Monitoring the quality of the astrometric solution

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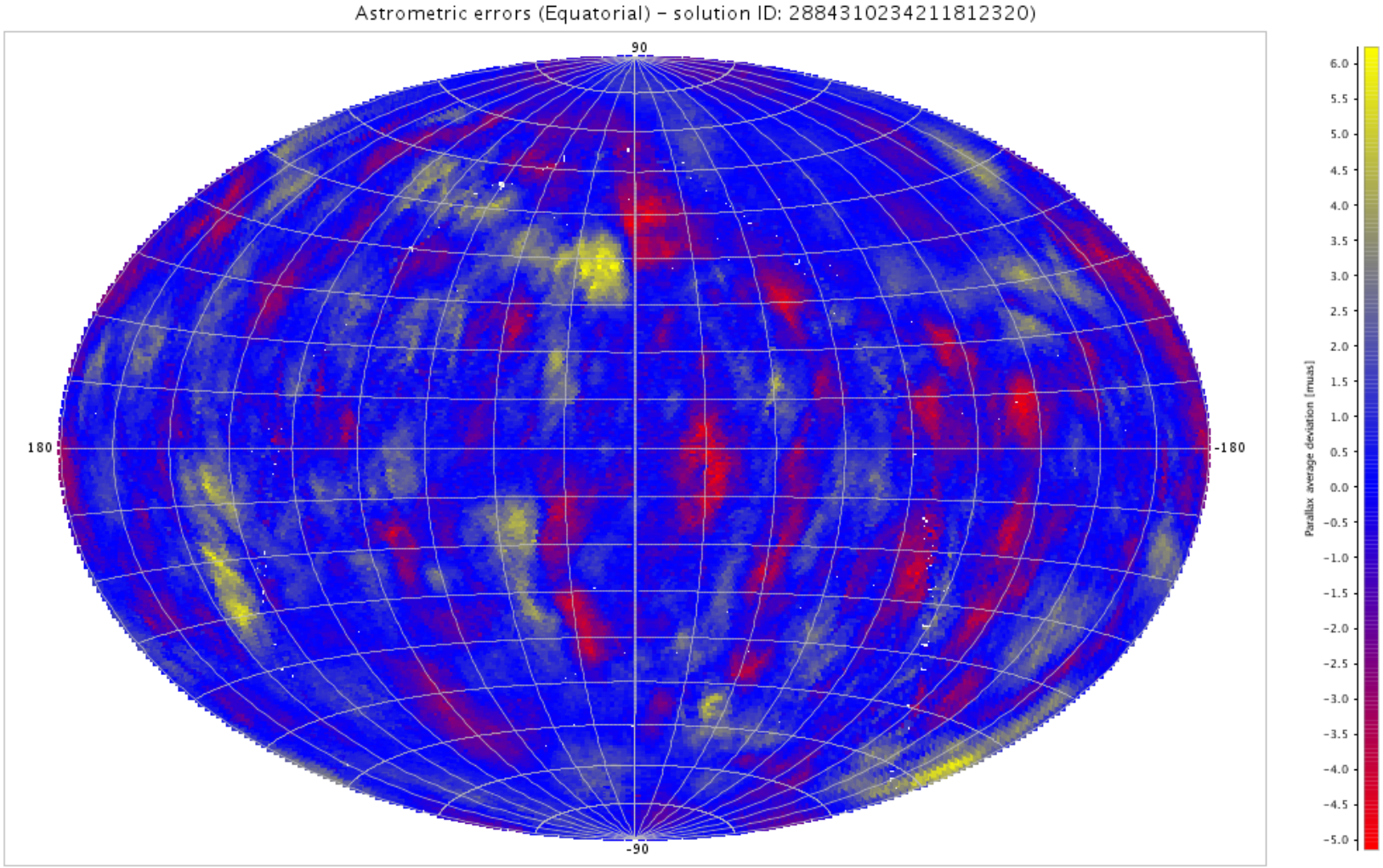
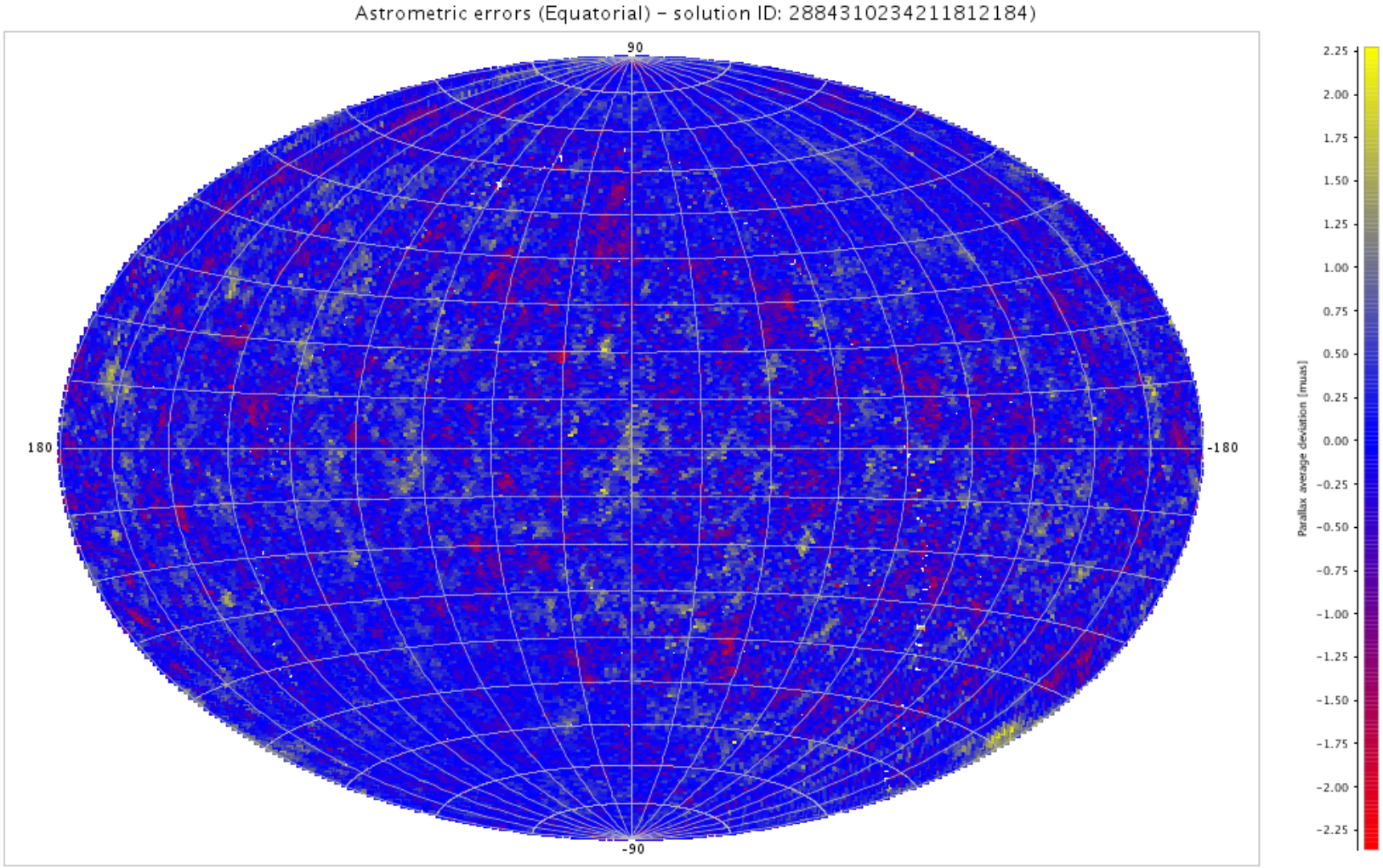
ELSA conference
Gaia: at the frontiers of astrometry
Sèvres, June 2010

Astrometric error map: before



By courtesy of Uwe Lamer

Astrometric error map: after



By courtesy of Uwe Lamer

Gaia's DPAC: an iterative process

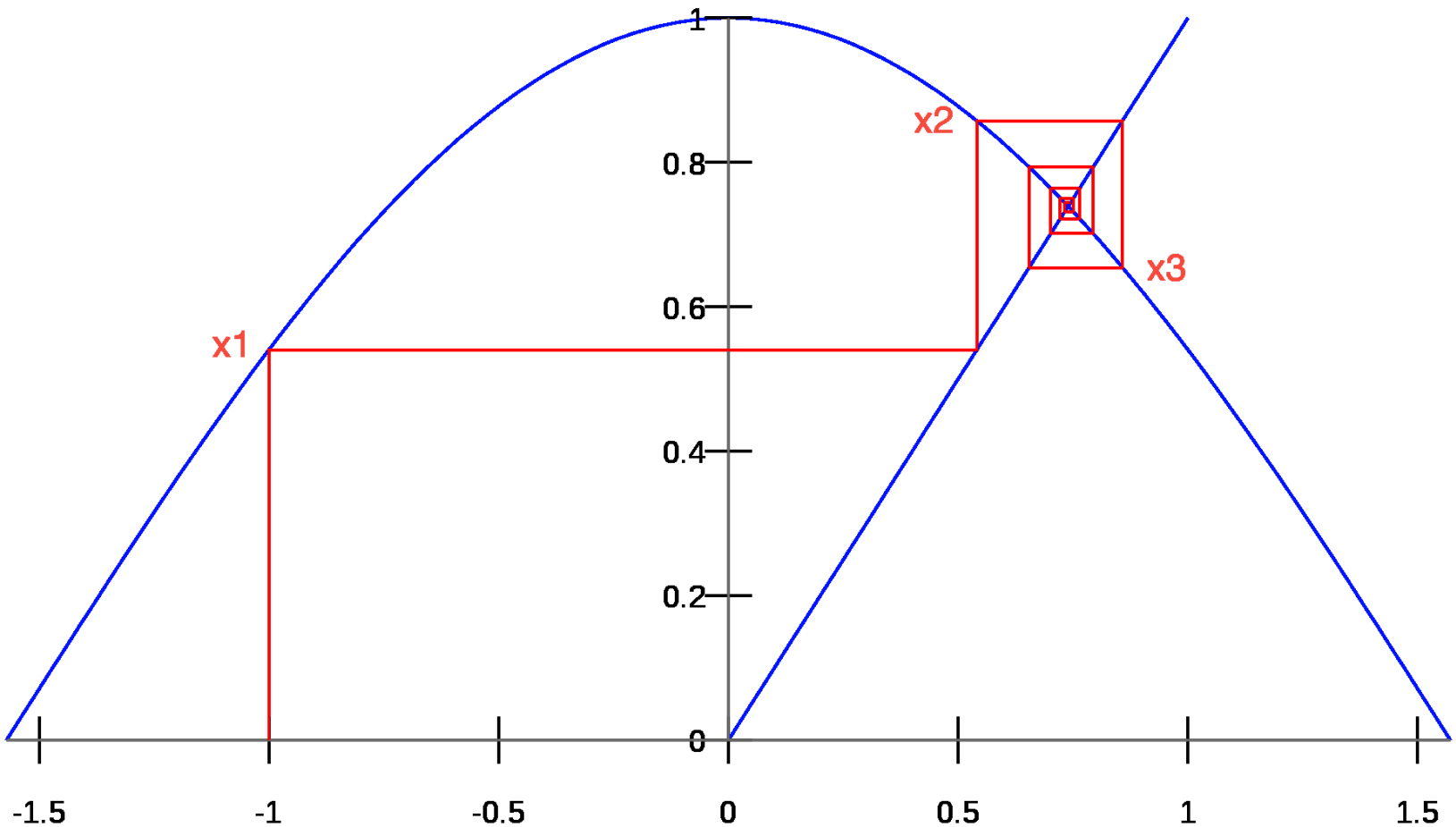
CU3: core processing

- IDT-FL (daily)
- AGIS (each 3 monthes)
- IDU (each 3 monthes)
- AGIS
- ...

Iterative scheme

$$f(x) = 0$$

Iterative scheme: exemple cosinus fixed point



Iterative scheme: conjugate gradient in AGIS

A residual reducing method,

i.e. converging!

Monitoring errors using simulated data

- True values $\overline{x}$: the ones used to generate true observations.
- Noisy observations: the true observations plus Gaussian noises.
- Computed values x_l: the ones computed from noisy observations.
- Errors $e_l = x_l - \overline{x}$: the difference between computed and true!

