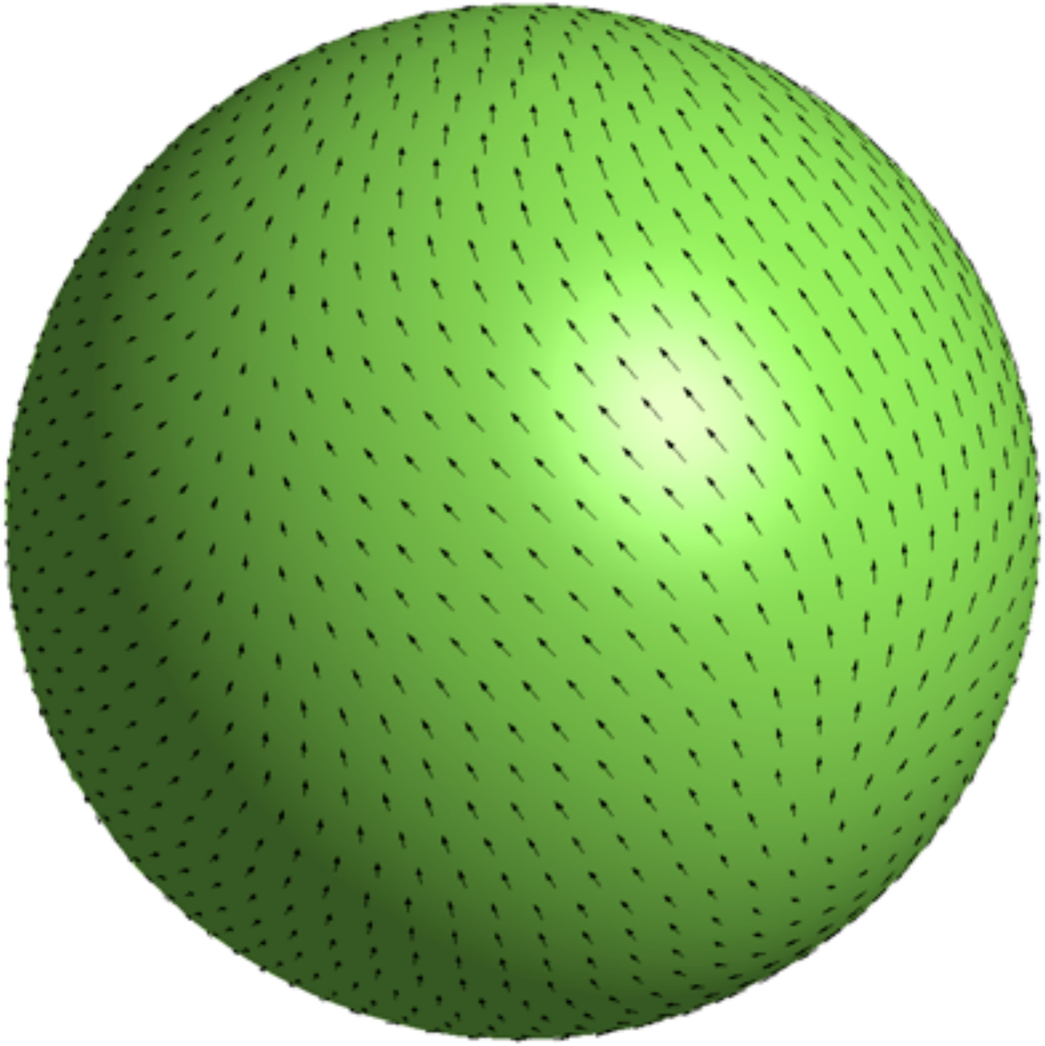
An accurate catalogue of stellar positions defines an **undistorted** (α, δ) grid

An example of a catalogue we don't want...



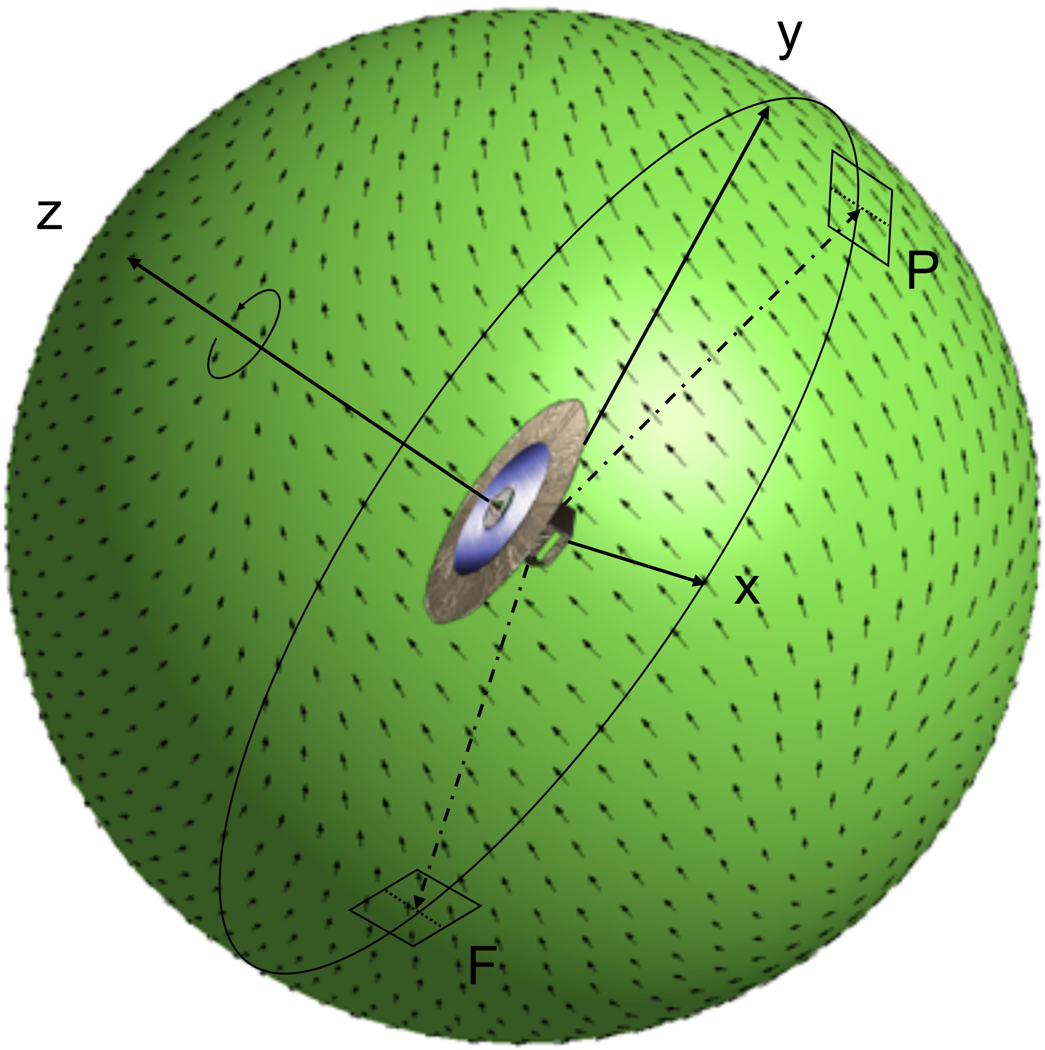
Systematic errors in the positions look like a **distorted** (α, δ) grid

The same catalogue shown differently



Systematic error mapped across the celestial sphere

How Gaia "sees" the systematic errors (1/3)



Stars are measured relative to the centres of the two fields of view

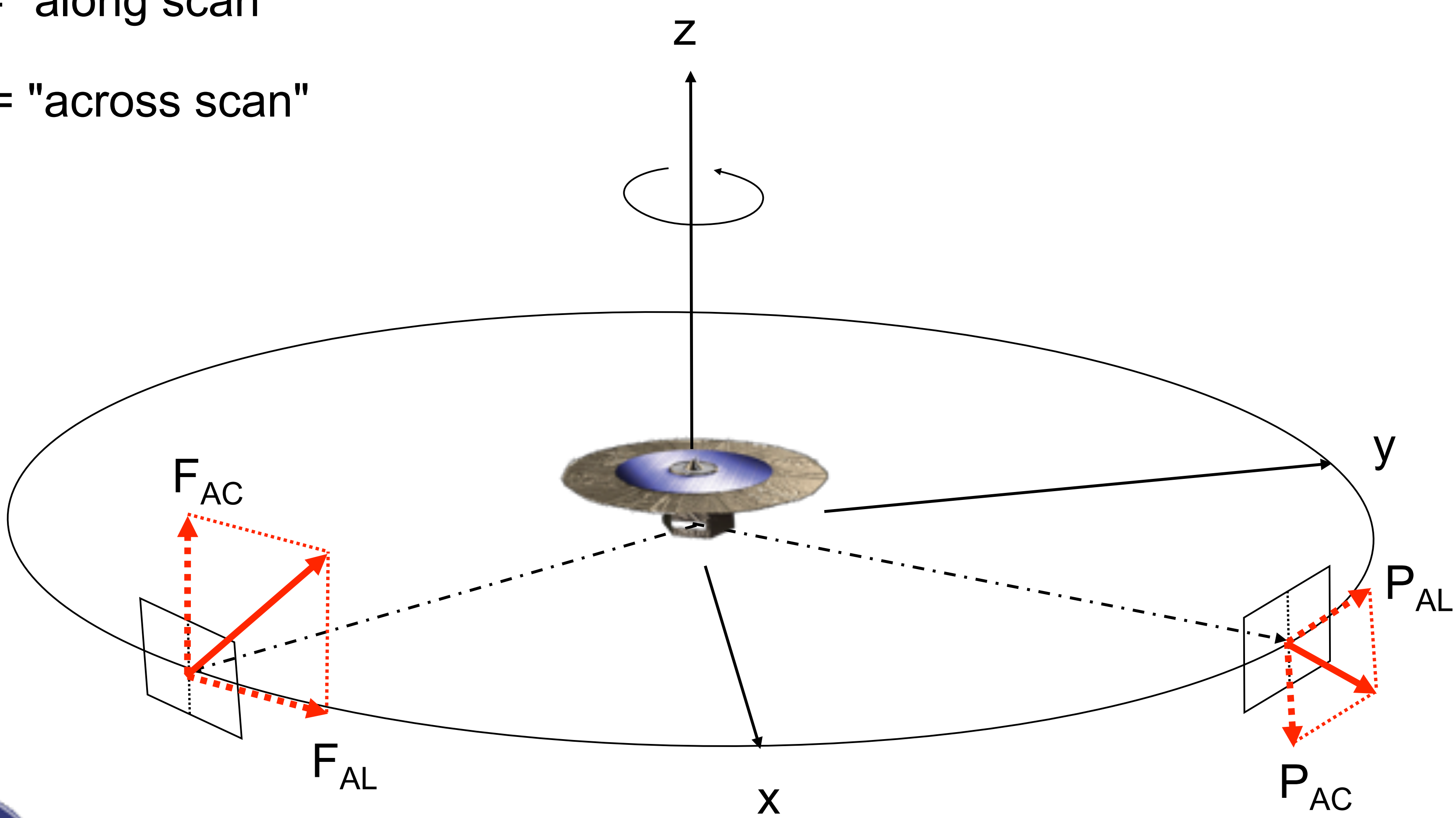
P = preceding FoV

F = following FoV

How Gaia "sees" the systematic errors (2/3)

AL = "along scan"

AC = "across scan"



How Gaia "sees" the systematic errors (2/3)

Across scan:

