Characterizing the astrometric errors in the Gaia catalogue

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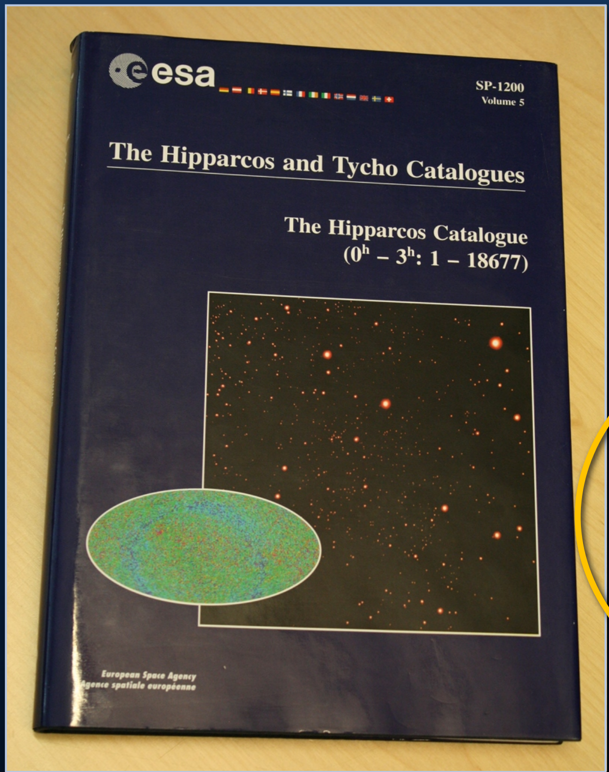
ELSA conference 2010, Sèvres, 7-11 June 2010

Outline

- Astrometric parameters
  - Hipparcos versus Gaia
  - ‘Nuisance’ parameters
  - How to compute
- Error characterization
  - Decomposing the problem
  - Estimating covariances
- Results
  - Composition of errors
  - Monte-Carlo covariance estimates
- Conclusions & future work

Hipparcos vs Gaia catalogue



5 astrometric parameters

- estimated values and their standard errors

$$\alpha \;\; \delta \;\; \varpi \;\; \mu_{\alpha*} \;\; \mu_\delta$$

$$\sigma_\alpha \;\; \sigma_\delta \;\; \sigma_\varpi \;\; \sigma_{\mu_{\alpha*}} \;\; \sigma_{\mu_\delta}$$