Error maps:

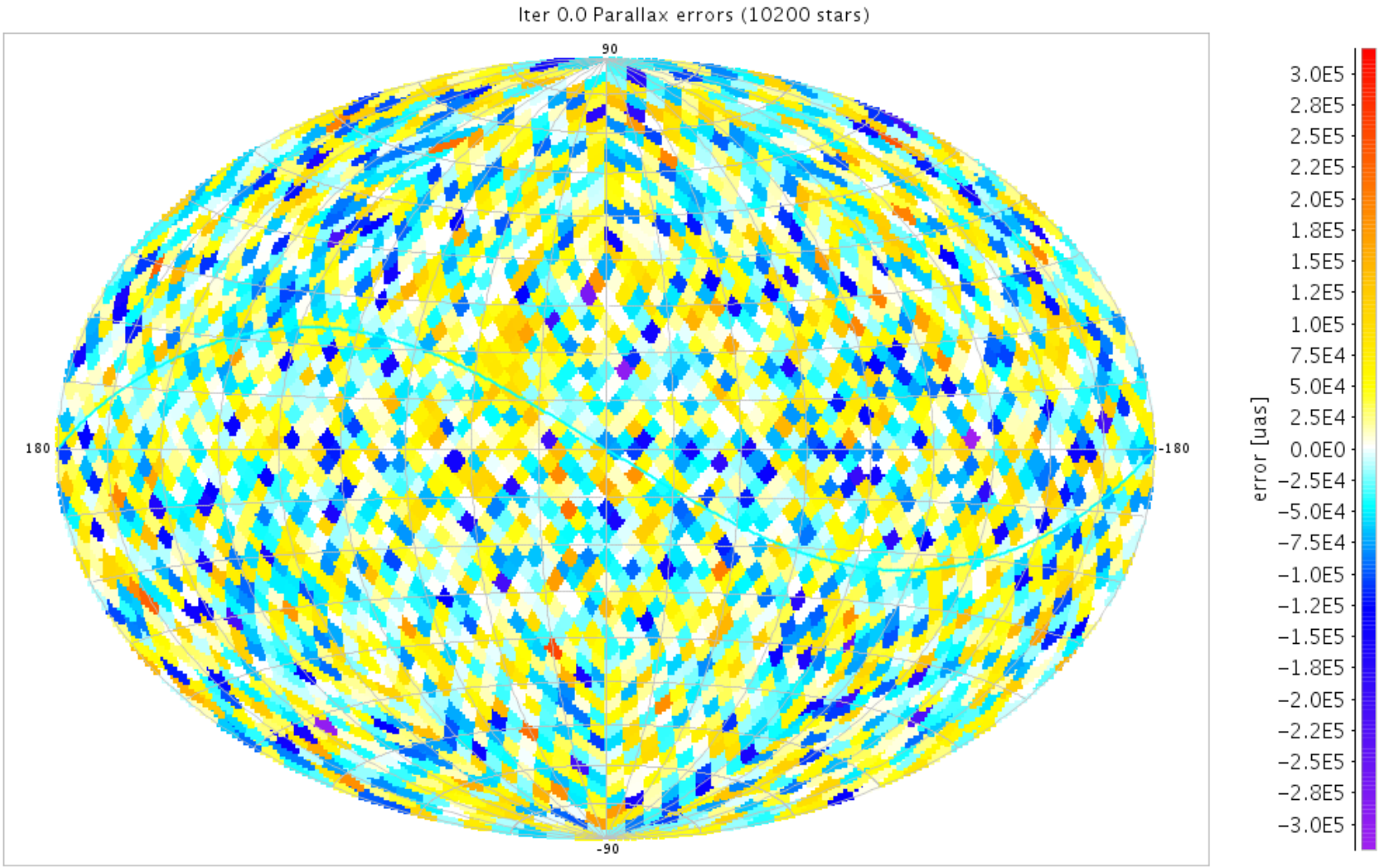
- galactic coordinates,
- median error for source parameters in beans.

About the Hipparcos Pleiades distance discrepancy

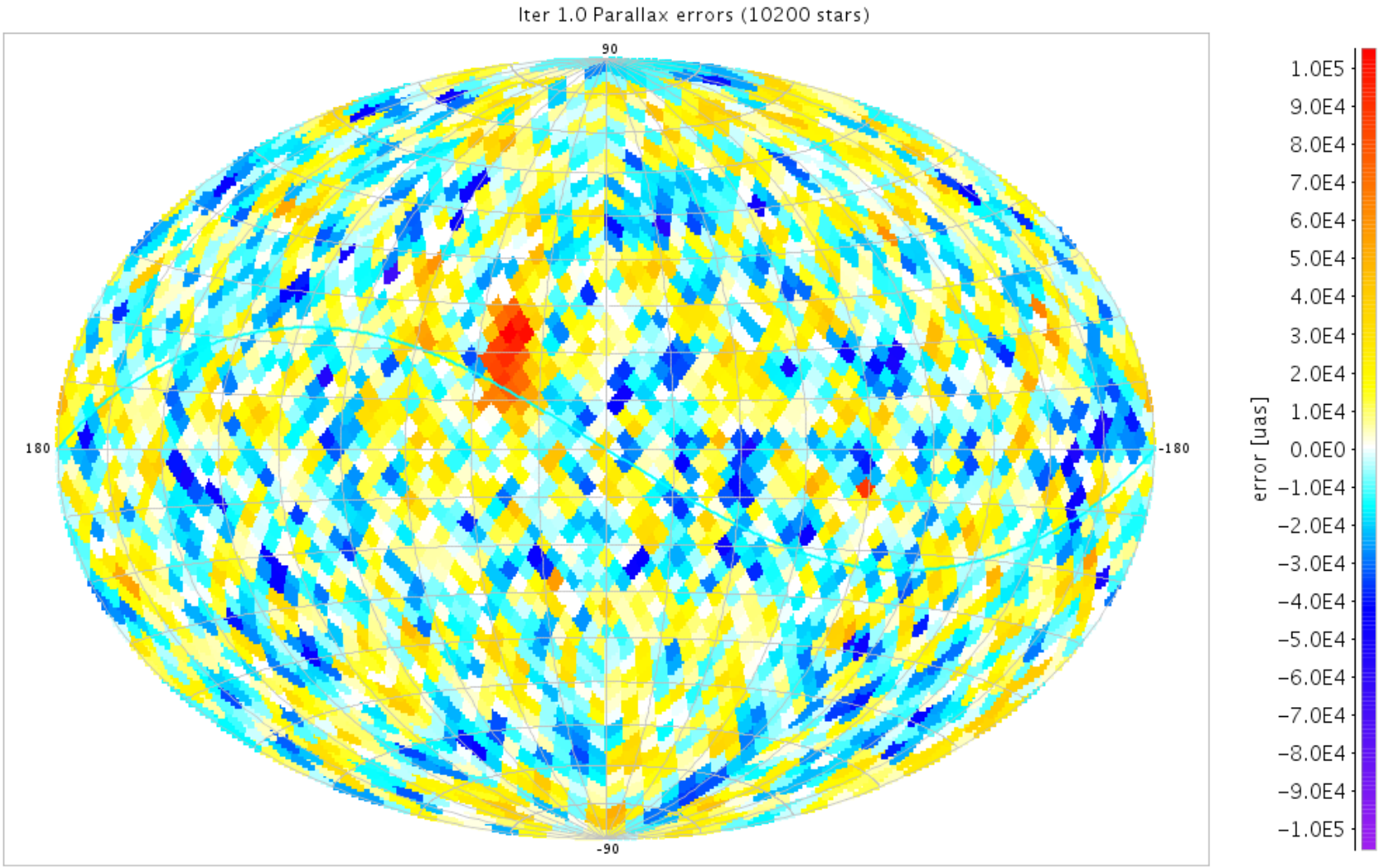
- Hipparcos (van Leeuwen, 2007): 122.2 ± 2 pc.
- Main-sequence fitting (An *et al.*, 2007): 135.5 ± 3 pc.

see
Parallaxes and proper motions for 20 open clusters as based on the new Hipparcos catalogue
F. van Leeuwen
Astronomy and Astrophysics 497 1 (2009) 209-242

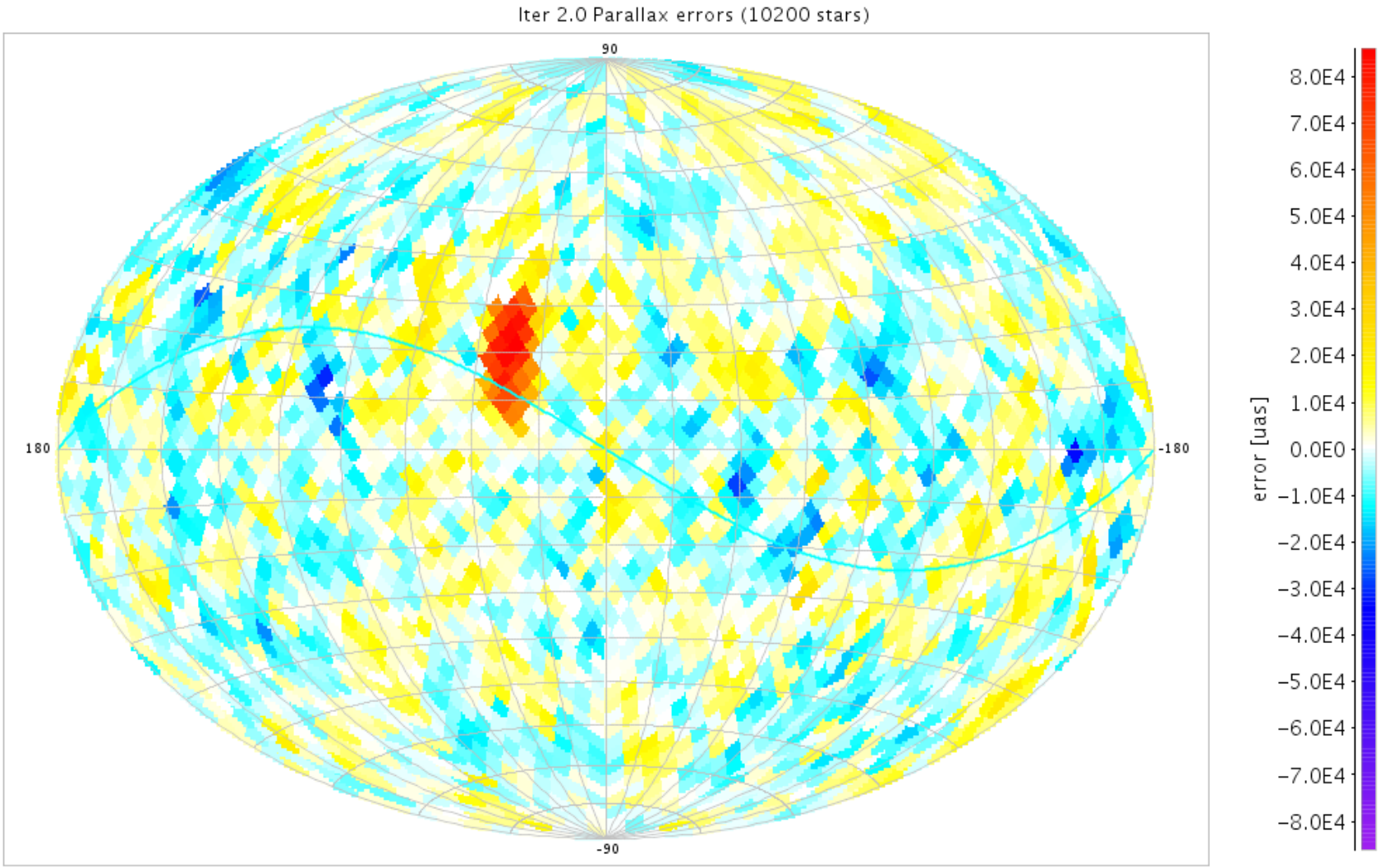
10200 sources, initialparallax error maps