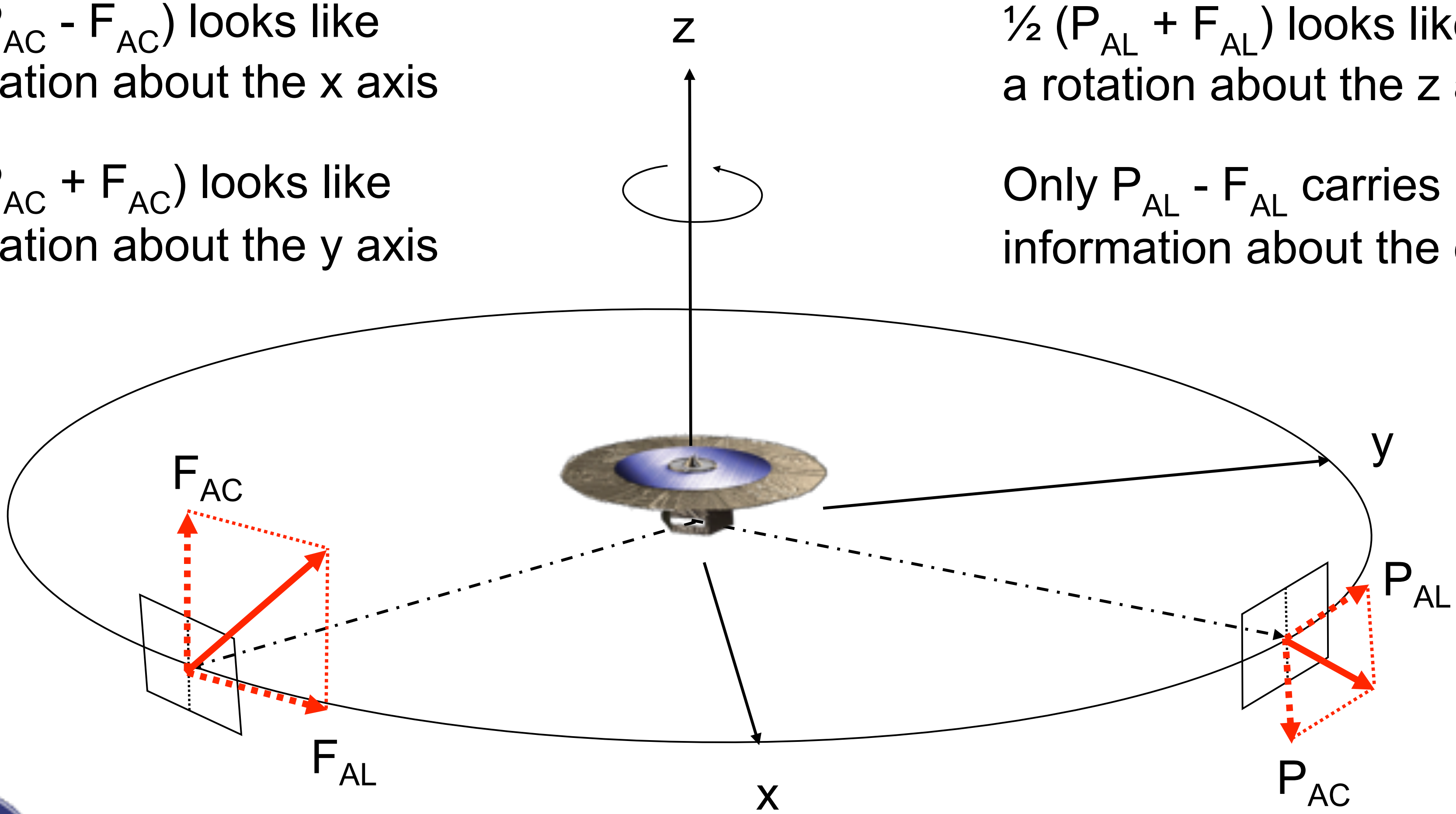
½ (P_{AC} - F_{AC}) looks like a rotation about the x axis

½ (P_{AC} + F_{AC}) looks like a rotation about the y axis

Along scan:

½ (P_{AL} + F_{AL}) looks like a rotation about the z axis

Only P_{AL} - F_{AL} carries real information about the errors



How Gaia "sees" the systematic errors (3/3)

Conclusion:

- Across-scan measurements contribute NOTHING to the global astrometry!
- Shown here for positions only, but the same conclusion holds for proper motions and parallaxes

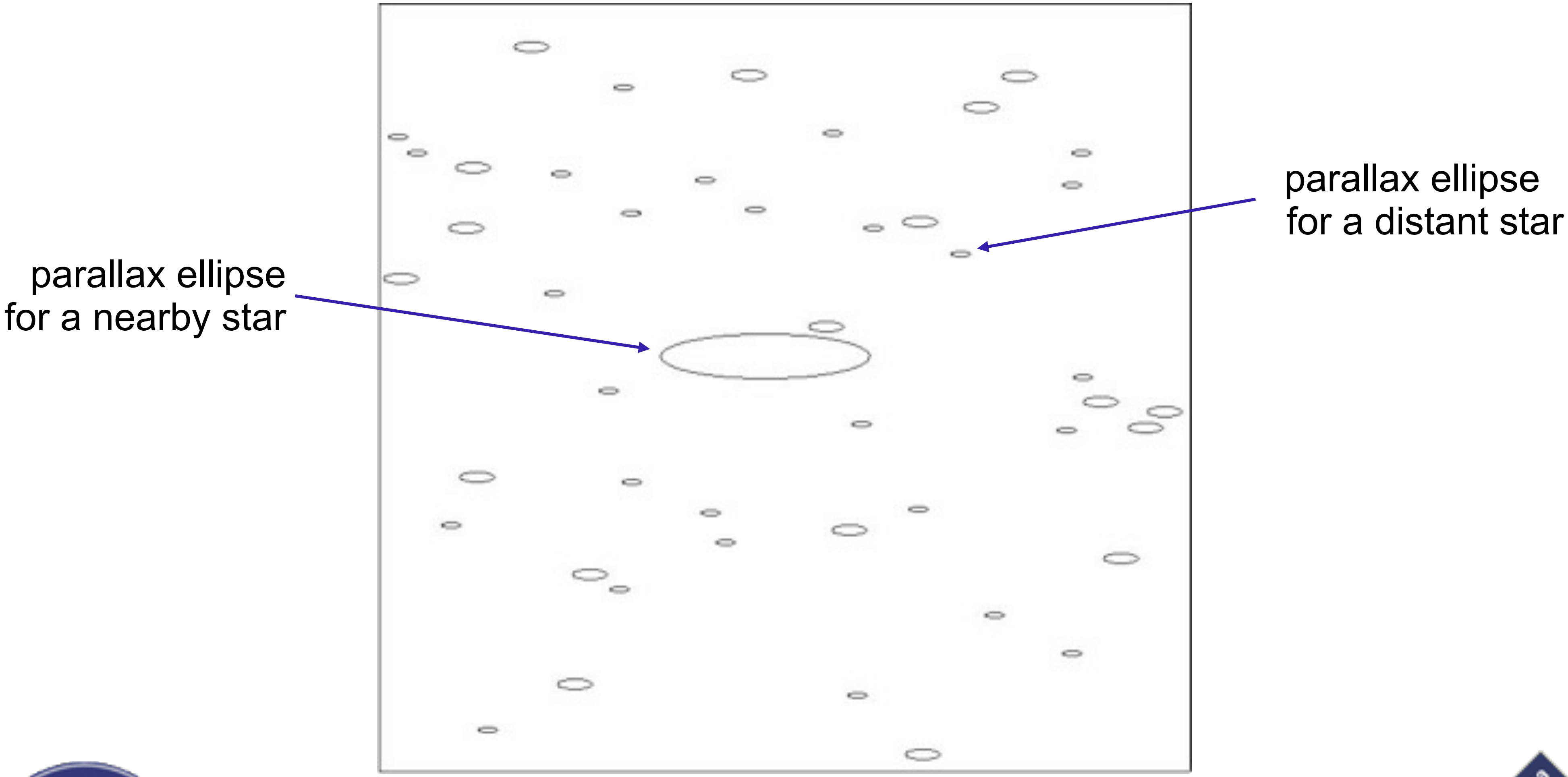
However:

- Orientation around **x** and **y** axes defines the AL direction and is therefore needed to lower precision (2nd order projection effect)
- Finite size of FoV allows some differential AC measurements