- Correlation between astrometric parameters of each star

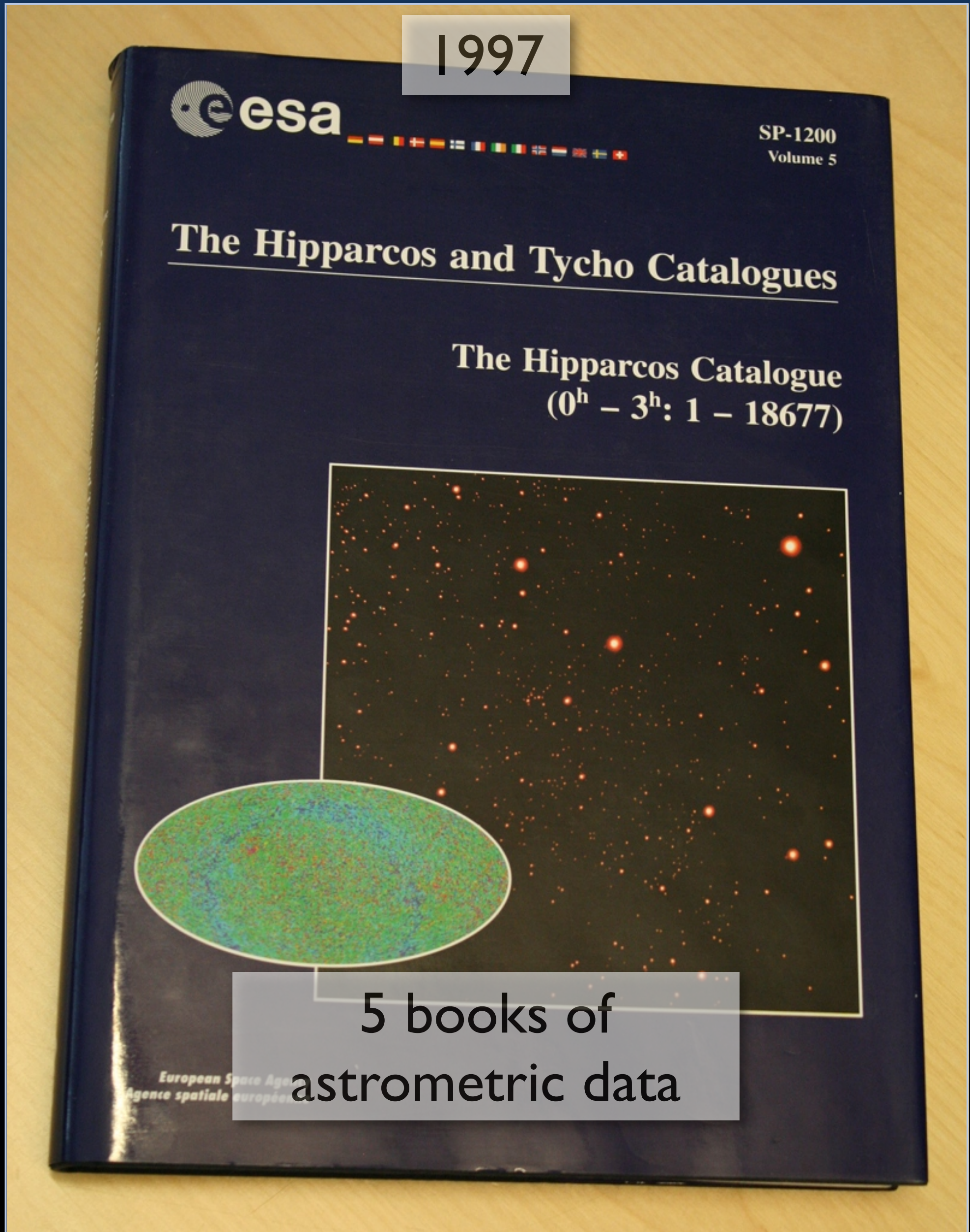
all 10 combinations

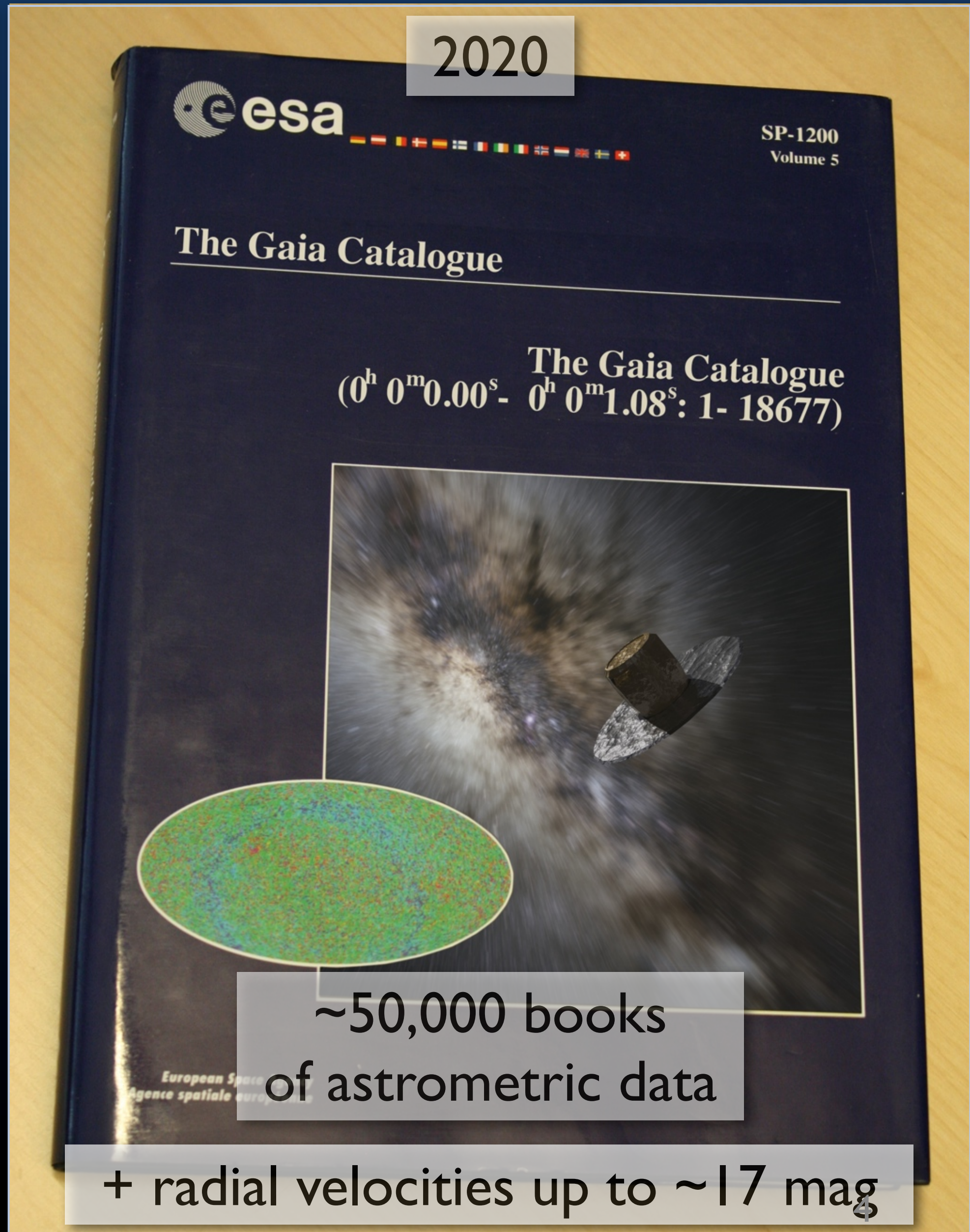
Covariances

Number: HIP (1)	(2)	Descriptor: epoch J1991.25 — RA h m s (3)	Dec ±° ′ ″ (4)	V mag (5)	(6)	(7)	Position: epoch J1991.25 (ICRS) — α deg (8)	δ deg (9)	(10)	Par. π mas (11)	Proper Motion $\mu_{\alpha*}$ mas/yr (12)	μ_δ mas/yr (13)
1		00 00 00.22	+01 05 20.4	9.10		H	0.000 911 85	+01.089 013 32		3.54	−5.20	−1.88
2		00 00 00.91	−19 29 55.8	9.27		G	0.003 797 37	−19.498 837 45	+	21.90	181.21	−0.93
3		00 00 01.20	+38 51 33.4	6.61		G	0.005 007 95	+38.859 286 08		2.81	5.24	−2.91
4		00 00 02.01	−51 53 36.8	8.06		H	0.008 381 70	−51.893 546 12		7.75	62.85	0.16
5		00 00 02.39	−40 35 28.4	8.55		H	0.009 965 34	−40.591 224 40		2.87	2.53	9.07

HIP (1)	Standard Errors: α* mas (14)	δ mas (15)	π mas (16)	$\mu_{\alpha*}$ mas/yr (17)	μ_δ mas/yr (18)	Astrometric Correlations (%): δ/α* (19)	π/α* (20)	π/δ (21)	$\mu_{\alpha*}$/α* (22)	$\mu_{\alpha*}$/δ (23)	$\mu_{\alpha*}$/π (24)	μ_δ/α* (25)	μ_δ/δ (26)	μ_δ/π (27)	μ_δ/$\mu_{\alpha*}$ (28)	Soln F1 % (29)	F2 (30)
1	1.32	0.74	1.39	1.36	0.81	+32	− 7	−11	−24	+ 9	− 1	+10	− 1	+ 1	+34	0	0.74