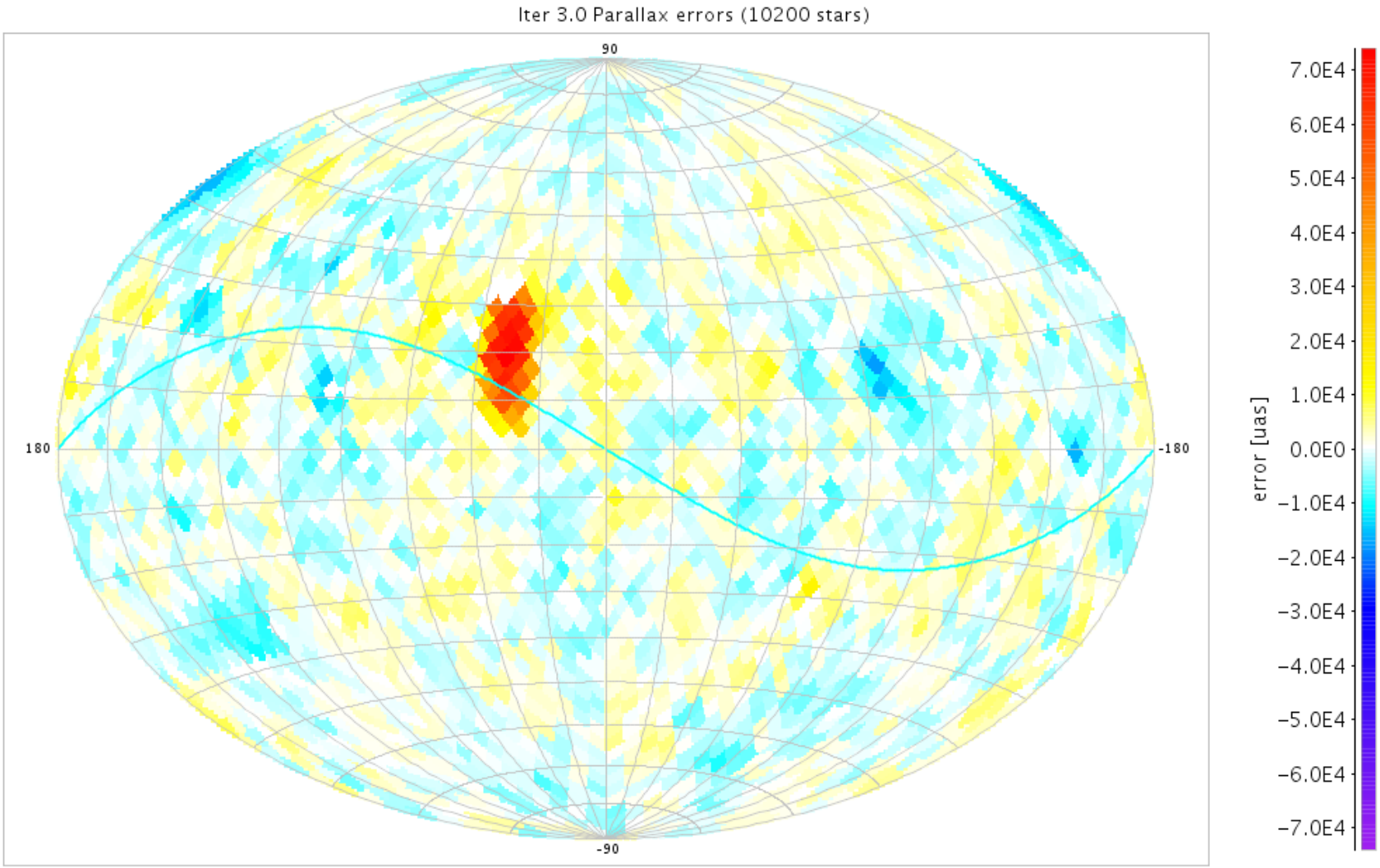
CG: parallax error maps iteration 1



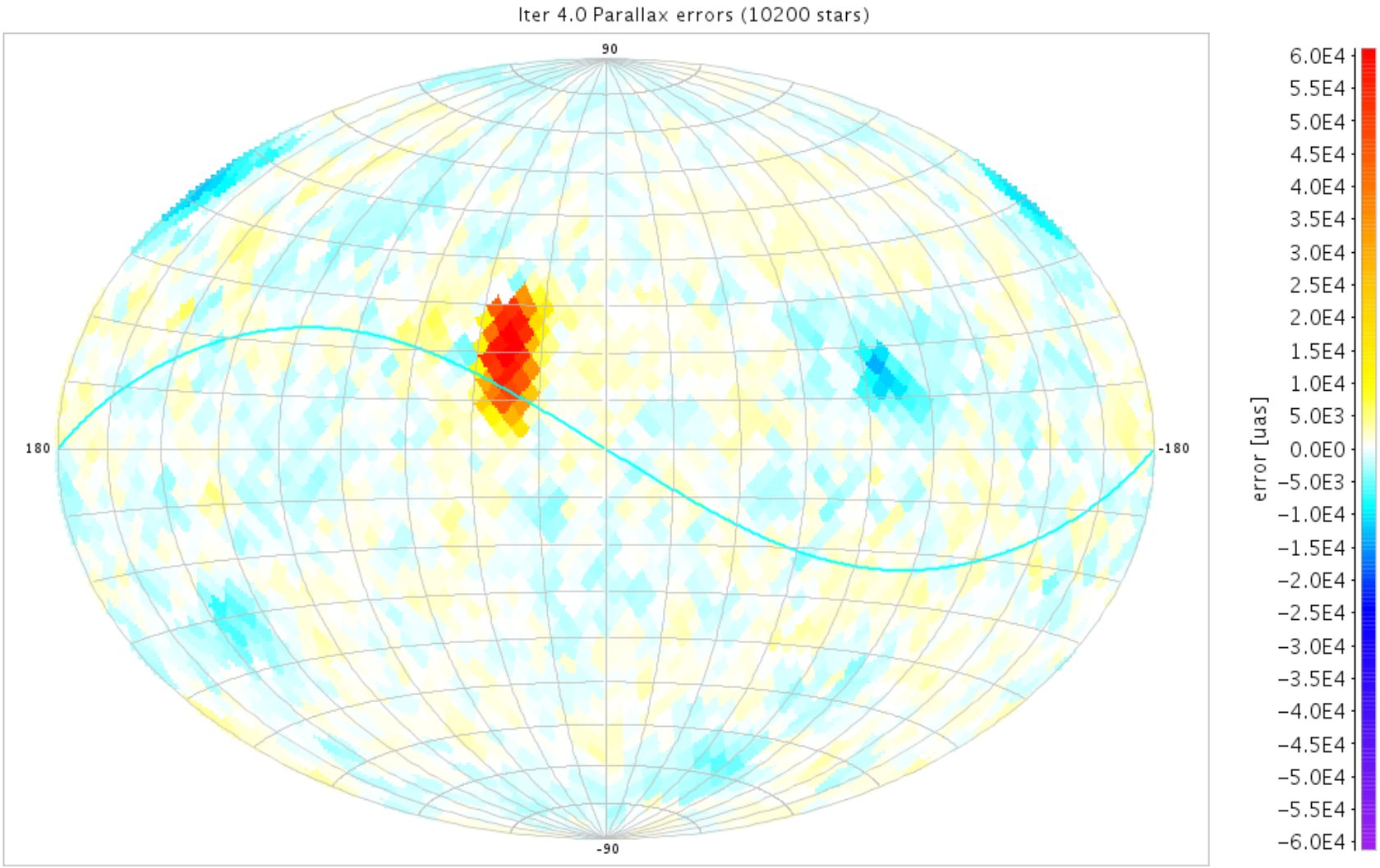
CG: parallax error maps iteration 2



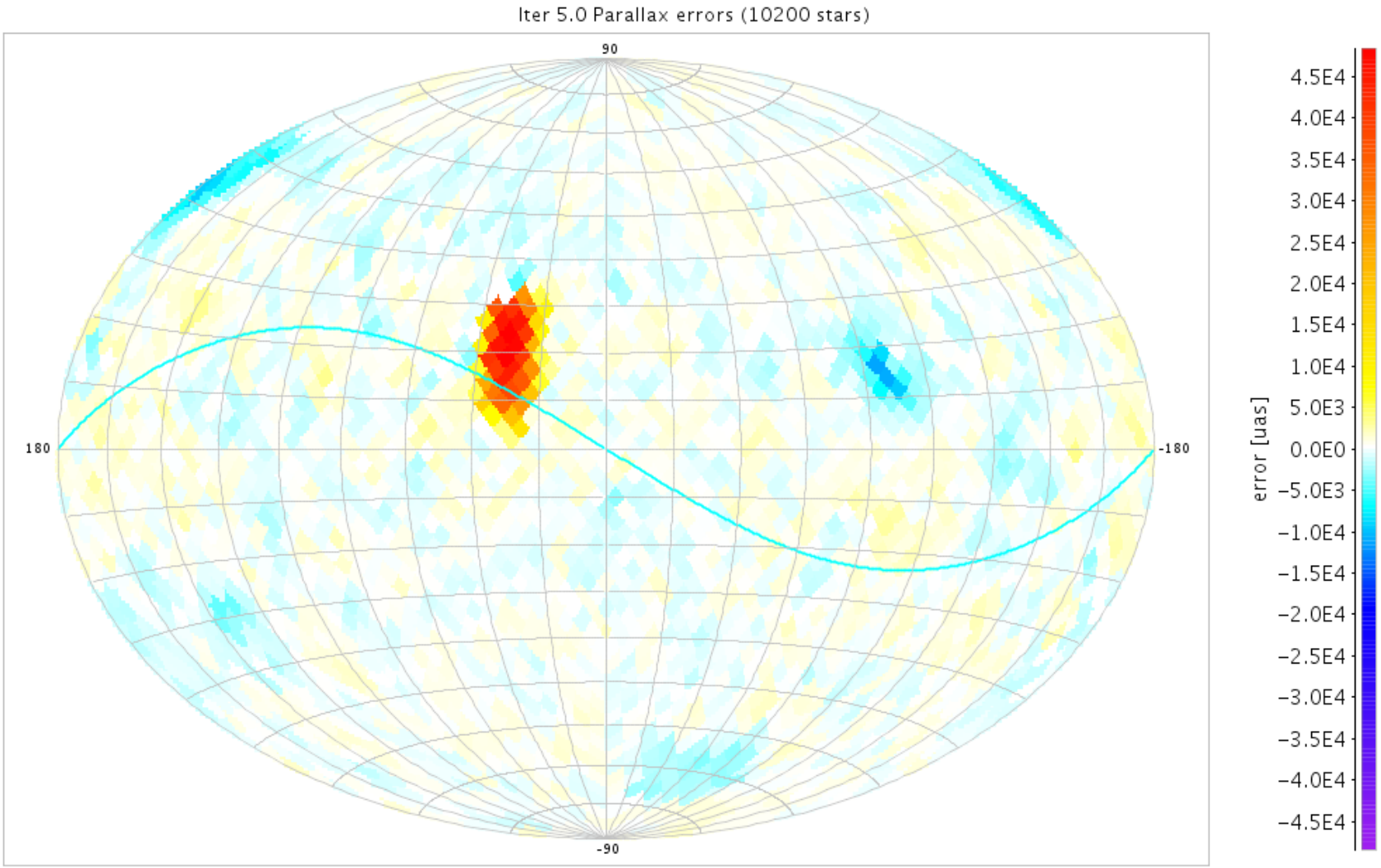
CG: parallax error maps iteration 3



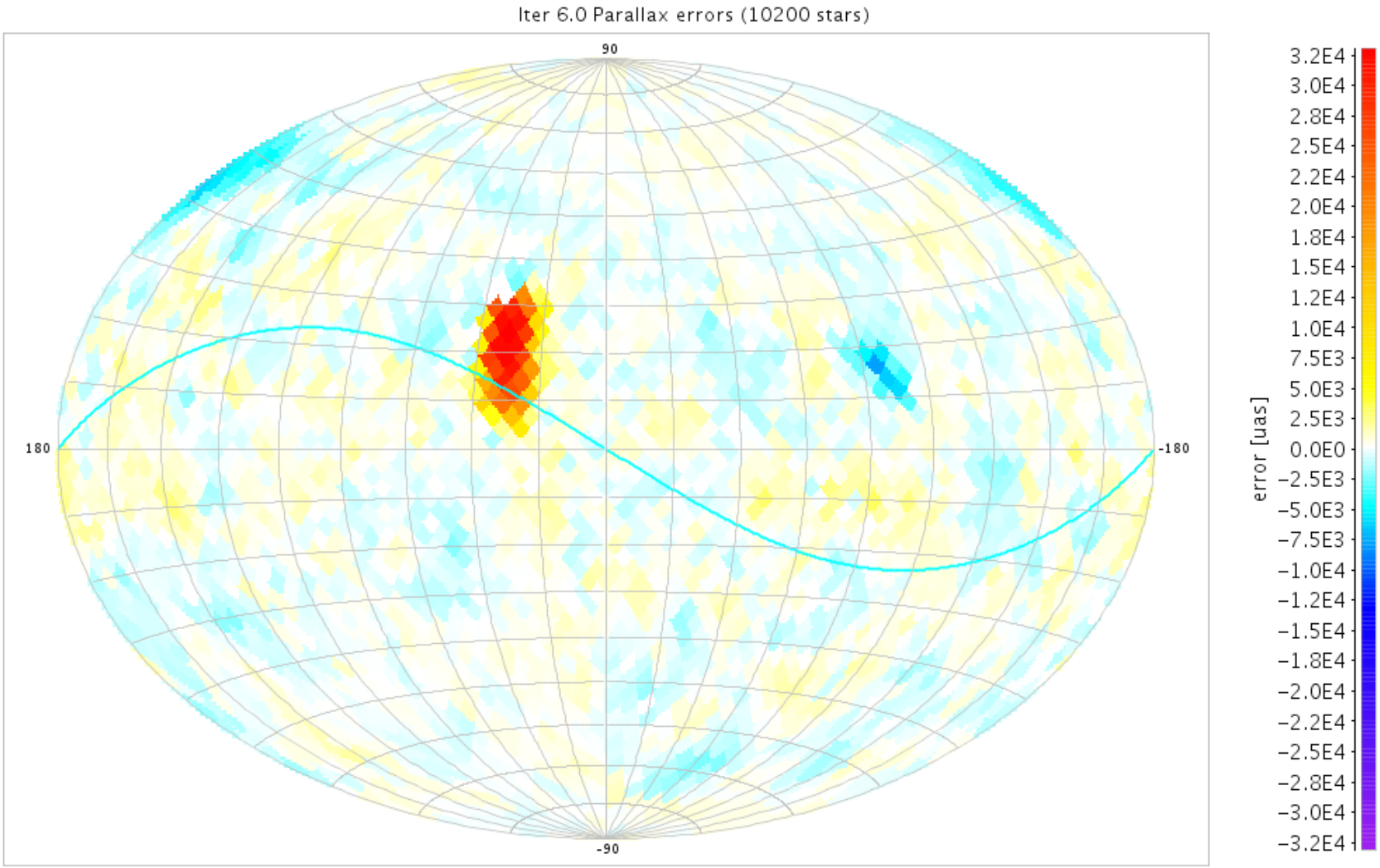
CG: parallax error maps iteration 4



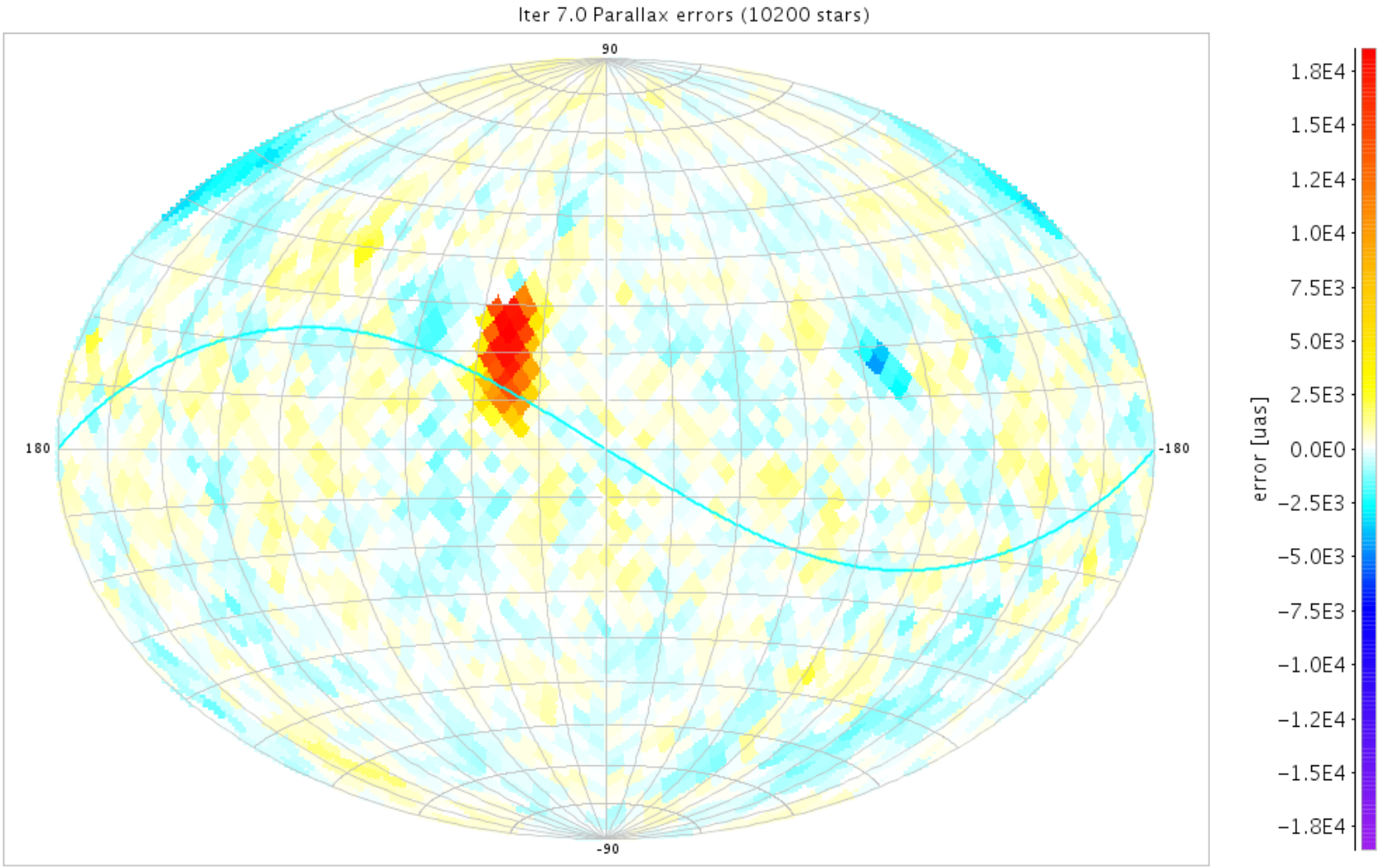
CG: parallax error maps iteration 5



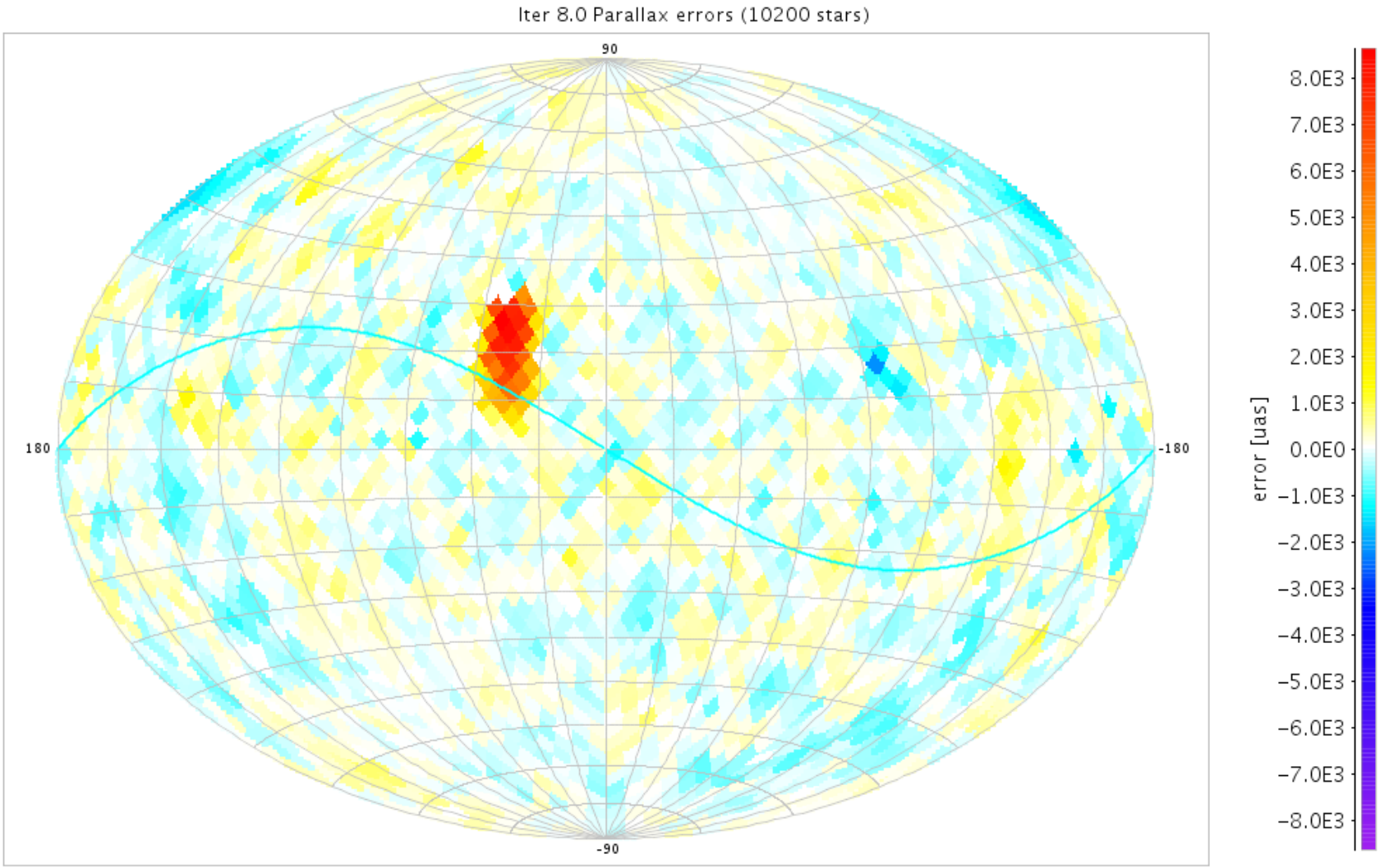
CG: parallax error maps iteration 6



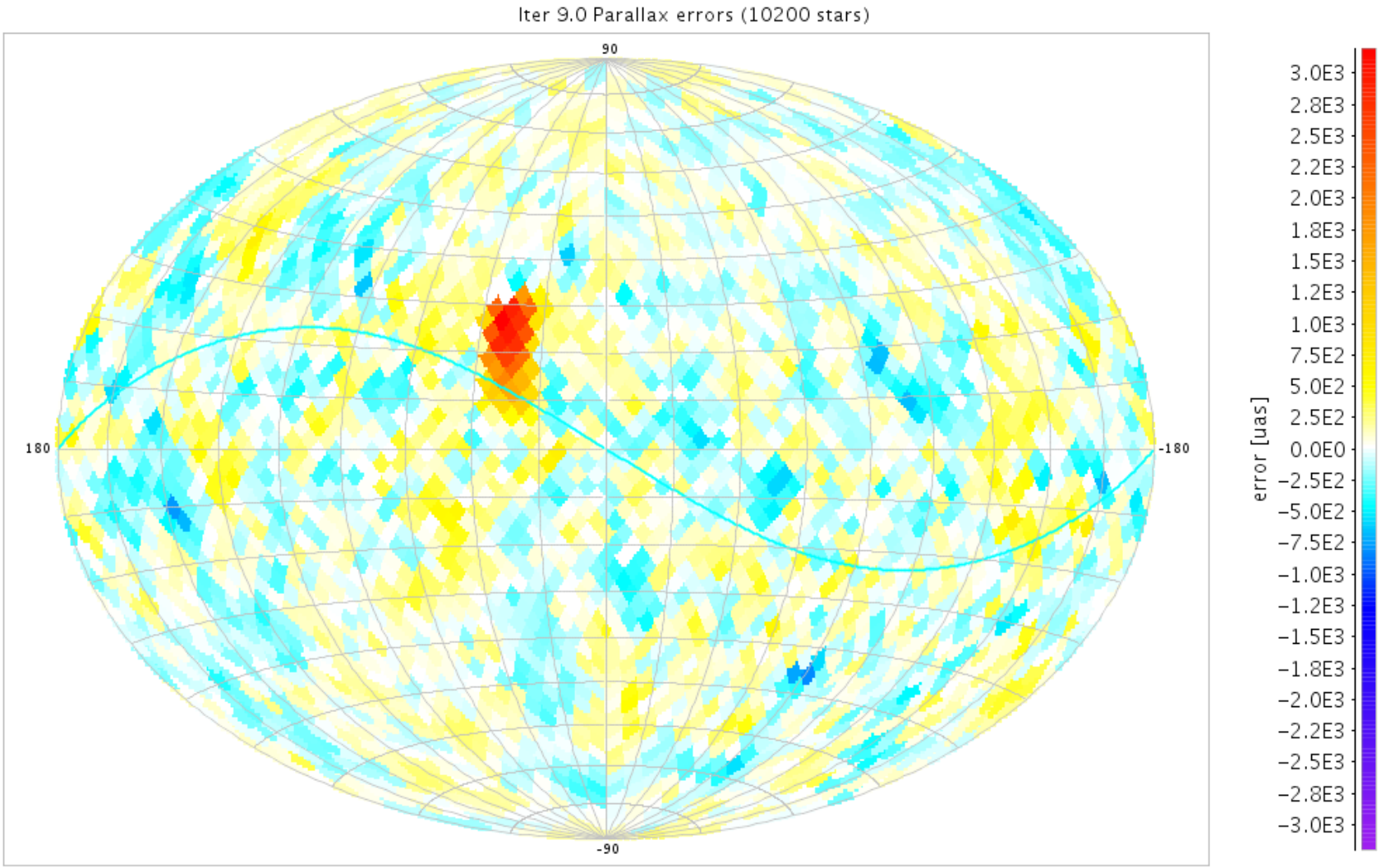
CG: parallax error maps iteration 7



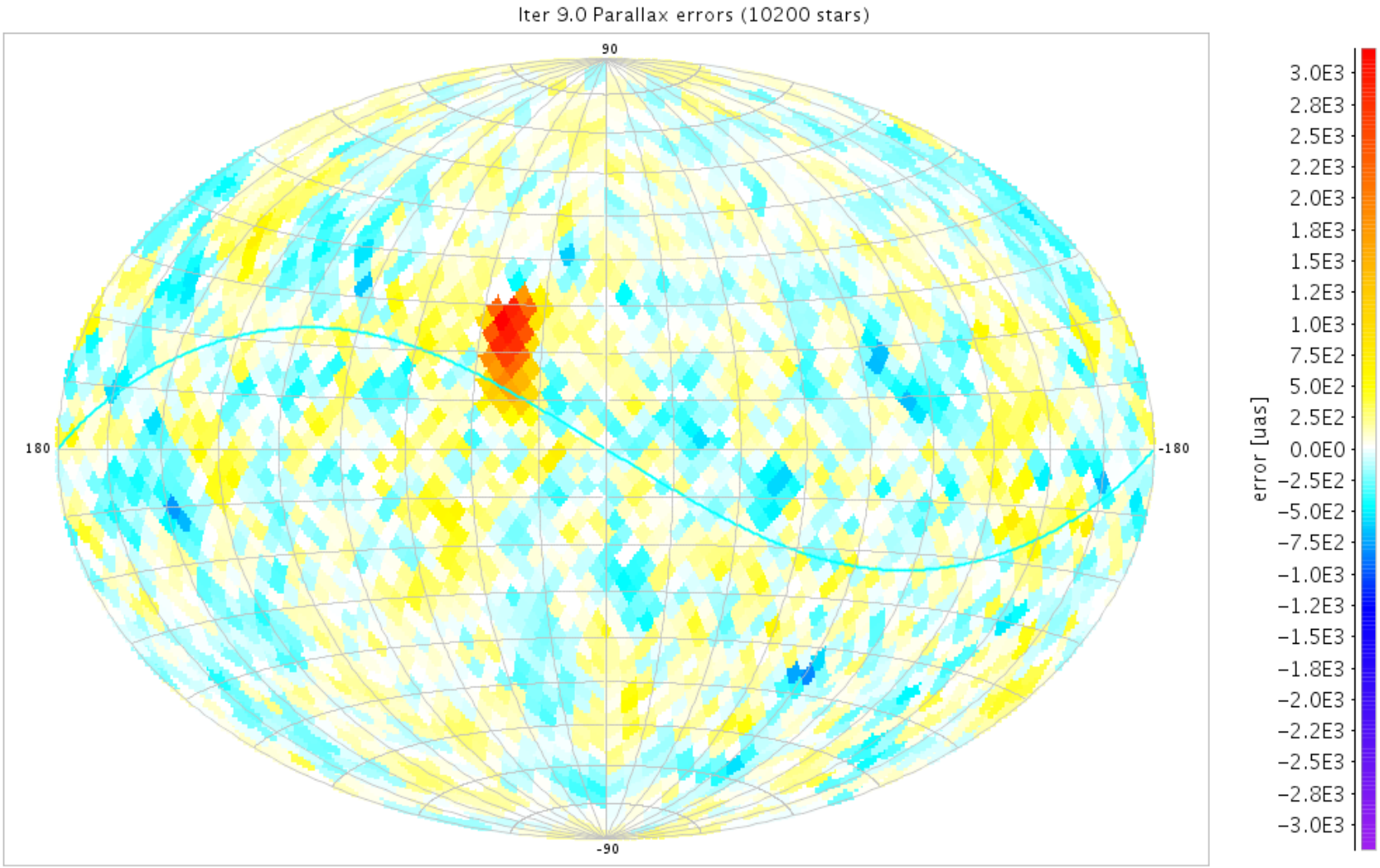
CG: parallax error maps iteration 8