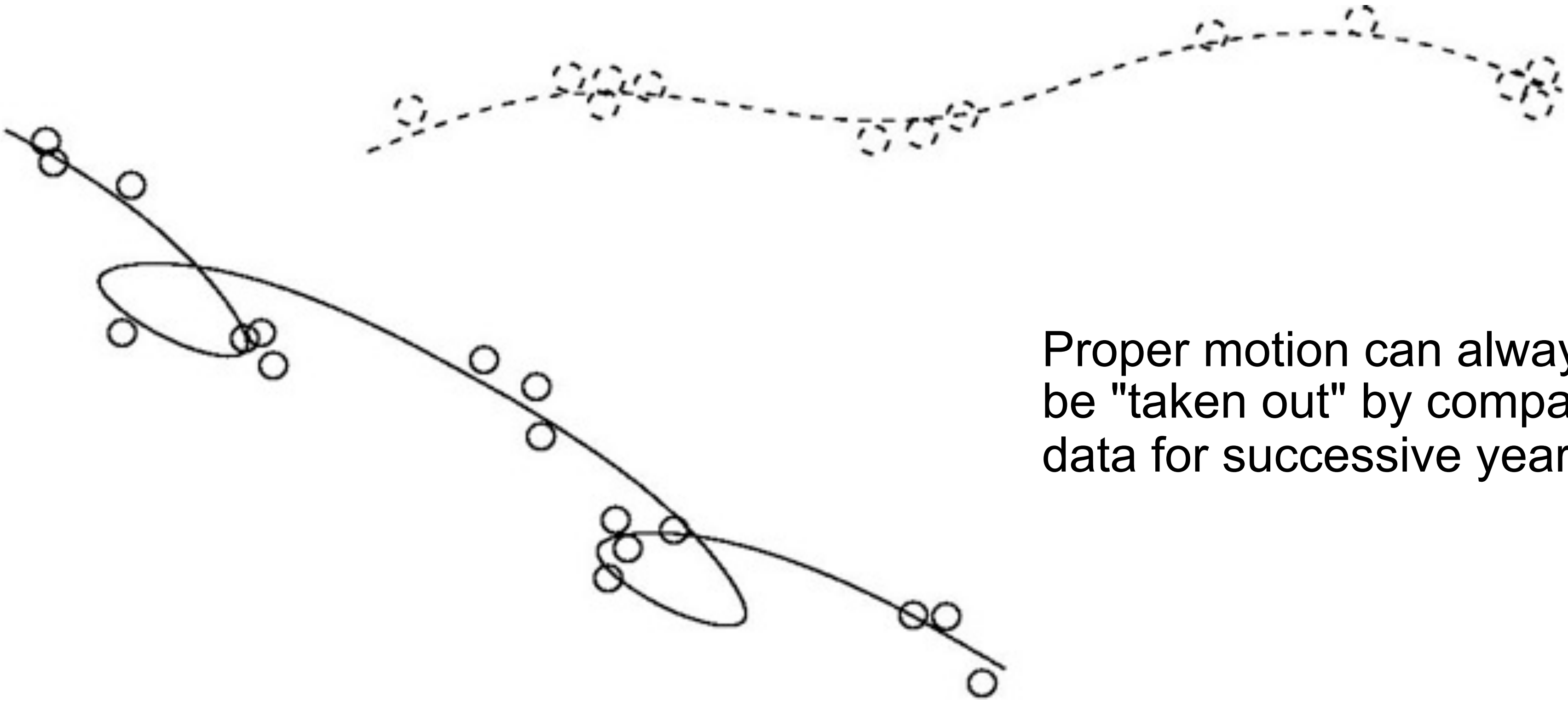
How to get absolute parallaxes

- What is the problem?
- Relative versus absolute parallaxes

Parallax mirrors the Earth's orbit as seen from the star

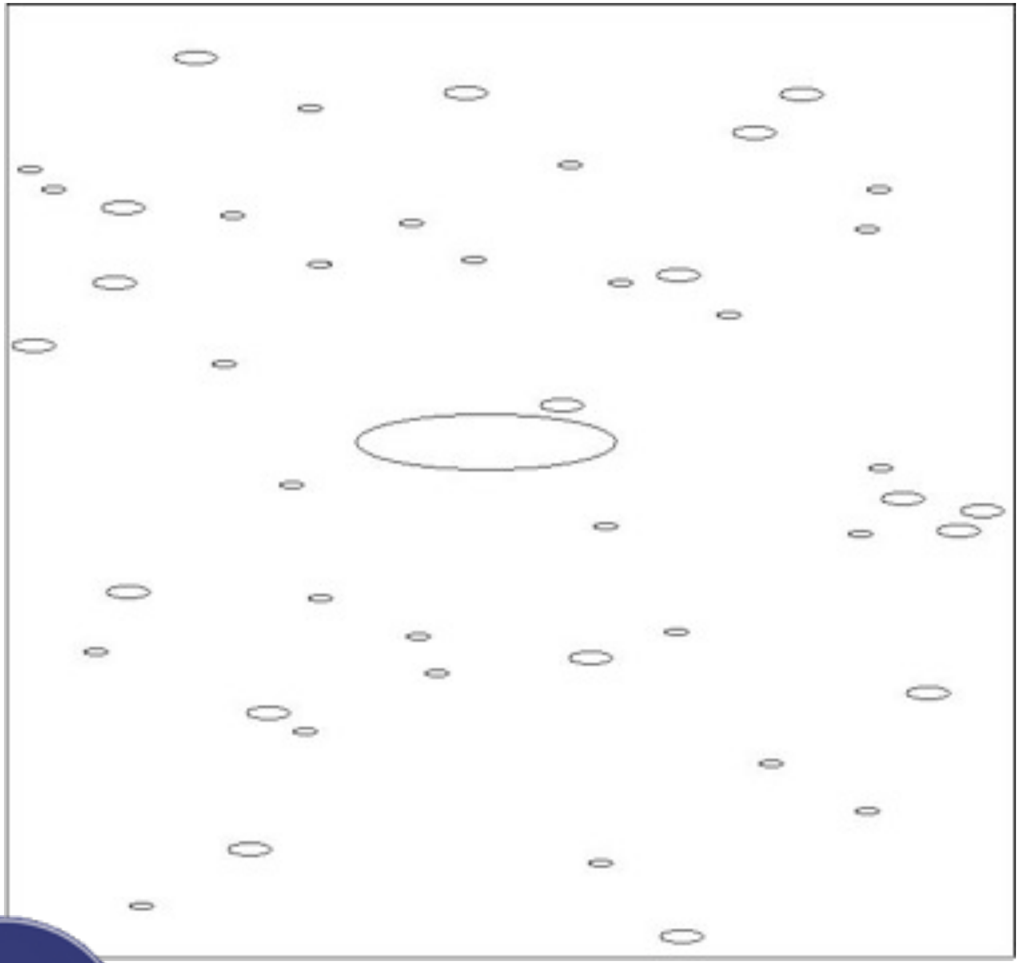


The actual motions are of course more complicated ...



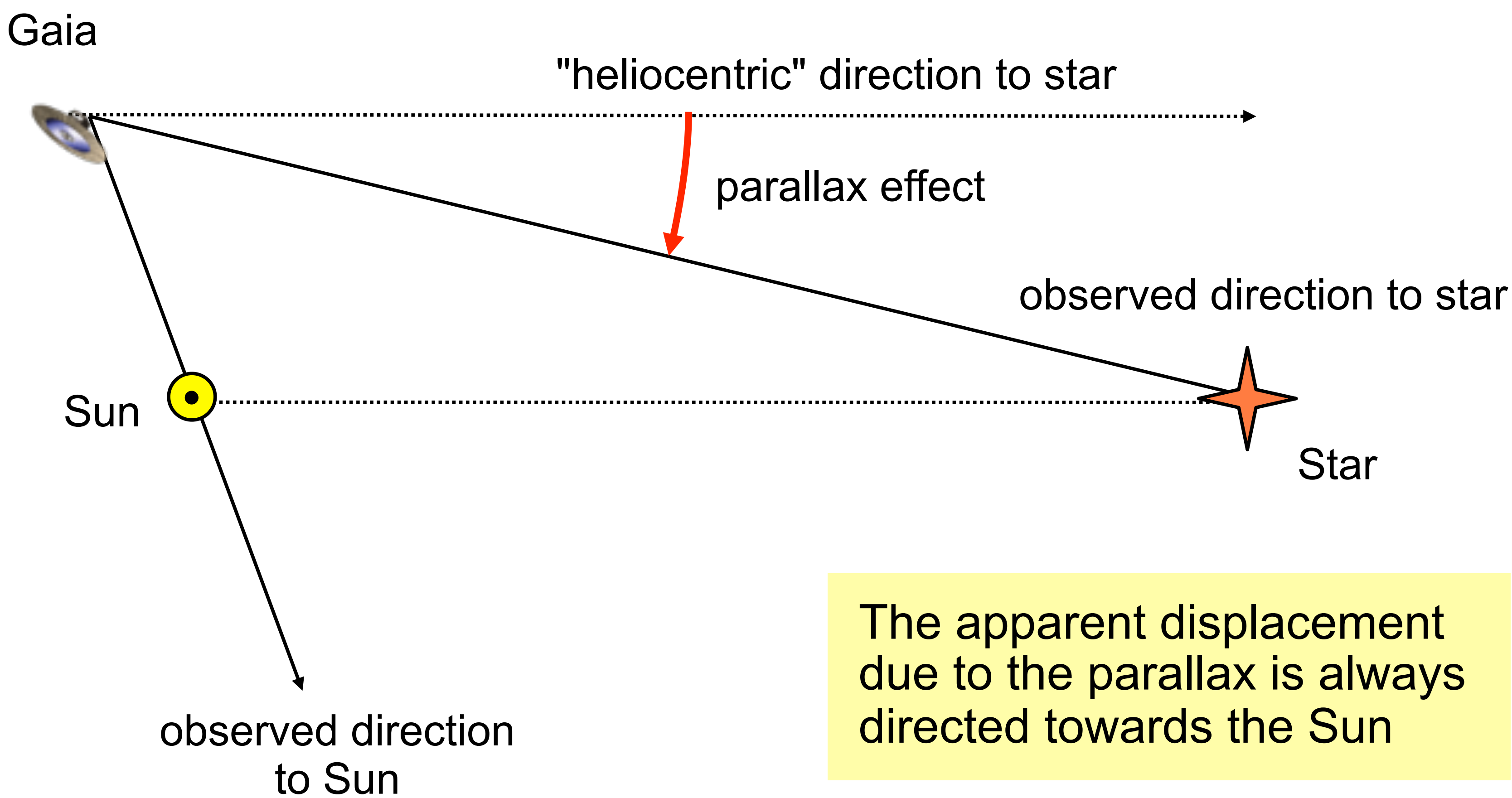
Proper motion can always be "taken out" by comparing data for successive years

Traditional parallax determination - ground-based or HST (in a field << 1 rad)



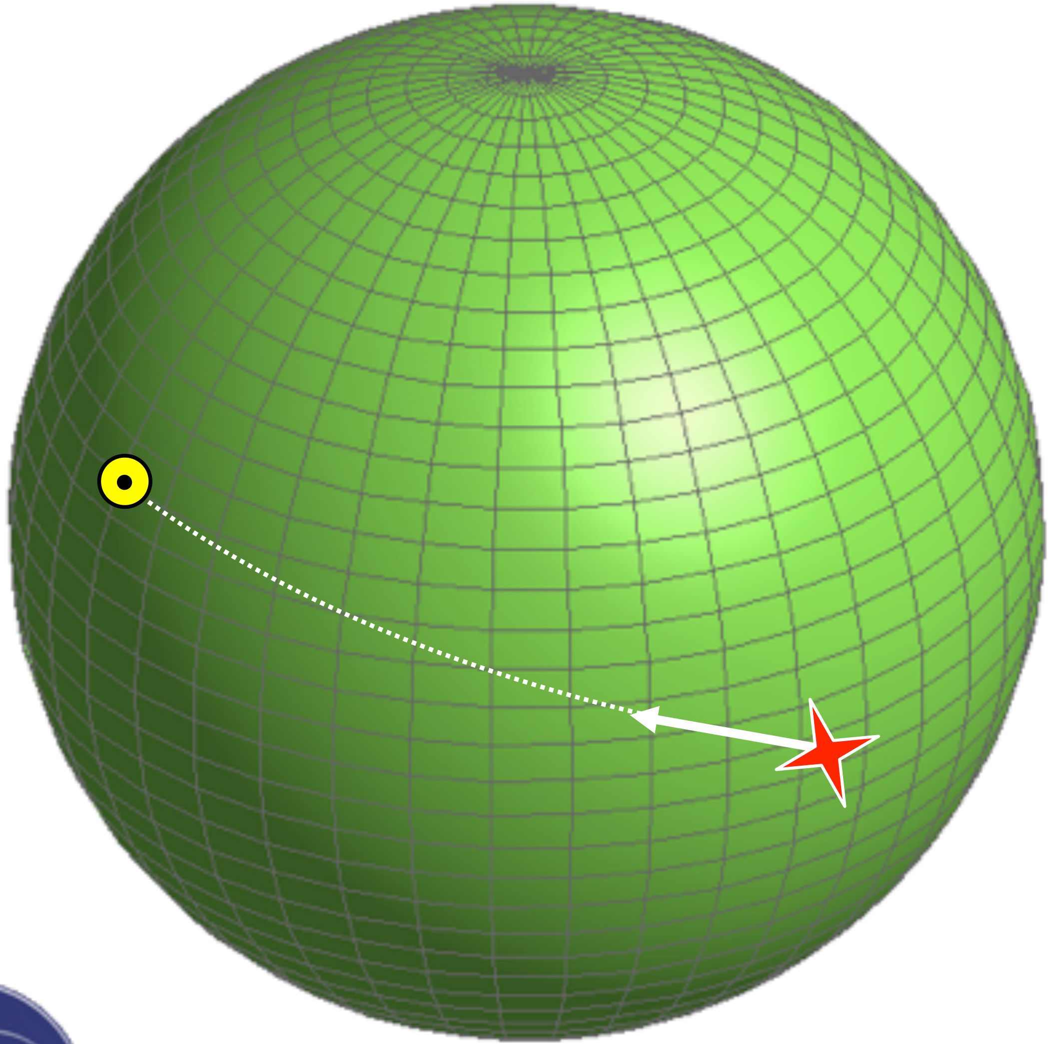
- All stars have similar parallactic motions, apart from a scaling factor (= parallax)
- Only relative parallaxes can be determined
- "Correction to absolute" requires a lot of guessing (unless there are enough quasars in the field)