2	1.28	0.70	3.10	1.74	0.92	+12	−14	−24	−29	+ 1	+21	− 2	−19	−28	+14	2	1.45
3	0.53	0.40	0.63	0.57	0.47	+ 6	+ 9	+ 4	+43	− 1	− 6	+ 3	+24	+ 7	+21	0	−0.45
4	0.53	0.59	0.97	0.65	0.65	−22	− 9	− 3	+24	+20	+ 8	+18	+ 8	−31	−18	0	−1.46
5	0.64	0.61	1.11	0.67	0.74	+10	+24	+ 6	+26	−10	+20	−16	−30	−19	+ 6	0	−1.24

Hipparcos vs Gaia catalogue





Covariance & correlation

$\hat{x}_i$ and $\hat{x}_j$ are estimated values
of an astrometric parameter for two stars *i* and *j*.

These values are correlated if:

$$Cov[\hat{x}_i, \hat{x}_j] = E[e_i e_j] \neq 0 \quad \text{for } i \neq j$$

$$e_i = (\hat{x}_i - x_{i,true}) \quad \text{the error}$$

$$E[e_i] \equiv 0 \quad \text{assume no systematic error}$$

$$\boxed{Cov[\hat{x}_i, \hat{x}_j] = \rho_{ij}\sigma_i\sigma_j \quad (= \sigma_i^2 \text{ for } i = j)}$$

$$\rho_{ij} = \frac{E[e_i e_j]}{\sqrt{E[e_i^2]E[e_j^2]}} \quad \text{correlation coefficient}$$

Covariance & correlation

Why important? Any question with 'uncertainty' in it:

- For single star statistics one needs covariances between astrometric parameters of the same star.
- For multi-star statistics one need covariances between astrometric parameters of different stars.