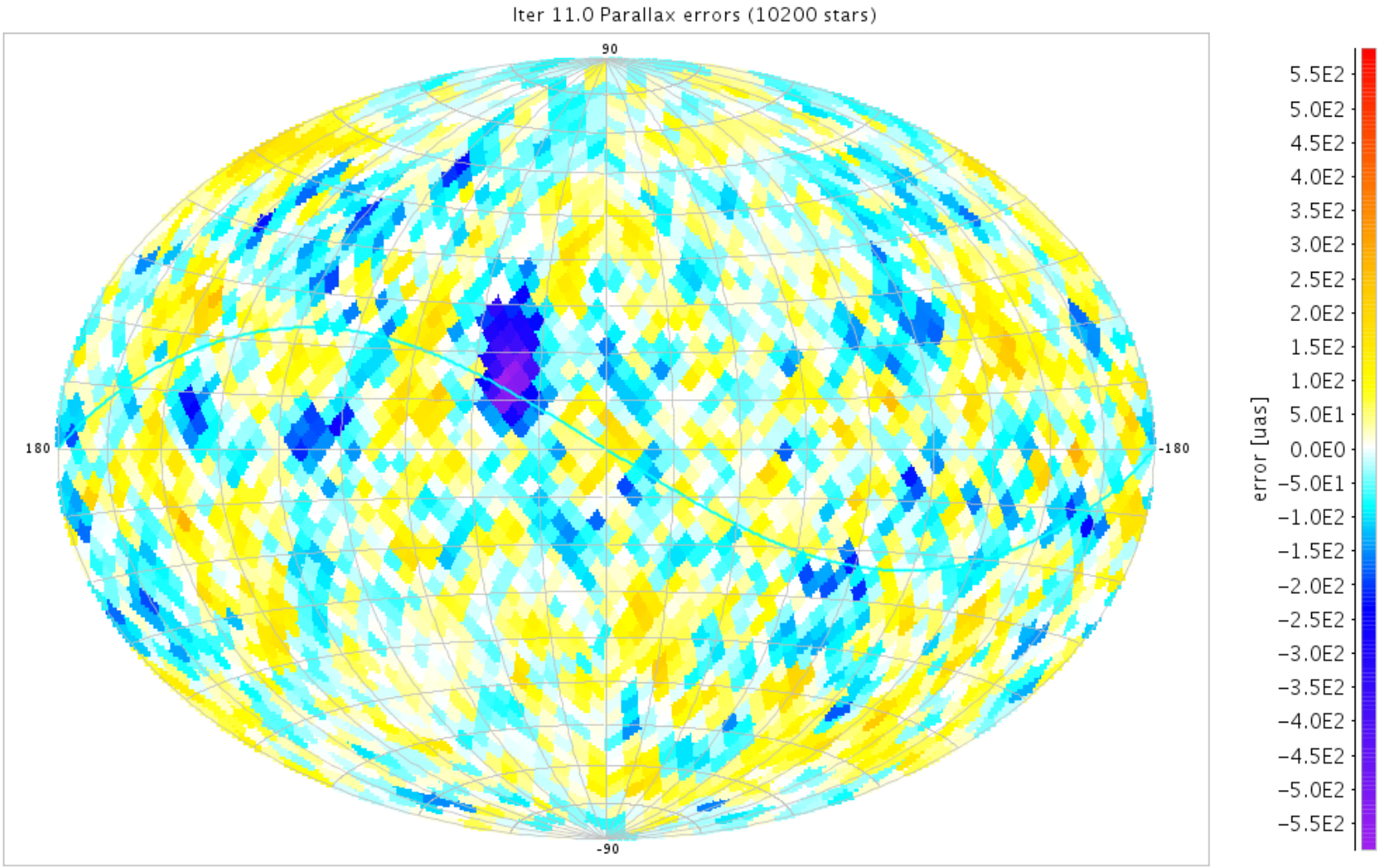
CG: parallax error maps iteration 9



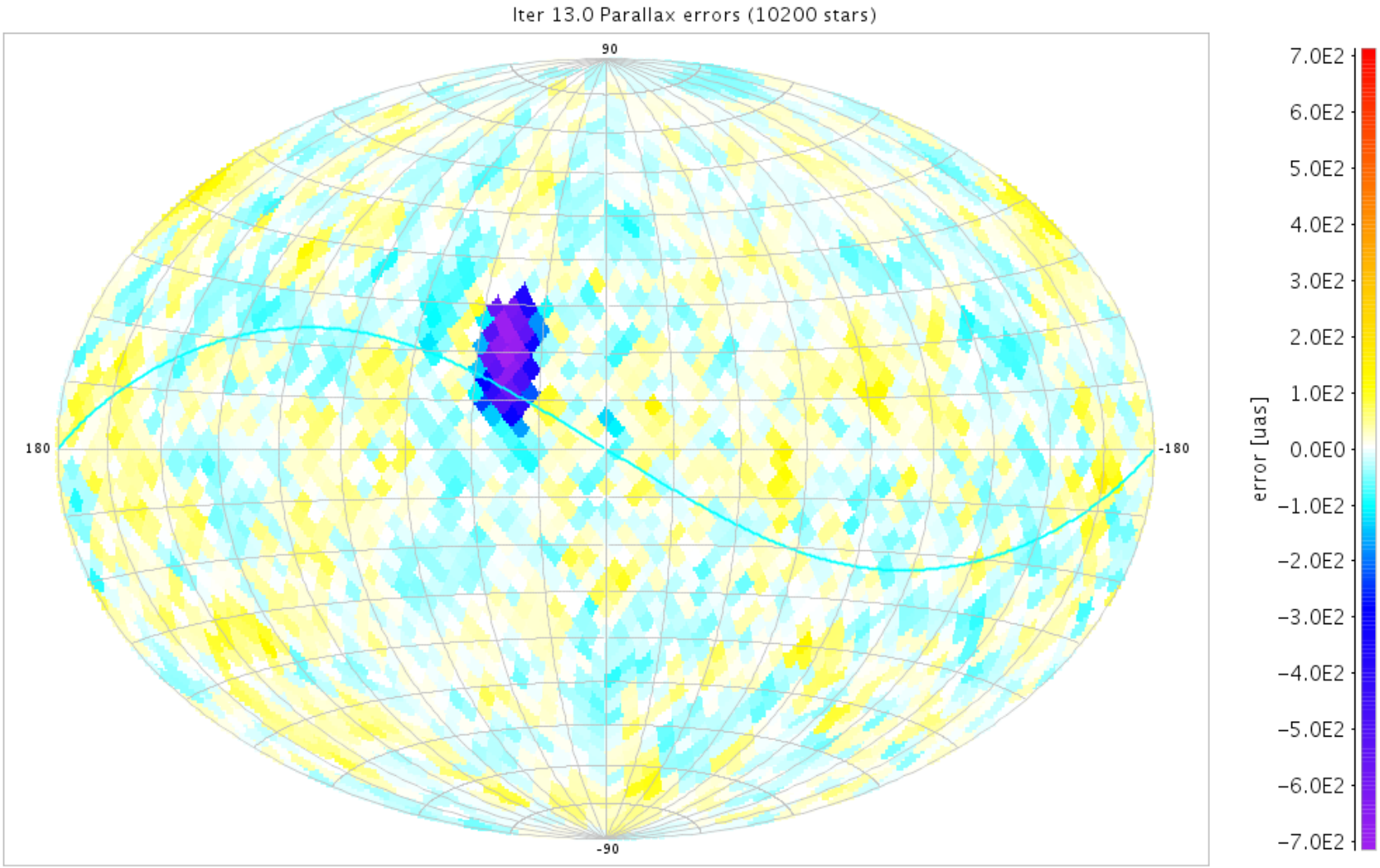
CG: parallax error maps iteration 10



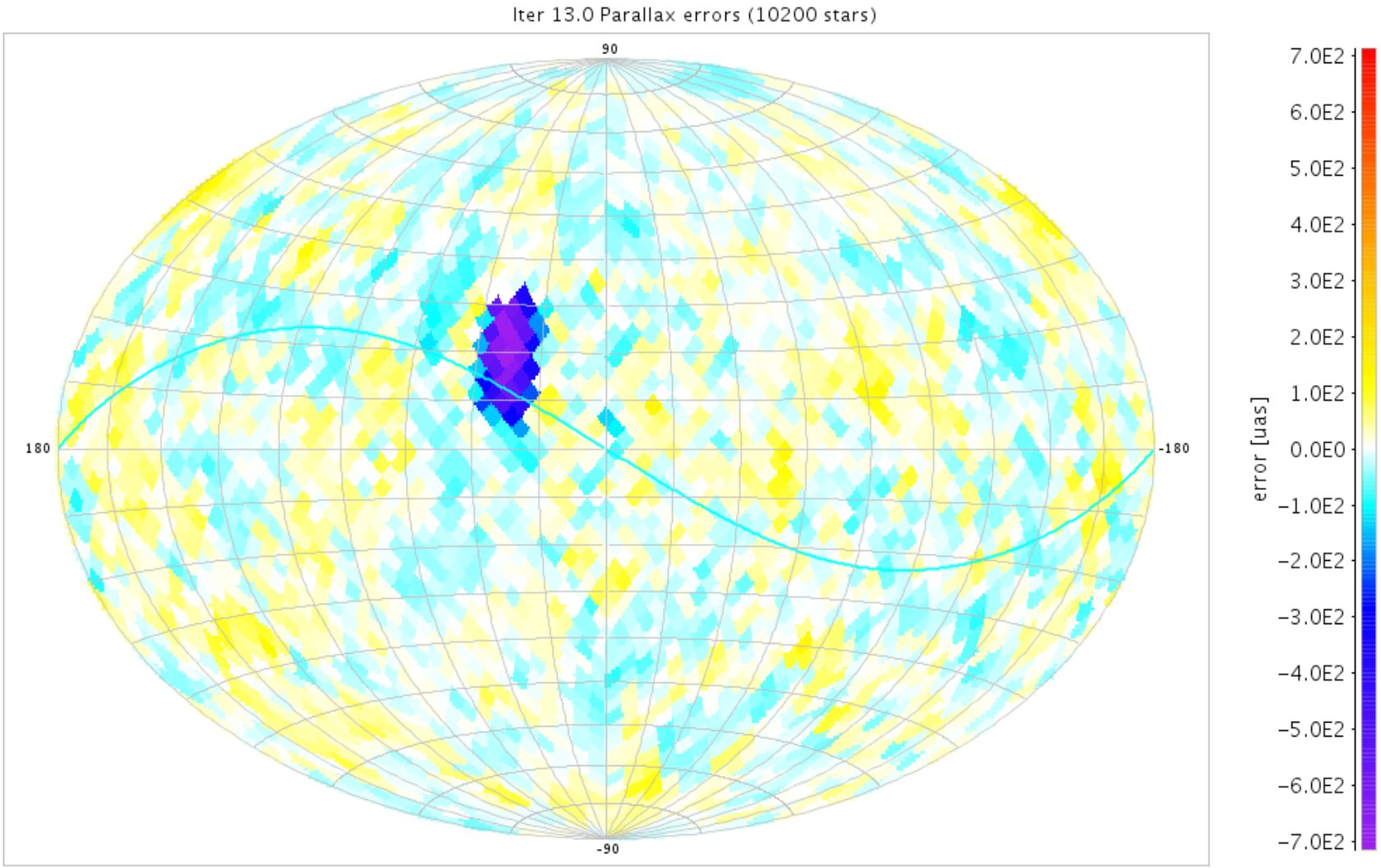
CG: parallax error maps iteration 11



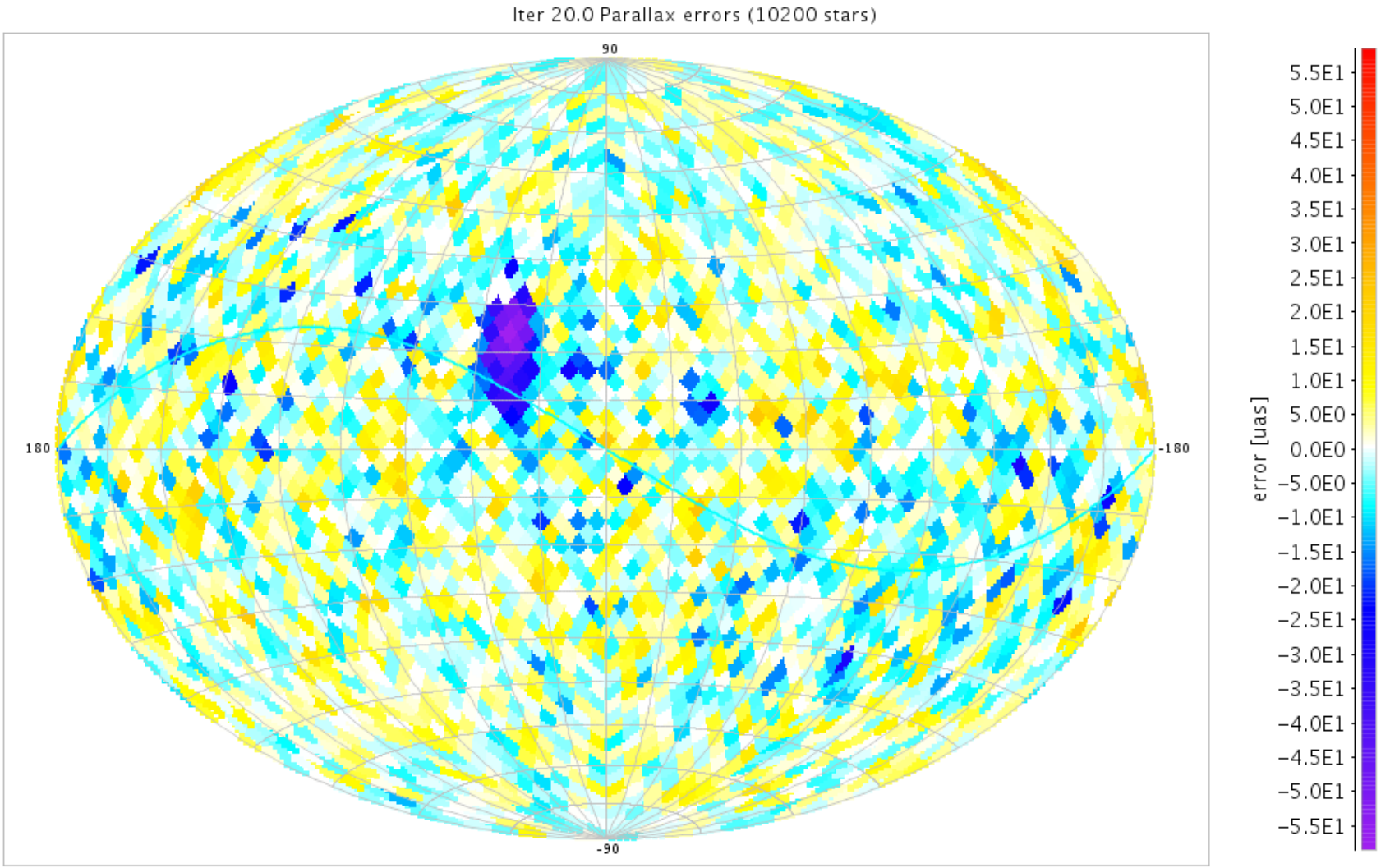
CG: parallax error maps iteration 12



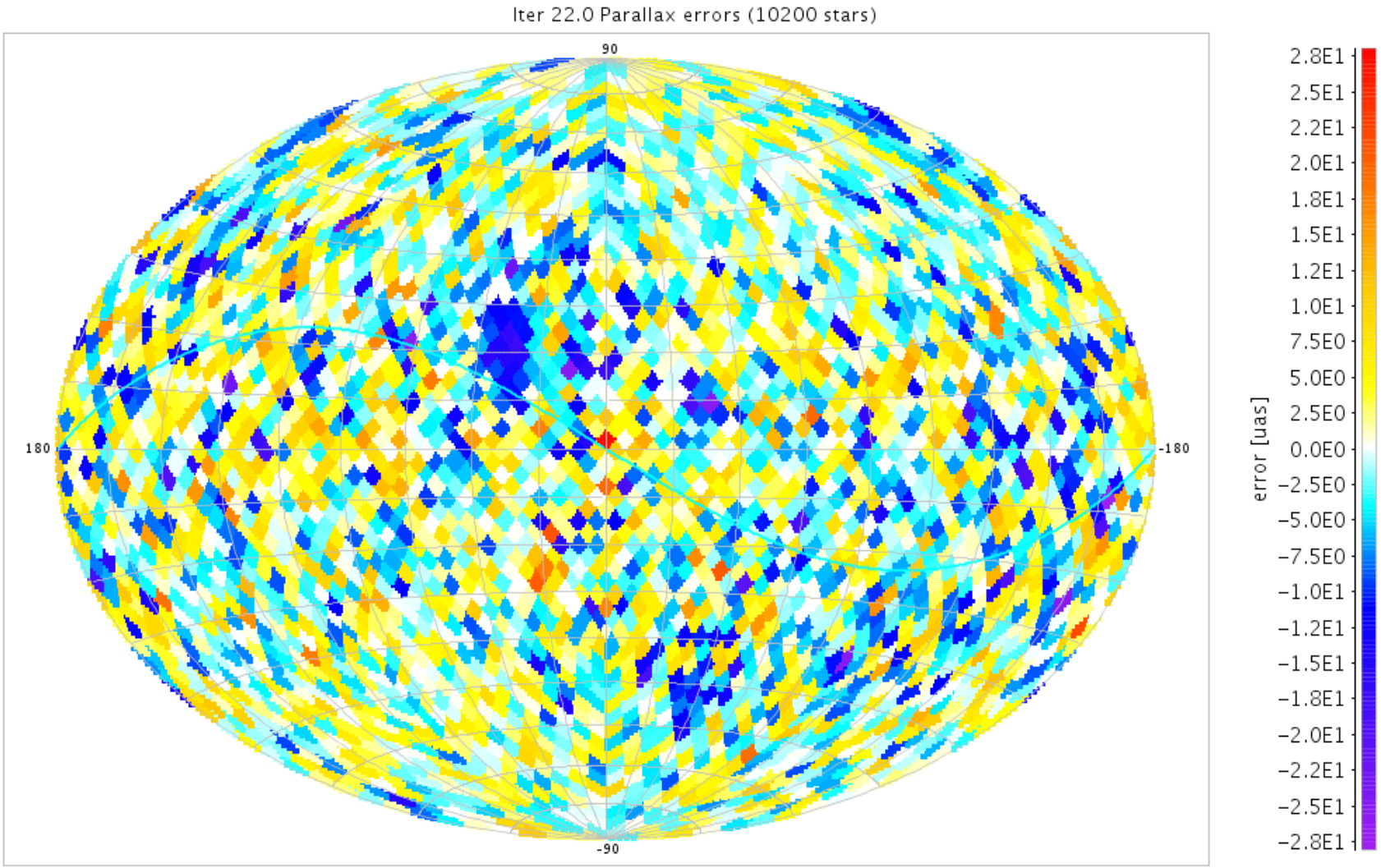
CG: parallax error maps iteration 13



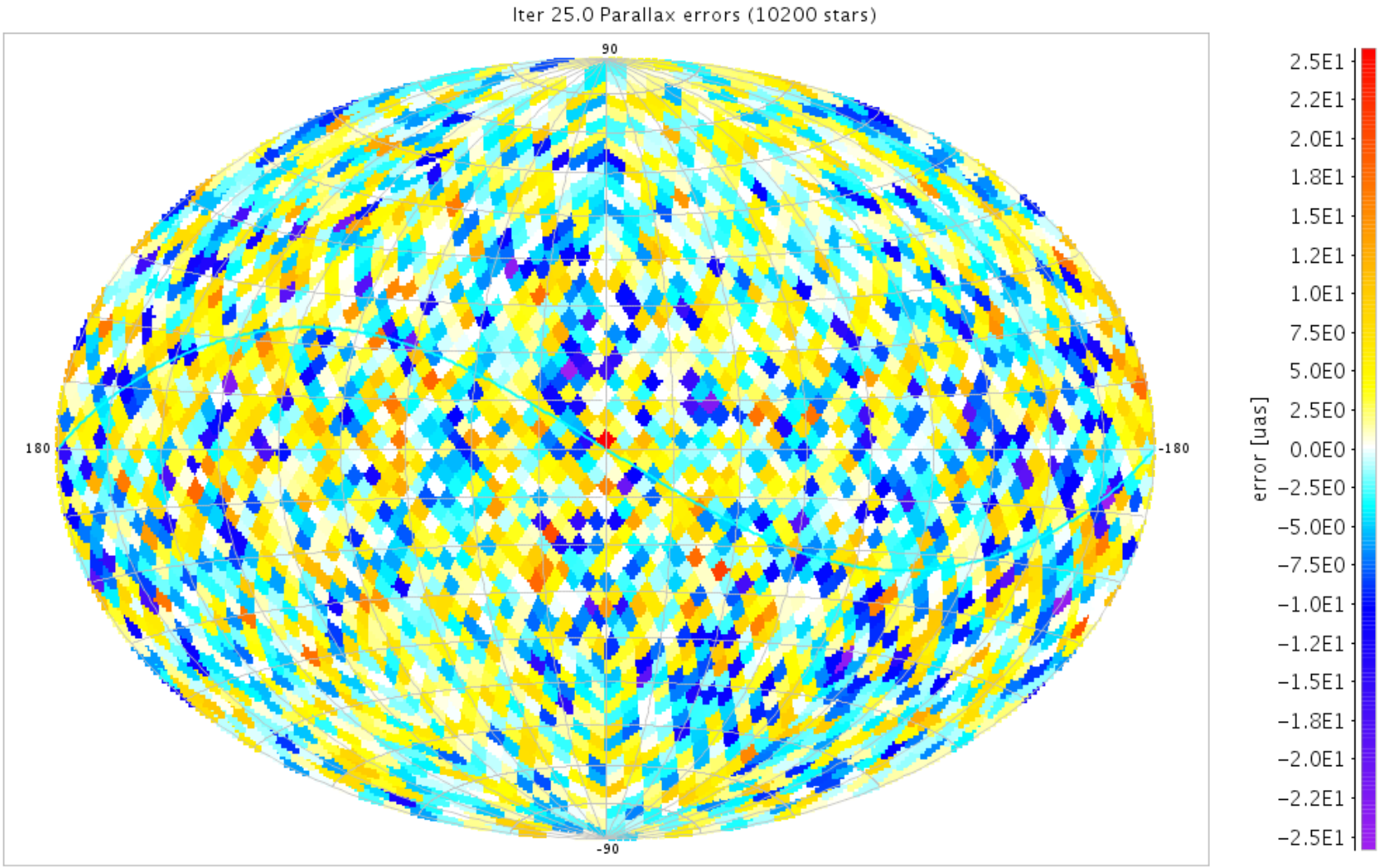
CG: parallax error maps iteration 20



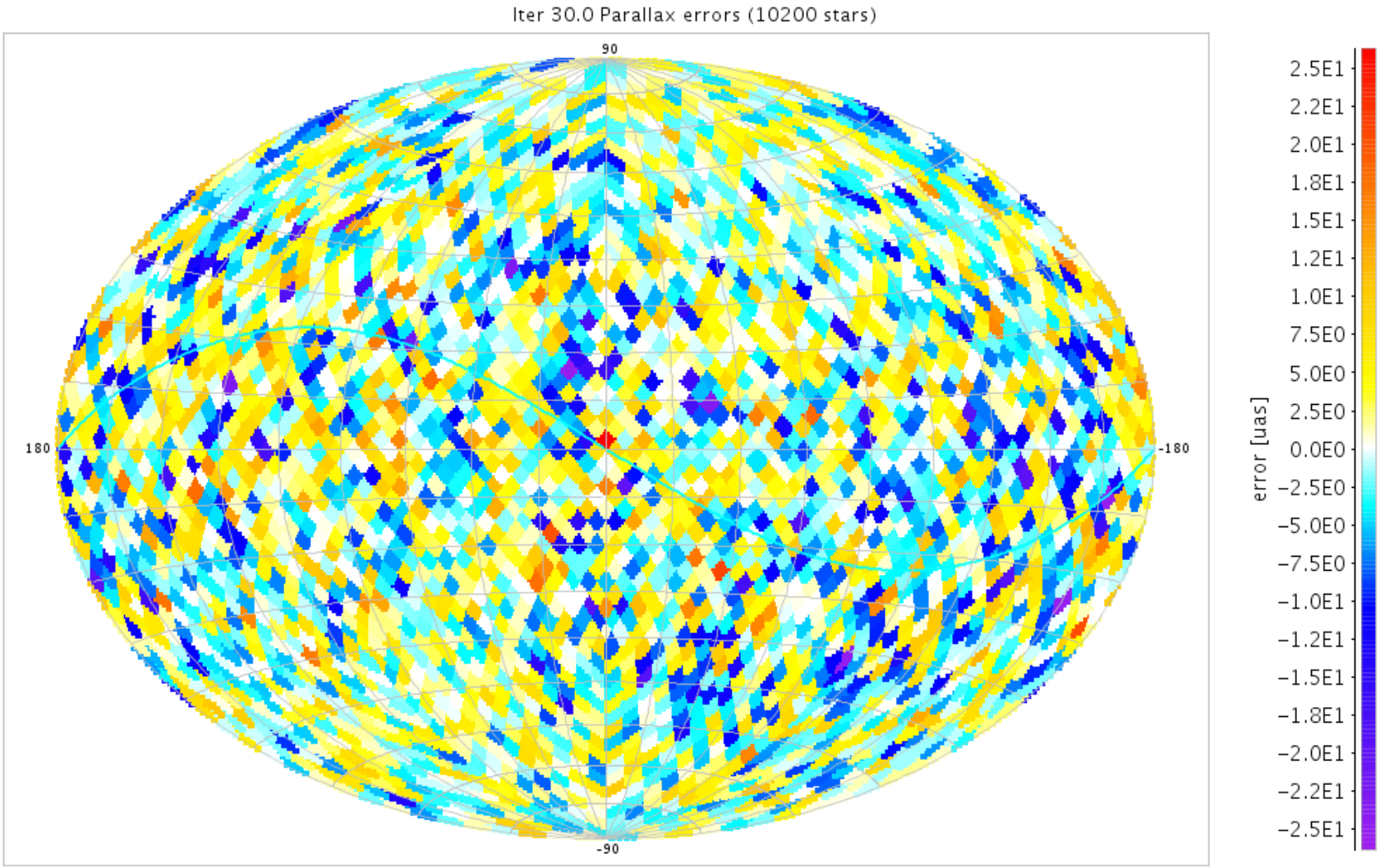
CG: parallax error maps iteration 22



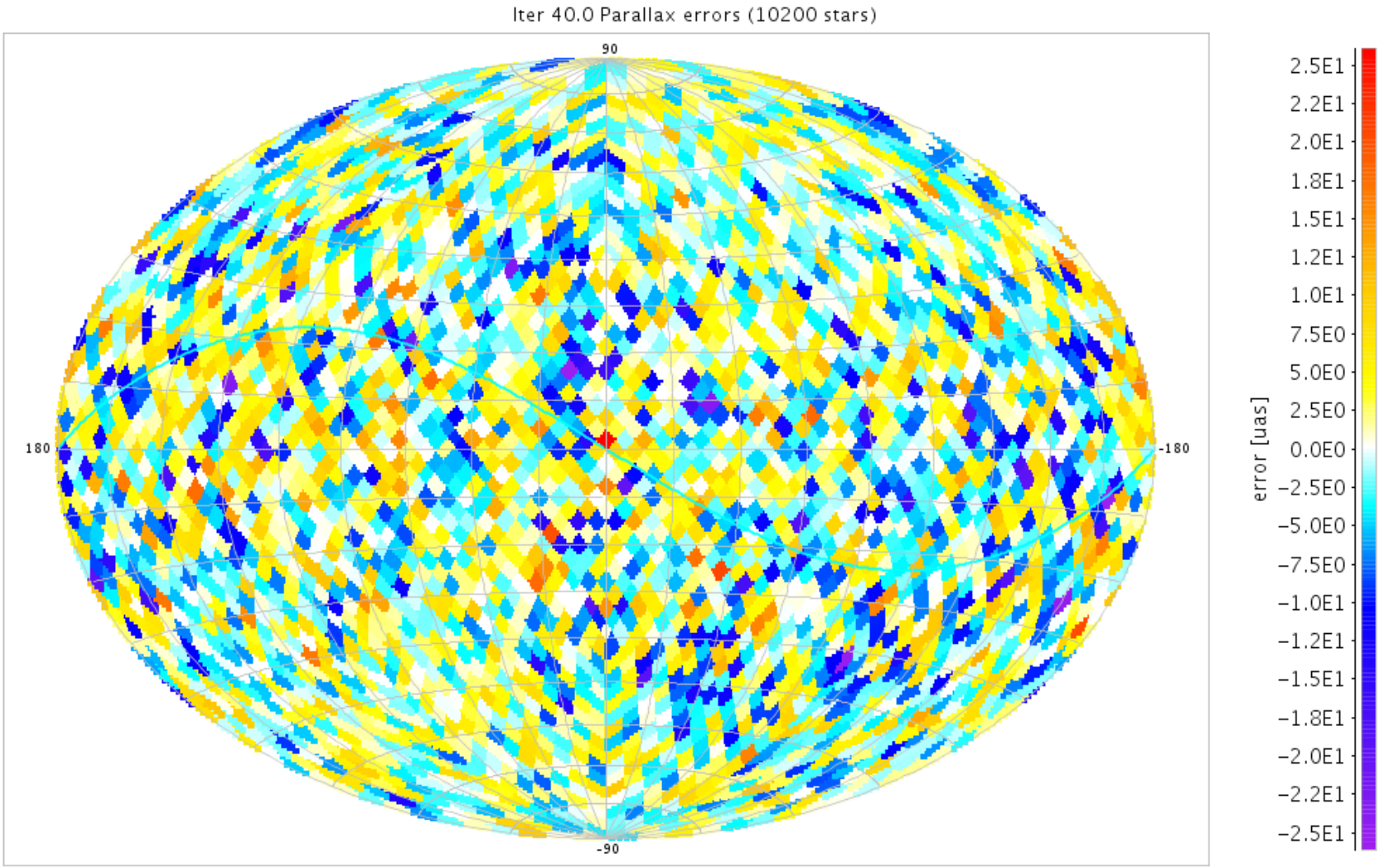
CG: parallax error maps iteration 25