Another way to look at parallax



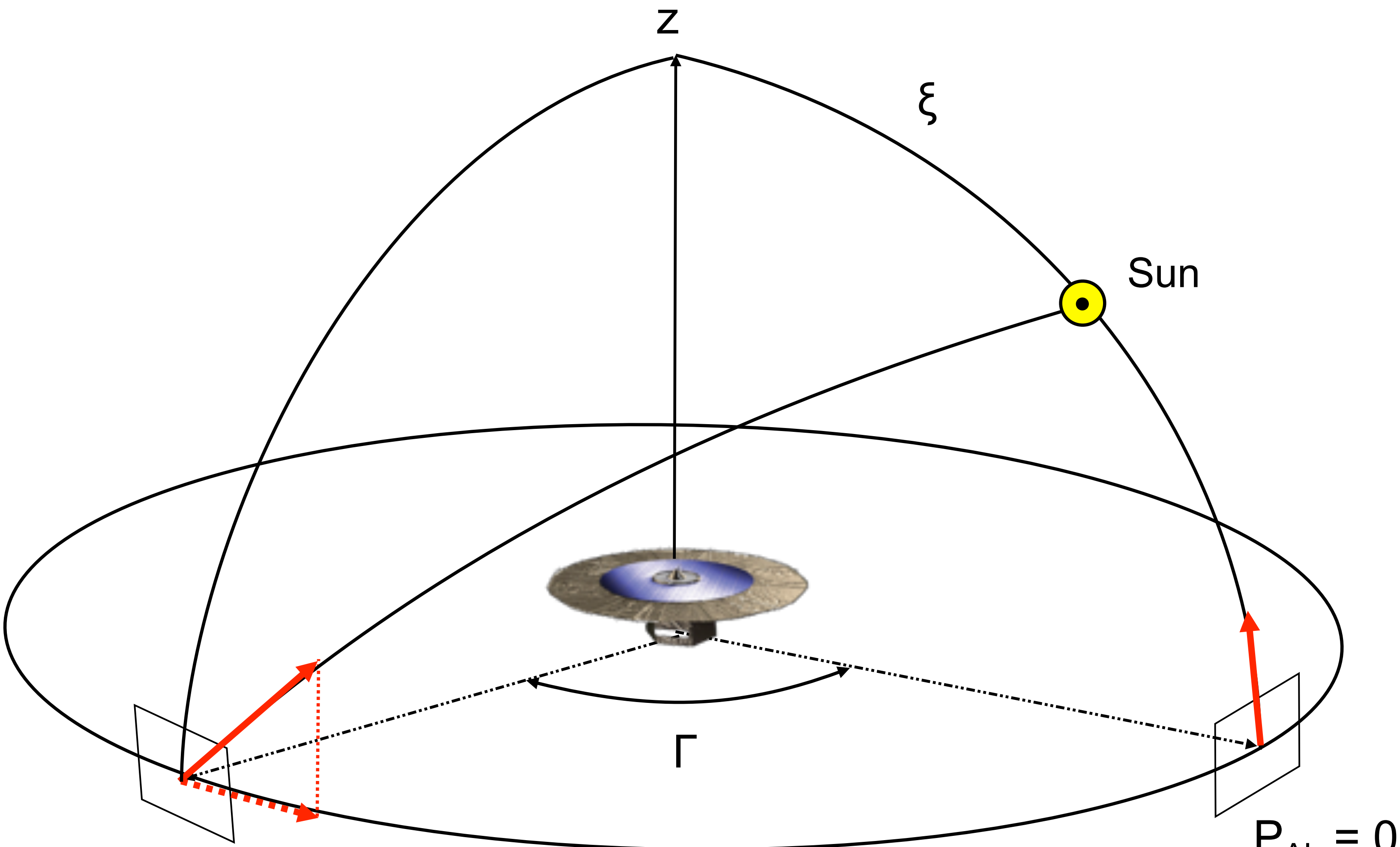
The apparent displacement due to the parallax is always directed towards the Sun

Same seen from "outside" the celestial sphere



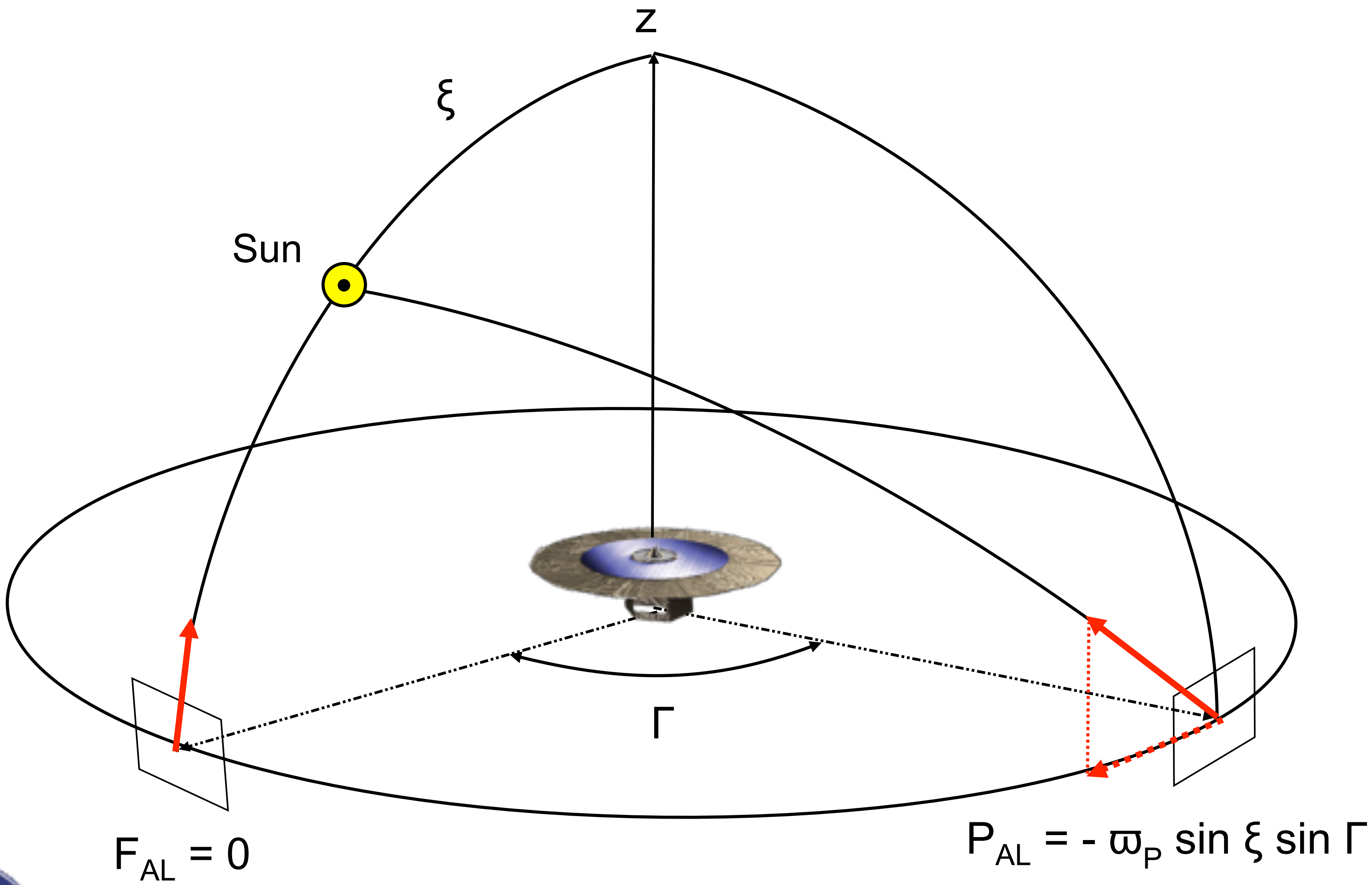
The apparent displacement due to the parallax is always directed towards the Sun

Absolute parallax measurement (1/3)



$F_{AL} = \varpi_F \sin \xi \sin \Gamma$

Absolute parallax measurement (2/3)



Absolute parallax measurement (3/3)

Conclusions:

- **Differential** along-scan measurements between the two FoV allows to determine **absolute** parallaxes

- Sensitivity is proportional to $\sin \xi \sin \Gamma$, where

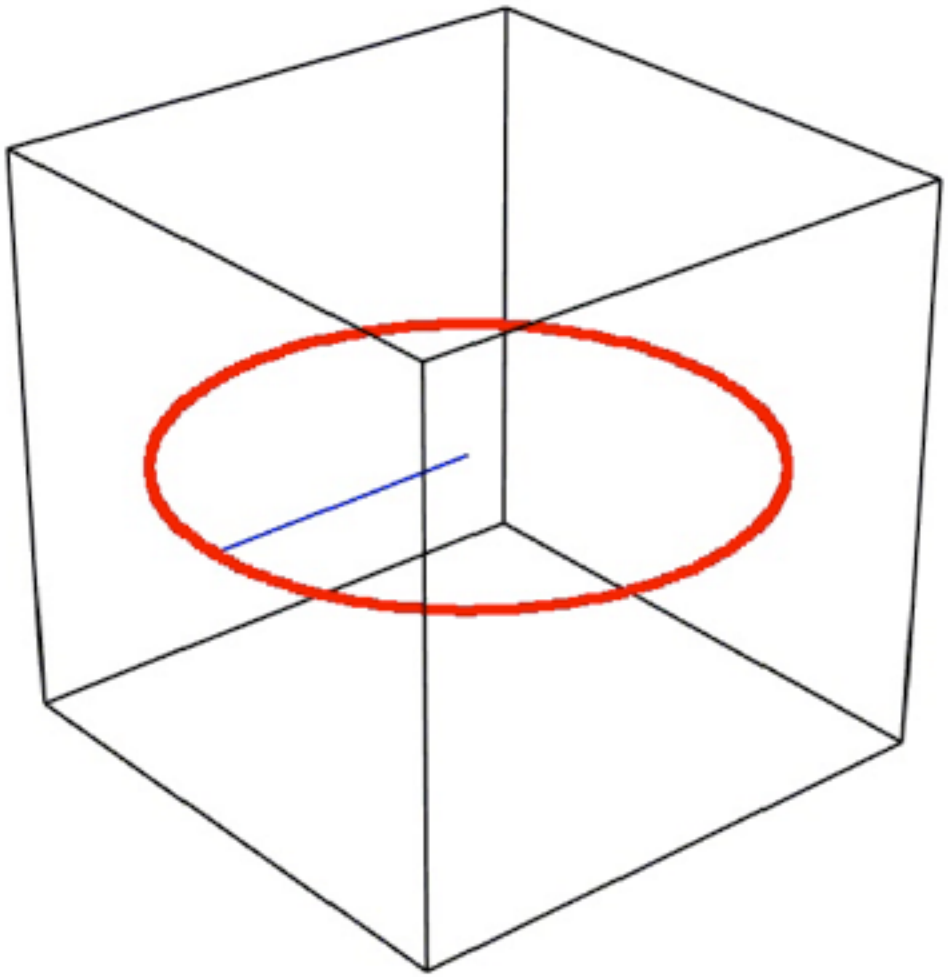
ξ = Sun-spin axis angle	= 45º	for Gaia
Γ = basic angle	= 106.5º	for Gaia

- Technical constraints limit ξ (earlier Gaia design had ξ = 55º)
- Γ = 90º ideal in principle, but undesirable for other reasons

Scanning law

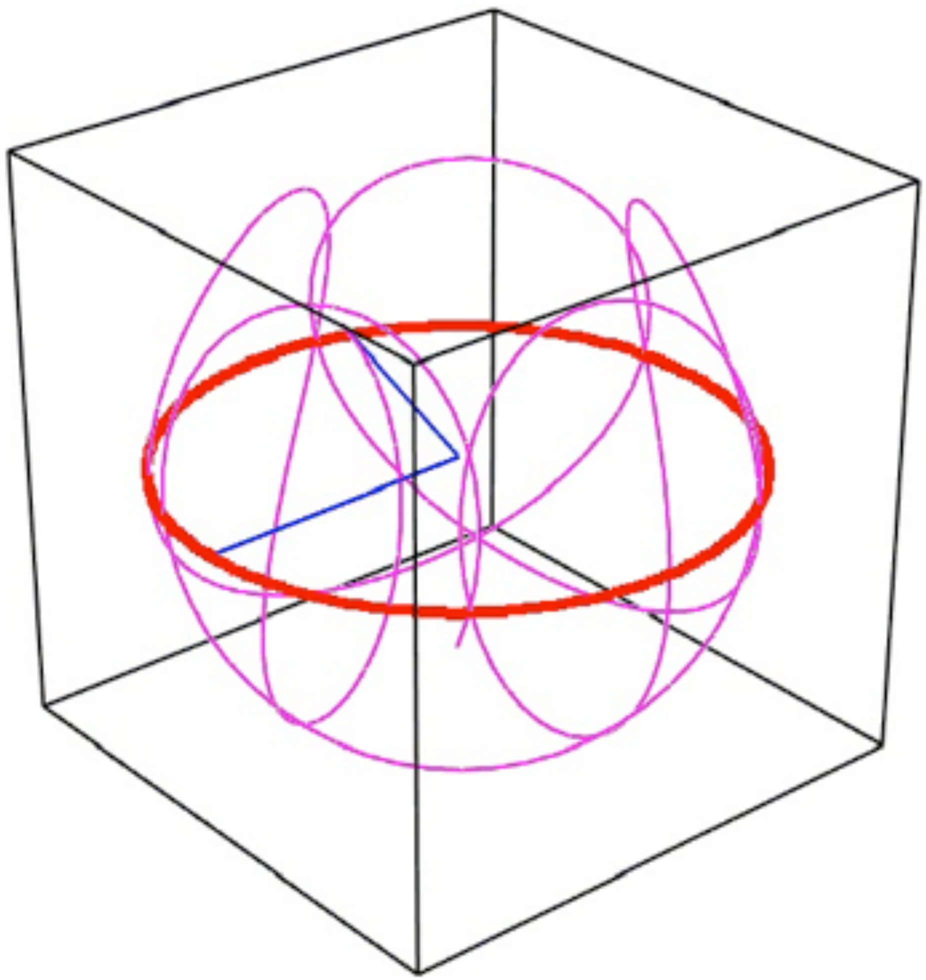
- A large (≈ 45º) Sun-spin axis angle (ξ) is necessary
- This angle must be kept constant to avoid variations of thermal load
- The scanning should cover as much sky as possible in a short time
- Successive great-circle scans must overlap (no gaps)
- These requirements leads to the **Nominal Scanning Law** used both for Hipparcos and Gaia
- It is a combination of 3 periodic motions (see animations)

Scanning law: 1st motion (P = 1 year)



Direction to Sun

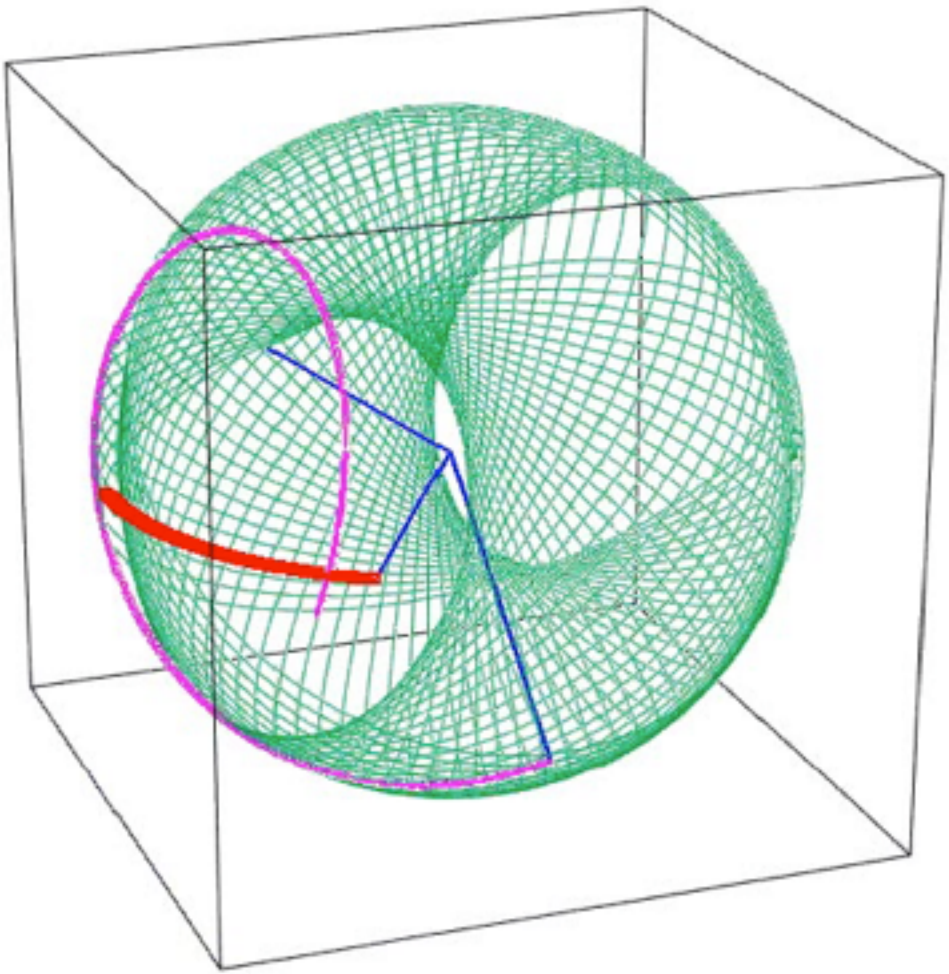
Scanning law: 2nd motion (P = 63 days)



Direction to Sun

Spin axis

Scanning law: 3rd motion (P = 6 hr)