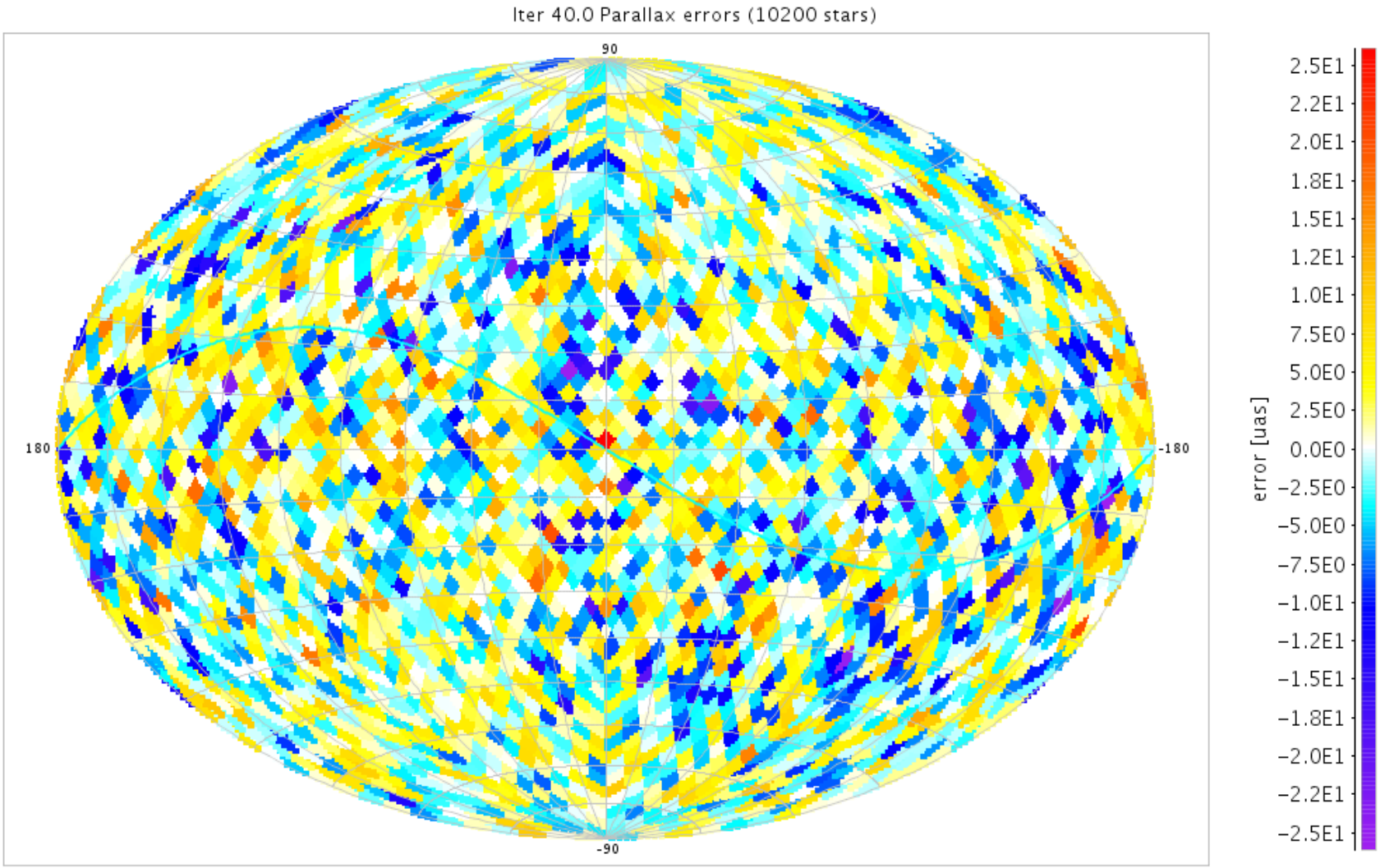
CG: parallax error maps iteration 30



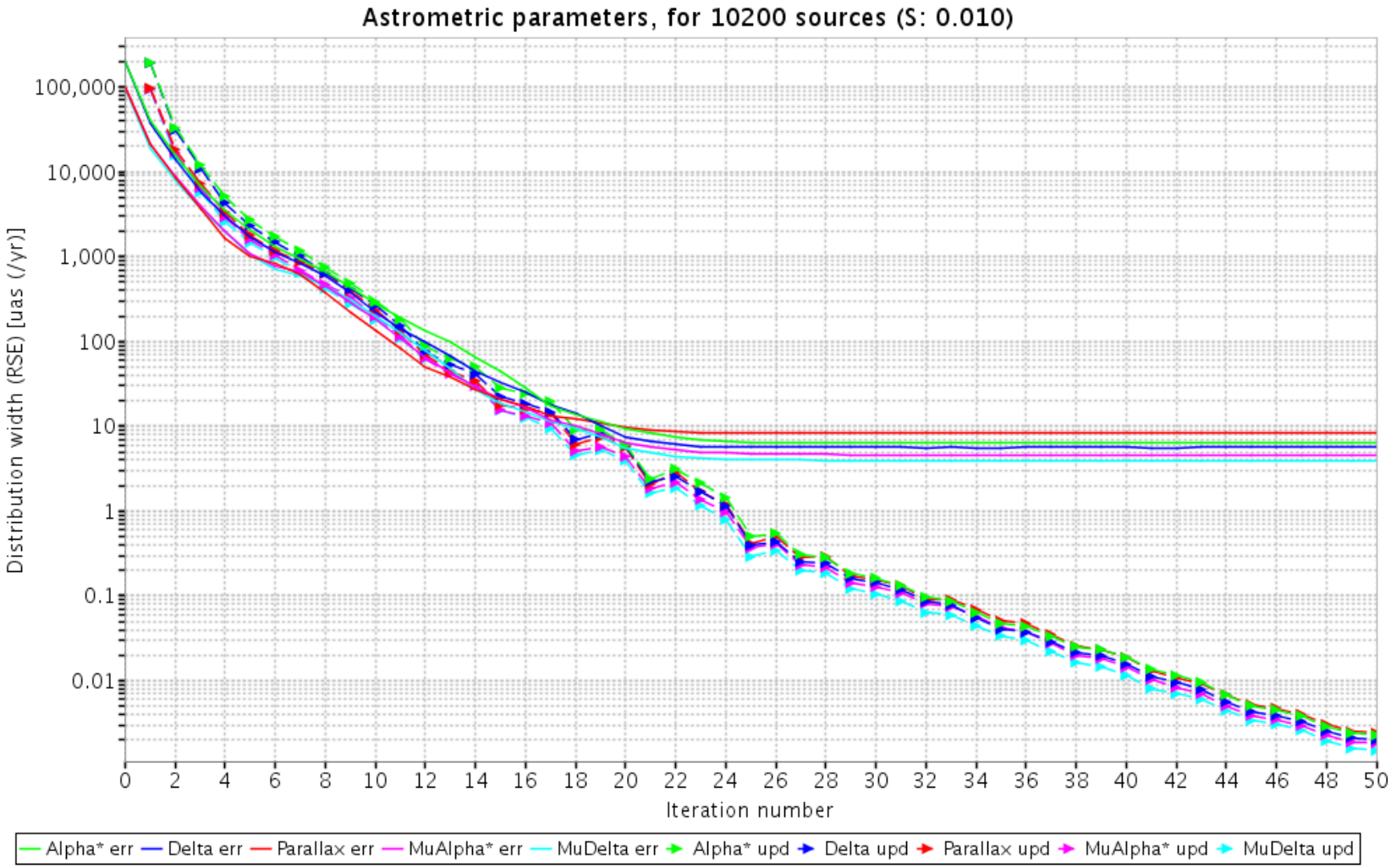
CG: parallax error maps iteration 40



CG: parallax error maps iteration 50



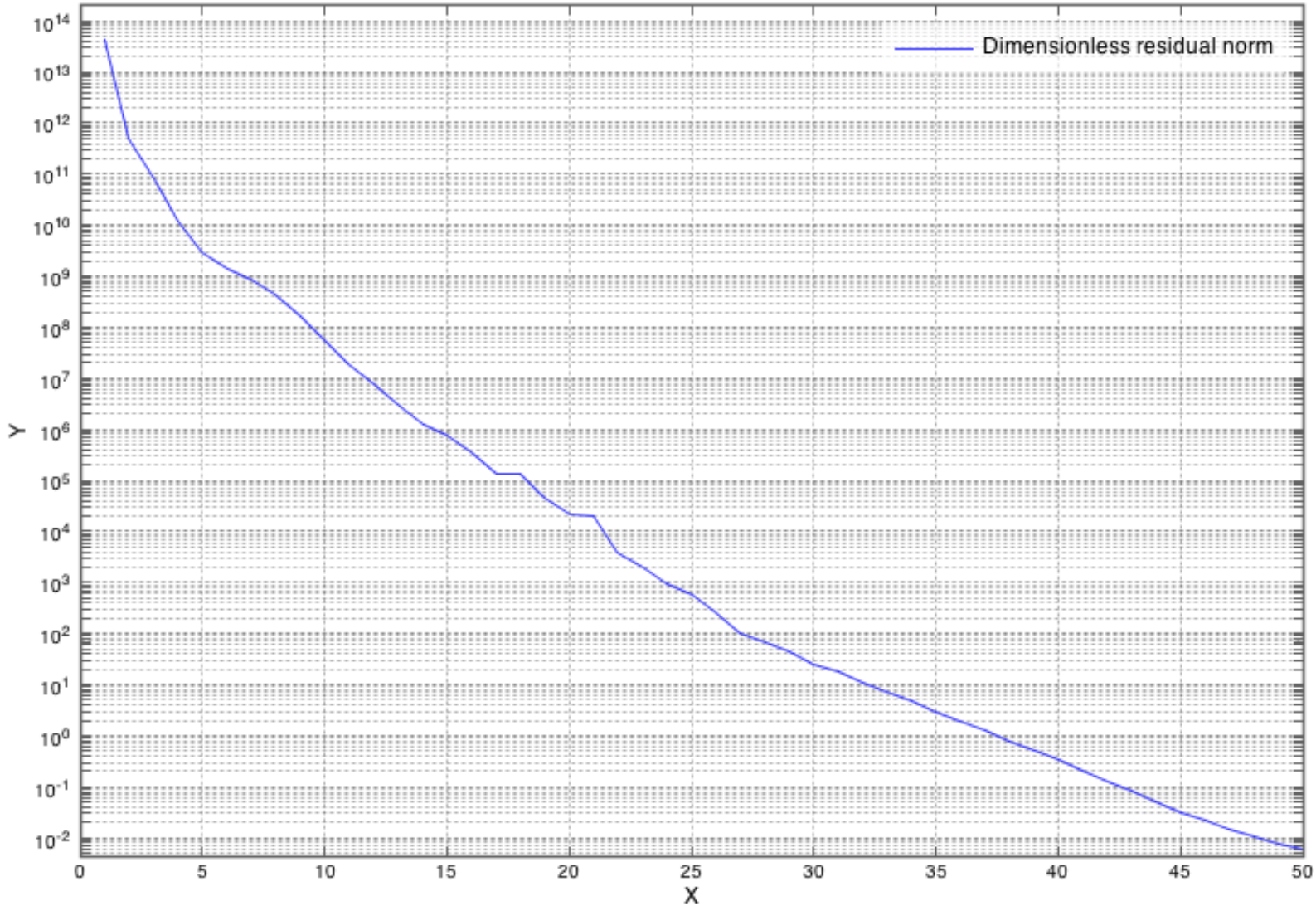
CG: Astrometric errors versus iteration



With real data?

- averaged parallax update or other updates,
- residuals,
- dimensionless norm of the residuals.

Convergence criterion: dimensionless norm of the residuals



Gauss-Markoff theorem (Aitken)

Consider: M with full-rank, $\overline{y}$ and $\overline{x}$ such that $\overline{y} = M\,\overline{x}$.

Assume: $y = \overline{y} + e$, with $E[e] = 0$, $\nu(e) = W^{-1}$, W positive definite