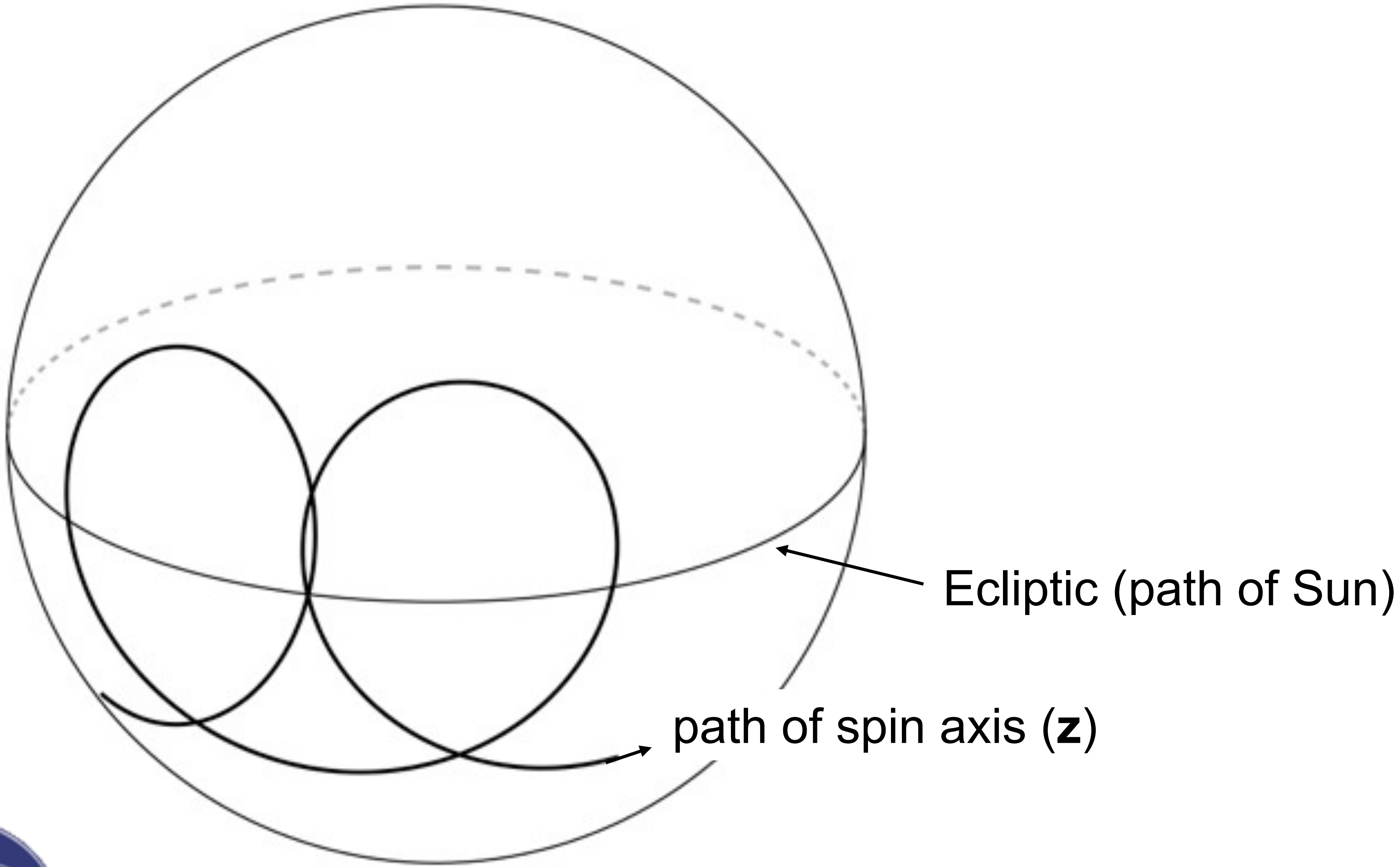


Direction to Sun

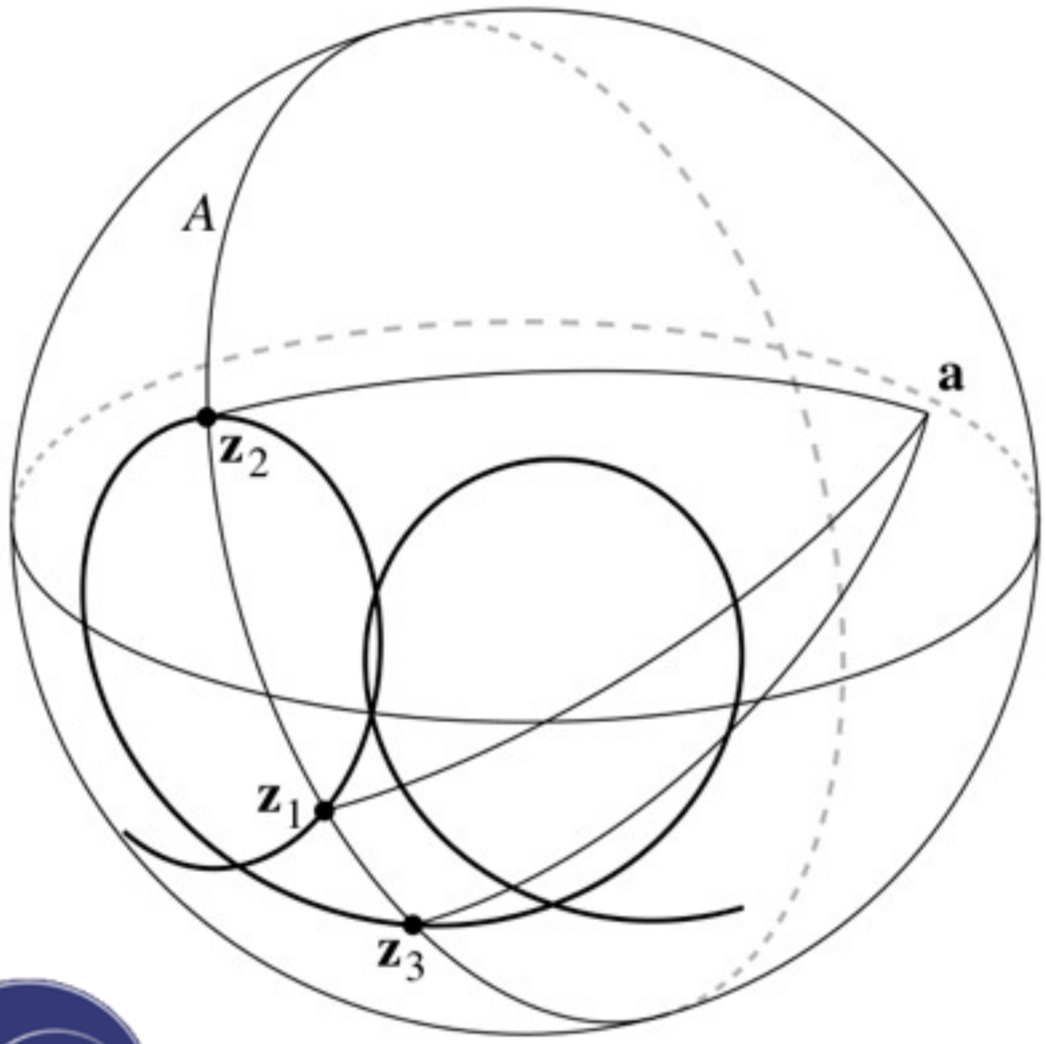
Spin axis

Field of view

Loops made by spin axis must overlap



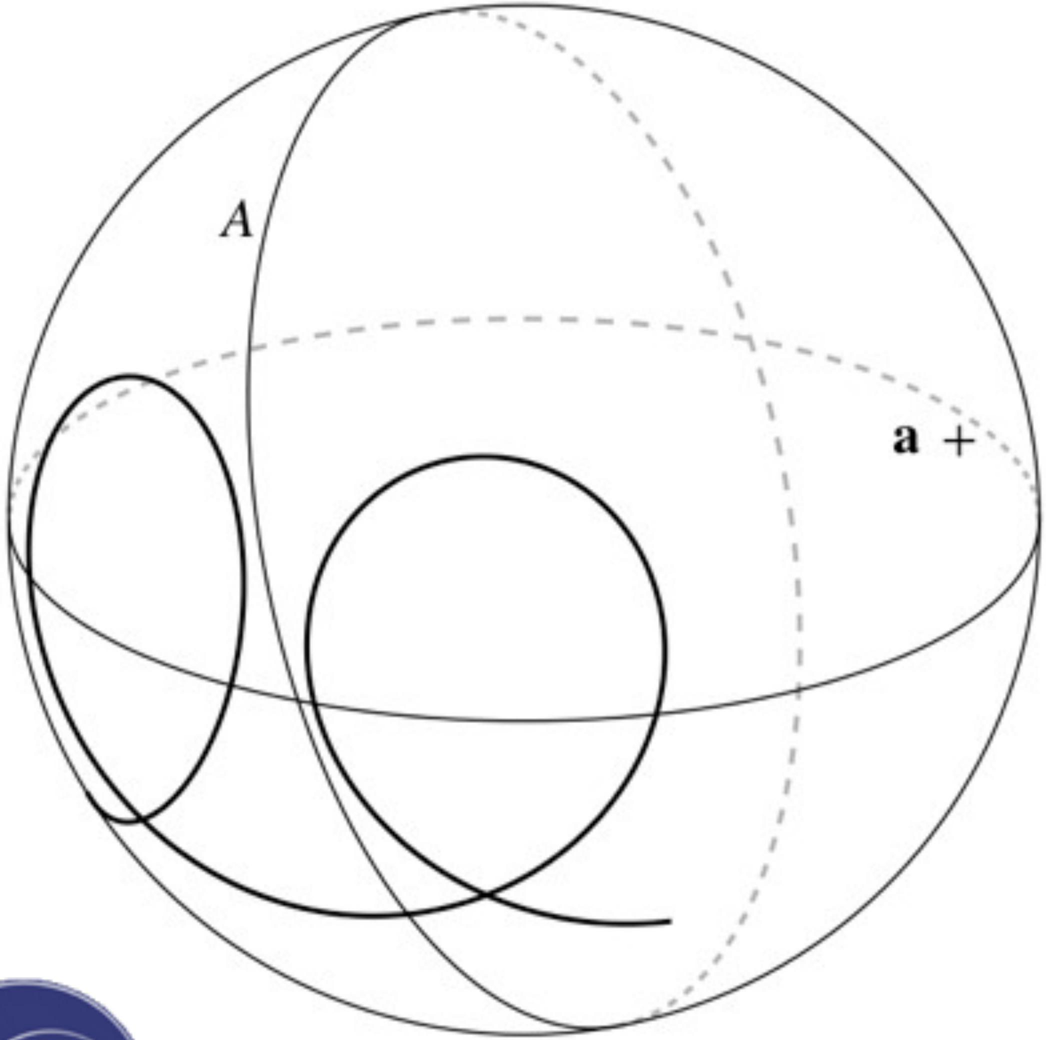
Sky coverage (1/3)



Potential measurements of **a** at $\mathbf{z}_1$ $\mathbf{z}_2$ $\mathbf{z}_3$

Speed of **z** must be small enough to ensure actual measurement at $\mathbf{z}_1$ $\mathbf{z}_2$ $\mathbf{z}_3$

Sky coverage (2/3)

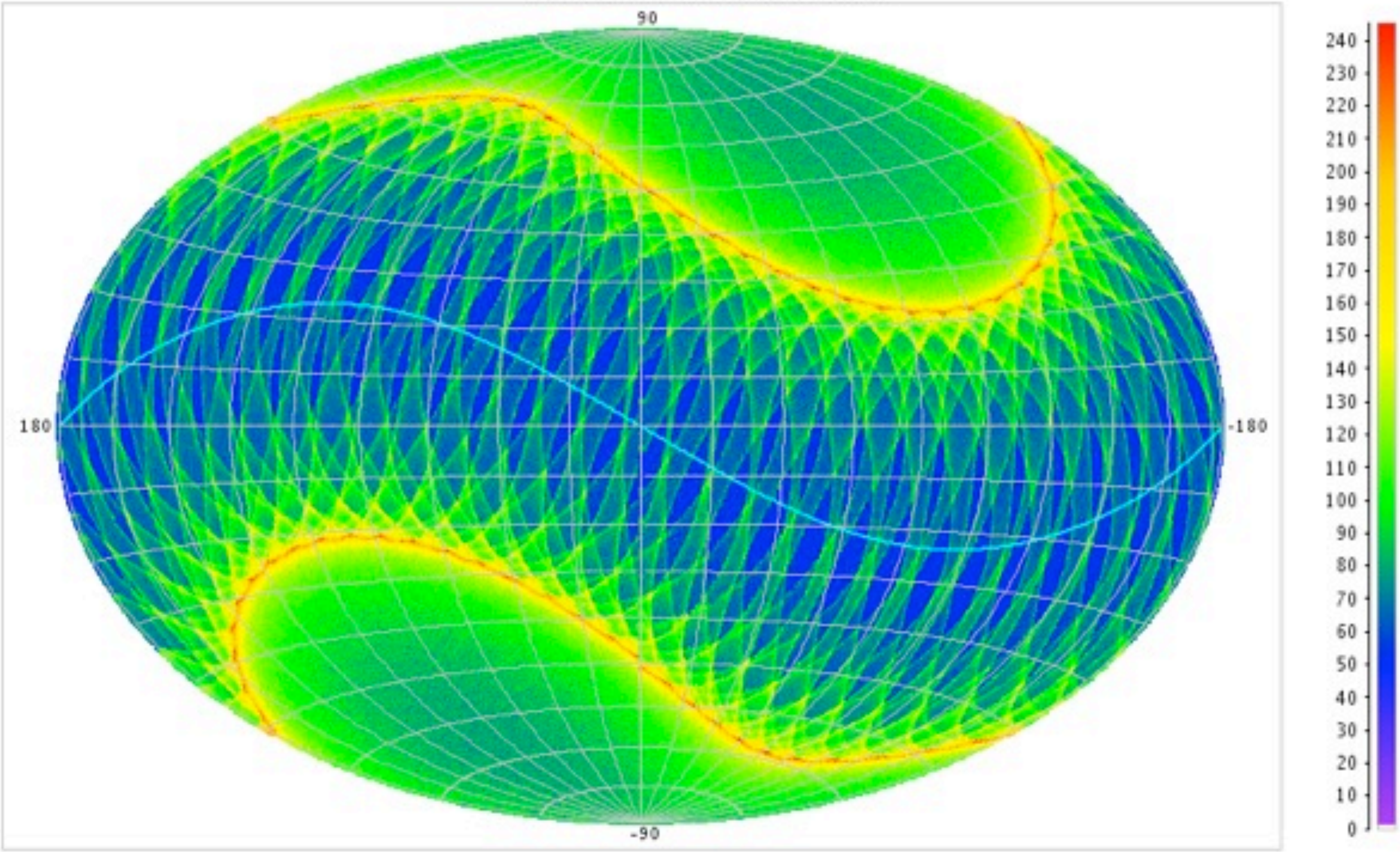


Potential measurements of **a'** at one **z** only

Speed of **z** must be large enough to ensure loop overlap

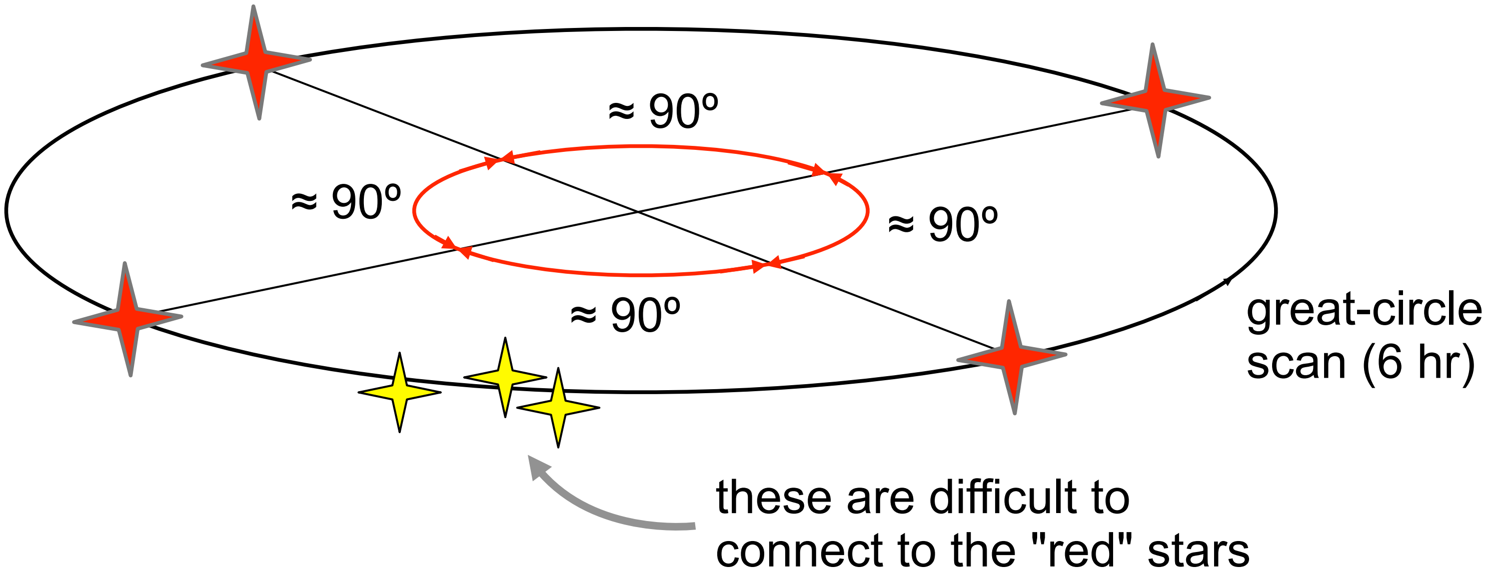
Sky coverage (3/3)

Number of FoV crossings per star (5 yr)



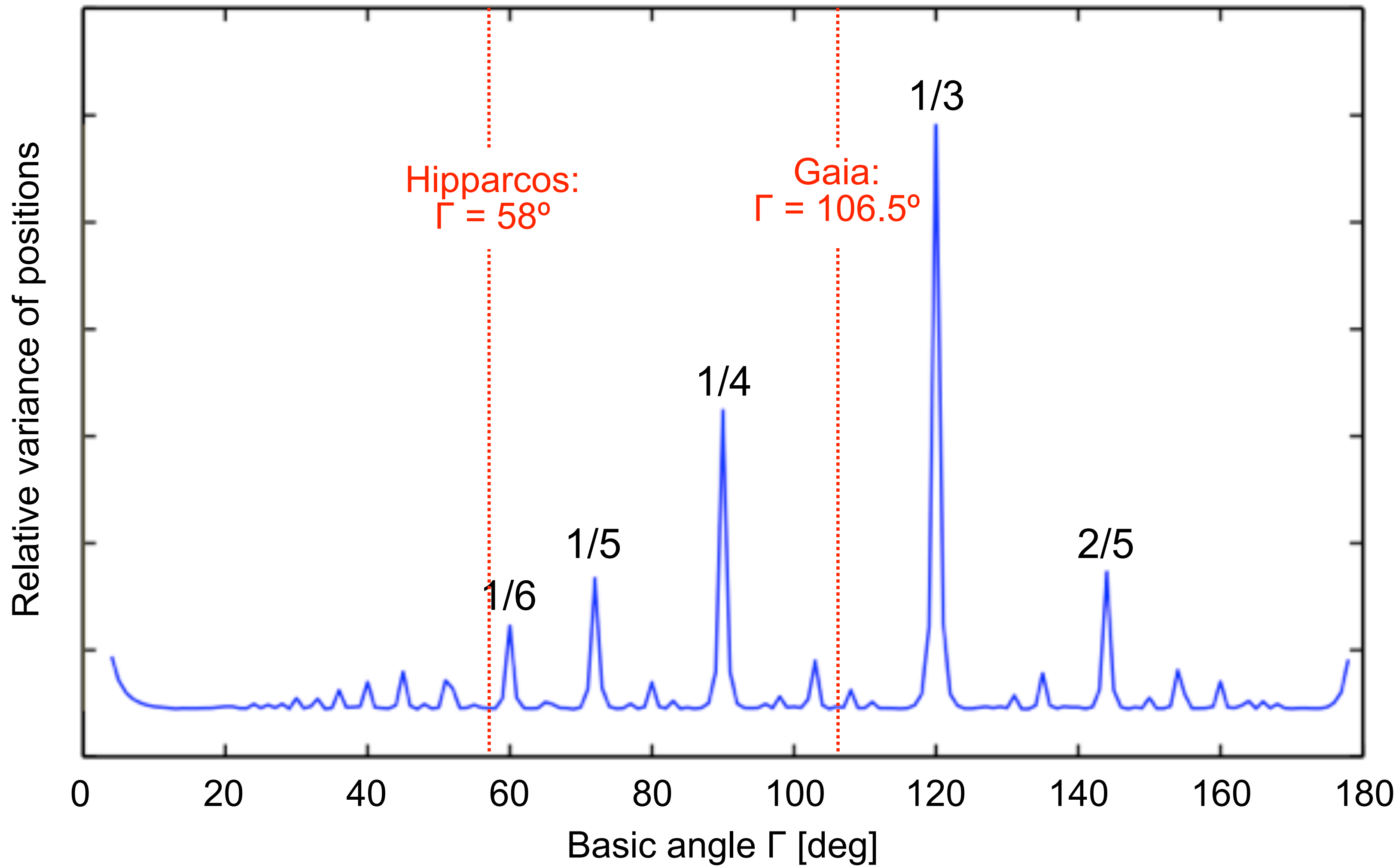
Equatorial projection

Choice of basic angle (Γ)



- Γ = 90º (ideal for parallaxes) gives bad connectivity along great circle
- In fact Γ = 360º × (n/m) should be avoided for small integers n, m

Choice of basic angle: 1D case (great circle)



Credit: S. Nzoke Baman

Choice of basic angle: 2D case (sphere)

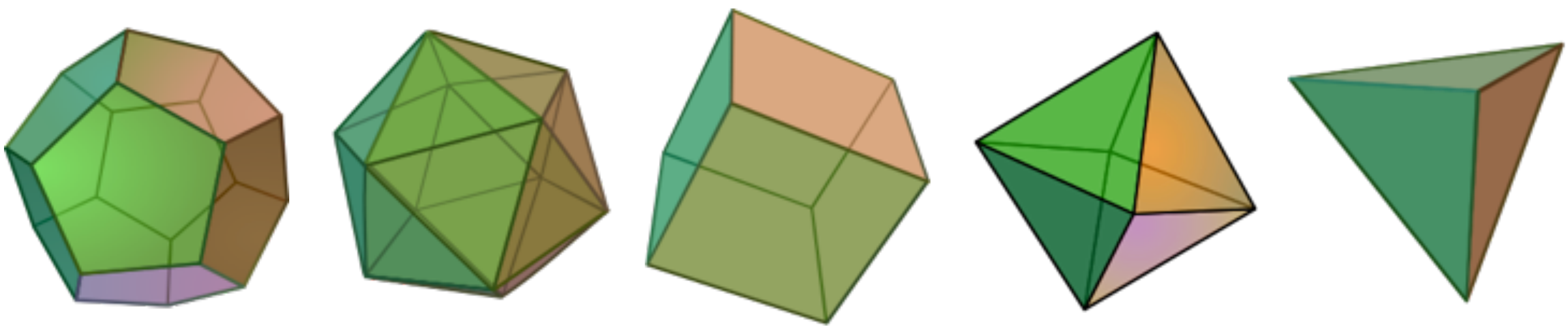
Great-circle scans are not processed in isolation -
the astrometric solution is global (whole celestial sphere)

Question:

Are there any "bad" angles that should be avoided
(equivalent to $360^\circ \times (n/m)$ on the circle)?

The five Platonic solids

The vertices of a Platonic solid define a grid of "equidistant" points on the circumscribed sphere

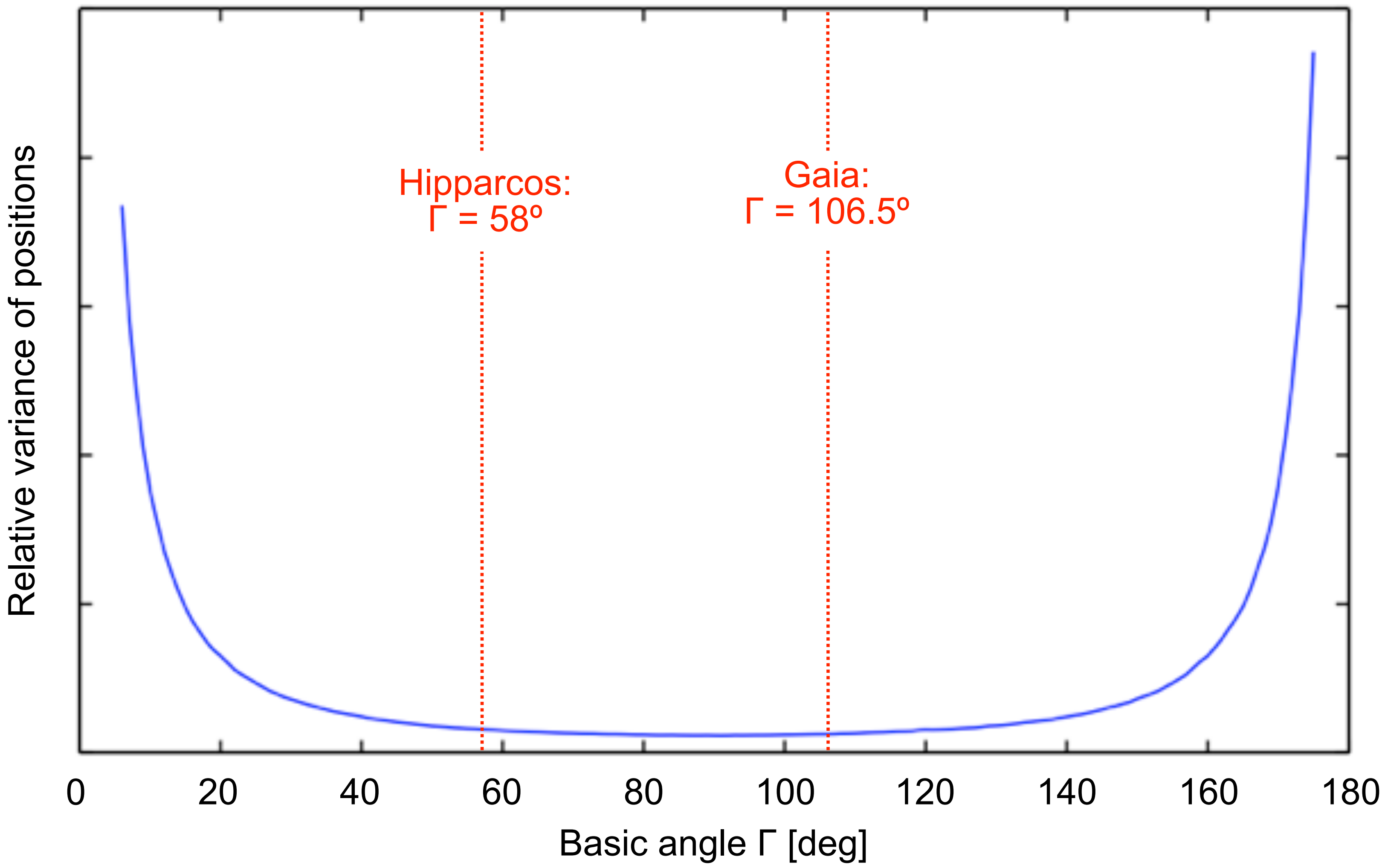


Dodecahedron	Icosahedron	Hexahedron	Octahedron	Tetrahedron
$\phi = 41.81^\circ$	$\phi = 63.43^\circ$	$\phi = 70.53^\circ$	$\phi = 90^\circ$	$\phi = 109.47^\circ$

Source: Wikipedia

Perhaps these angles (ϕ) should be avoided?

Choice of basic angle: 2D case (sphere)



Credit: S. Nzoke Baman

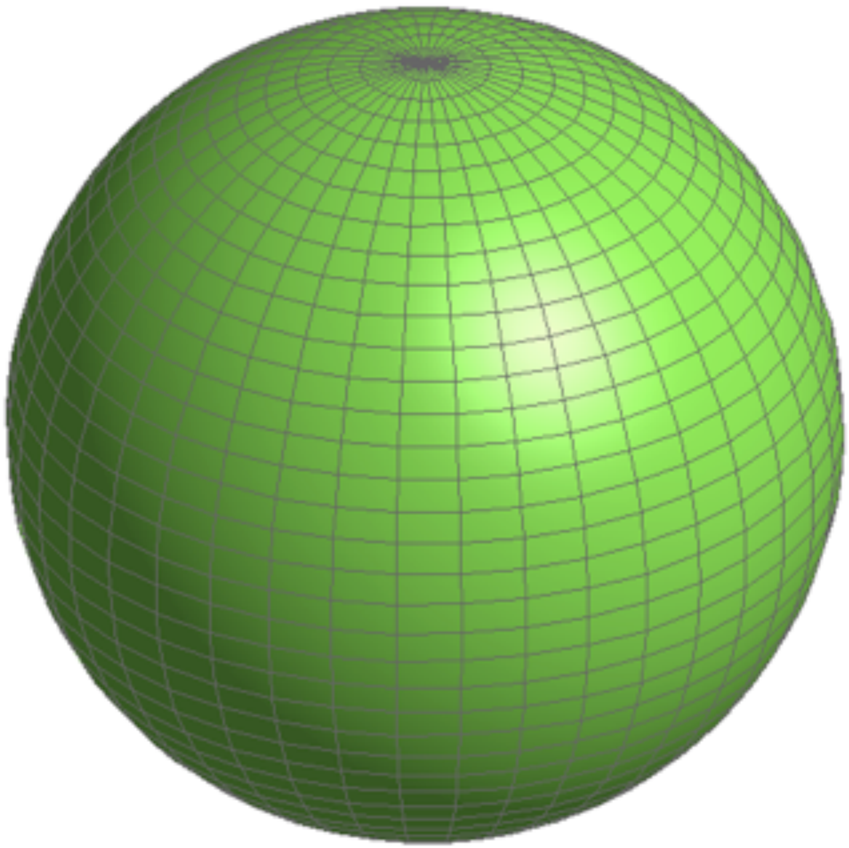
Cf. Makarov (A&A 340, 309, 1998)

Choice of basic angle (Γ)

Conclusions:

- Γ = 90º is optimal from a "global" viewpoint
- Nevertheless a slightly different value (106.5º) was selected for Gaia
- This avoids (n/m) congruence for a partial "great-circle reduction"
- It allows the "One Day Astrometric Solution" (ODAS) as a first-look check of instrument health