Then the best unbiased homogeneous linear estimator for the parameter $\overline{x}$ is the solution,

$$\hat{x} = (M^{\mathrm{T}} W M)^{-1} M^{\mathrm{T}} W y,$$

of the weighted least squares problem,

$$\min_{x} (M\,x - y)^{\mathrm{T}} W (M\,x - y).$$

A random vector $\hat{x}$ function of the observation y is an unbiased estimate of the parameter $\overline{x}$ if $E[\hat{x}] = \overline{x}$.

Error bars

Variance matrix

$$\nu(\hat{x}) = (M^{\mathrm{T}} W M)^{-1}.$$

It is not possible to compute $(M^{\mathrm{T}} W M)^{-1}$ nowdays.

Estimated source parameters rms normalized errors

$$\sqrt{\frac{1}{N}\sum\left(\frac{\hat{x}-x_{true}}{\hat{\sigma}_x}\right)^2}$$

number of sources	α	δ	ϖ	μ_{α^*}	μ_δ
$50\,000$	1.053	1.054	1.055	1.052	1.053
$80\,000$	1.060	1.082	1.067	1.066	1.081

Conclusion

- Conjugate gradient:
  - converge efficiently,
  - relatively low complexity per iteration,
  - well defined stopping criteria,
  - suitable to remove zonal errors from the initial star catalogue.
- Solution:
  - the more observations the better!
  - Variance? $\longrightarrow$ Next talk!
  - Outliers?

Papers

Complexity of the Gaia astrometric least-squares problem and the (non-)feasibility of a direct solution method,
A. Bombrun, L. Lindegren, B. Holl and S. Jordan,
A&A on line

A conjugate gradient algorithm for the astrometric core solution of Gaia,
A. Bombrun, L. Lindegren, B. Holl, D. Hobbs, U. Lammers and U. Bastian,
soon...

AGISLab: down scaled global astrometric problems

B. Holl, D. Hobbs, L. Lindegren.

Keep constant :

- mission duration : 5 years,
- number of transits per star: ~ 80,
- number of star in focal plane,
- number of star per attitude interval,
- motion of the spin axis.

number of stars	$N\epsilon$	$10\,000$
focal length	$f\sqrt{\epsilon}$	$3.5\,m$
time interval between attitude knots	τ/ϵ	$1\,500\,s$
scan rate	$w\sqrt{\epsilon}$	$6\,arsecond.s^{-1}$