Self-calibration



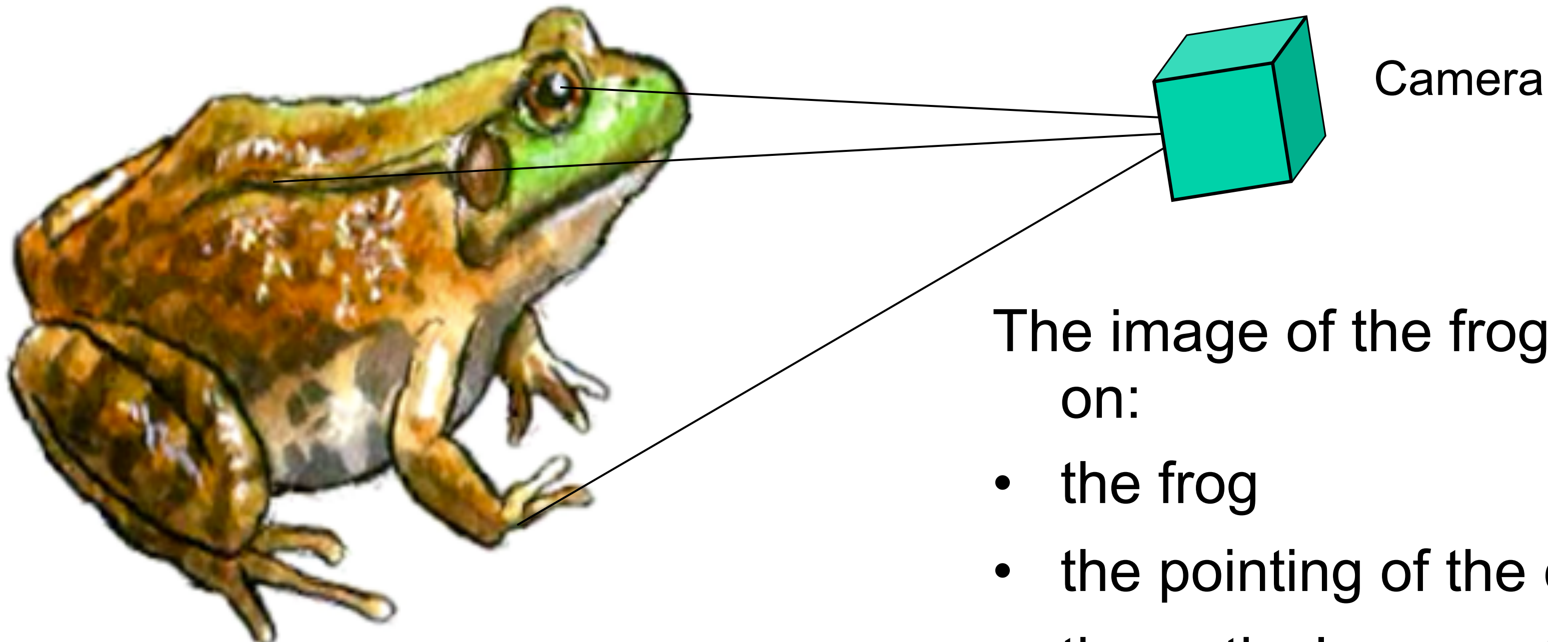
The measurements (observation times) depend on:

- the astrometric parameters of the stars
- the instantaneous pointing of the instrument (attitude)
- the geometrical calibration parameters
- several global parameters (e.g. PPN γ)

These "nuisance parameters" are determined from the same data as the astrometric parameters

Self-calibration

Analogous camera calibration problem in computer vision

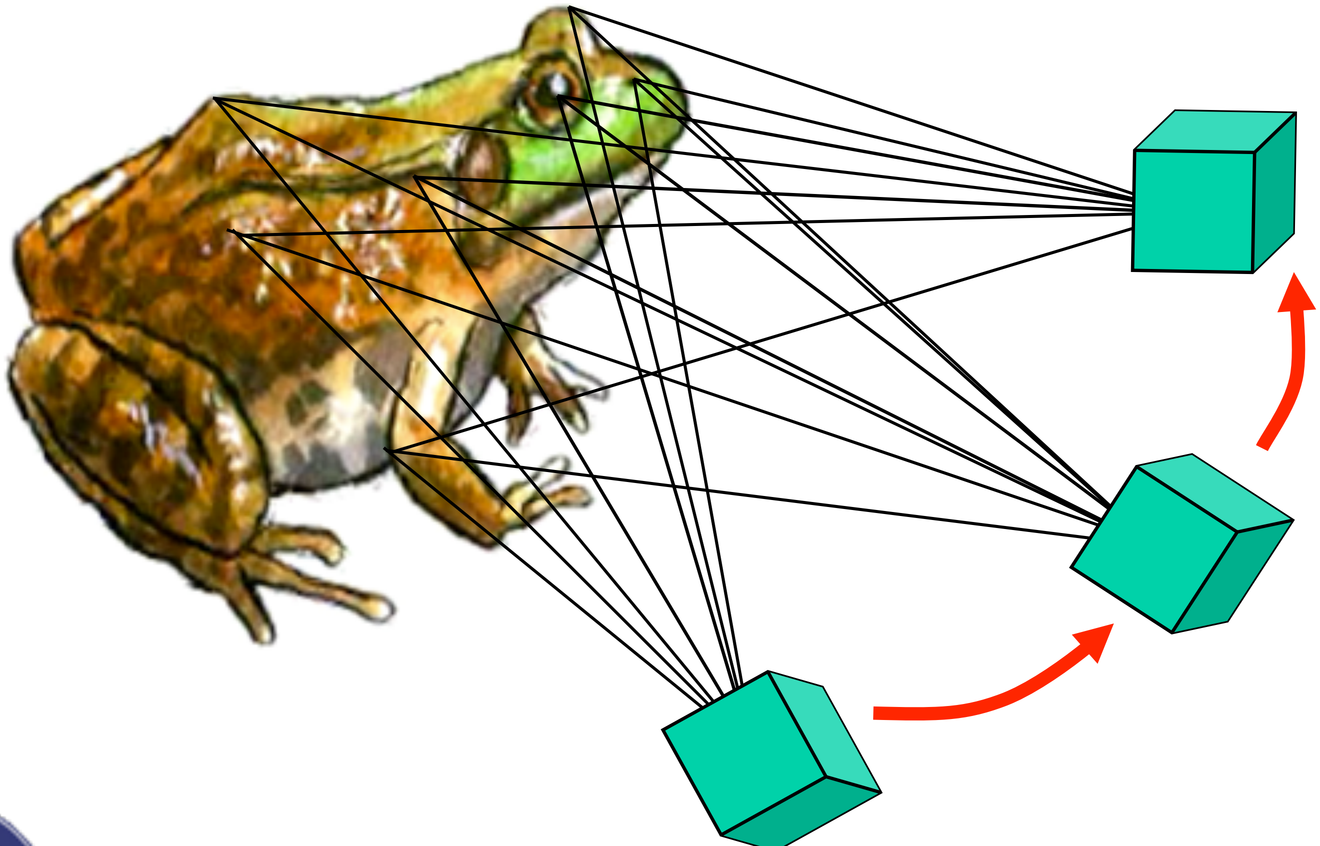


The image of the frog depends on:

- the frog
- the pointing of the camera
- the optical parameters and pixel geometry of the camera

Self-calibration using a moving camera

With ≥ 7 point correspondences and ≥ 3 camera positions, all parameters can be recovered up to a scale factor (Maybank & Faugeras 1992).



The resulting numerical problem

The observation equation for one observation, t_{obs}:

$$\frac{\partial t}{\partial s_i}\Delta s_i + \frac{\partial t}{\partial a_j}\Delta a_j + \frac{\partial t}{\partial c_k}\Delta c_k + \frac{\partial t}{\partial g}\Delta g \simeq t_{\text{obs}} - t_{\text{calc}}(s_i, a_j, c_k, g)$$

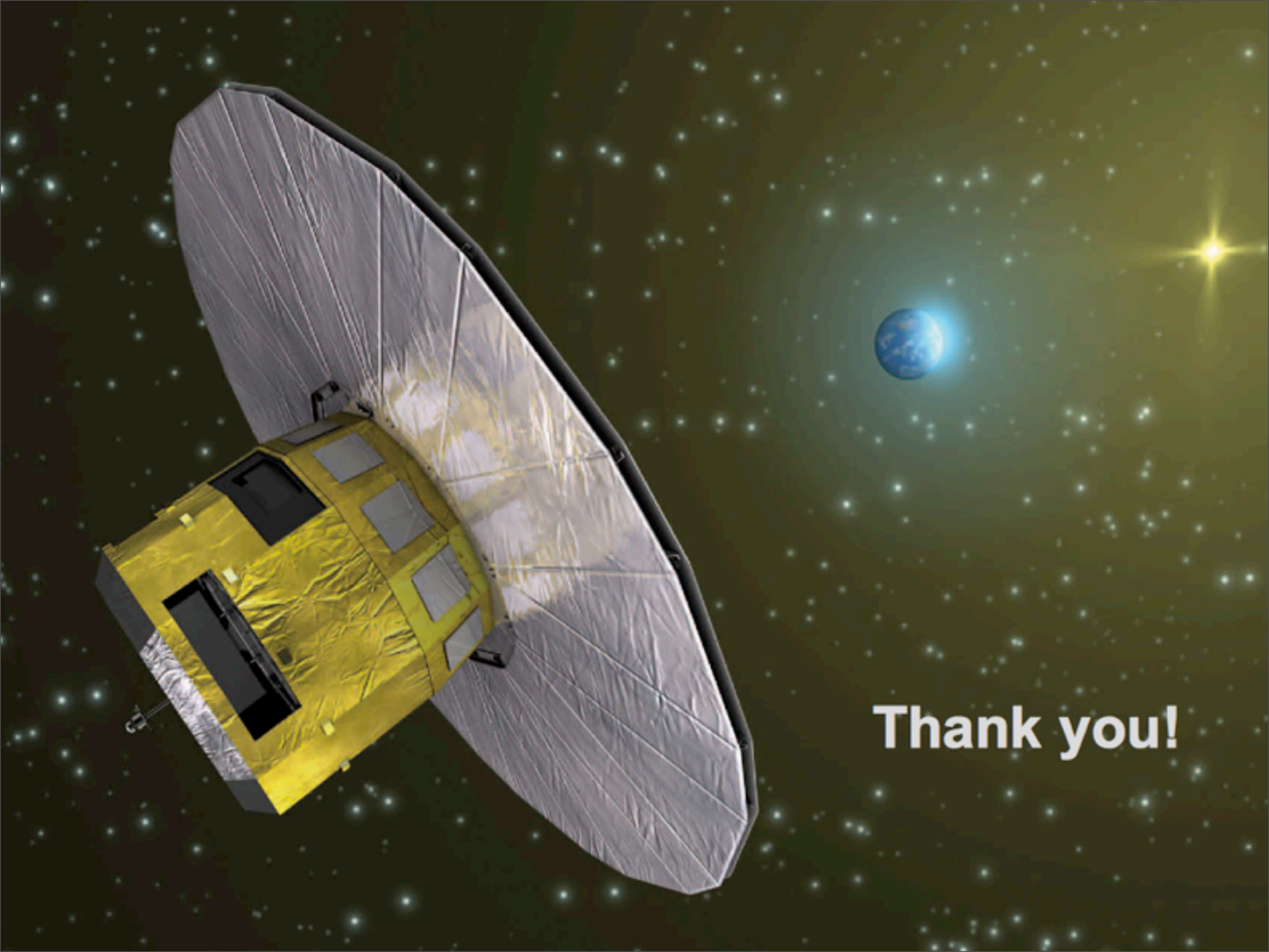
depends on

- the astrometric parameters s_i of the star (i)
- the attitude parameters a_j for the relevant time interval (j)
- the calibration parameters c_k for the relevant CCD, FoV, etc (k)
- the global parameters g

Every observation carries information on
some astrometric, attitude and calibration parameters

Conclusions

- The goal: an astrometric catalogue with **negligible systematic errors** (undistorted position/proper motion grid, absolute parallaxes)
- Measurements are **differential** w.r.t. the spinning instrument
- Astrometric accuracy on a global scale requires differential measurements over **large angles** (~ 90º)
- 1D measurements, basic angle, scanning law are carefully designed to allow **self-calibration**
- Self-calibration principle necessitates a **global reduction** of all the observations w.r.t. the astrometric, attitude and calibration parameters
- The entanglement of all data makes it **a hard numerical problem** - but it also makes the solution strong!


Thank you!