Example: Calculate mean parallax of *N* stars in a cluster.

- ✦ If all stars have roughly same magnitude: $\sigma_{\varpi_i} \cong \sigma_{\varpi}$
- ✦ If area of cluster is small on the sky: $\rho_{ij} \cong \rho$

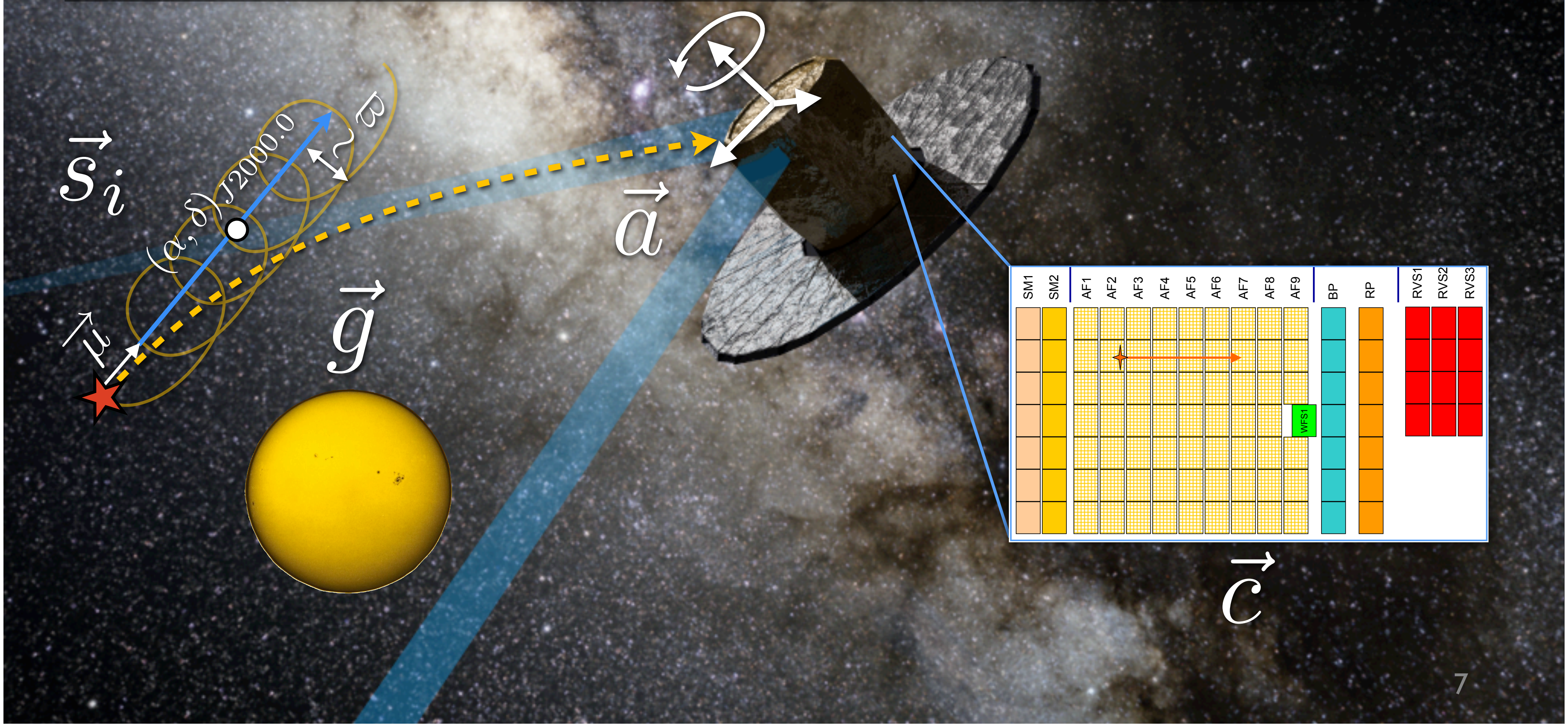
Then we find: $\sigma^2_{<\varpi>} = \sigma^2_{\varpi} \left(\frac{1}{N} + \frac{N-1}{N} \rho \right)$

- ✦ Limited accuracy by averaging *N* stars in small area: $\sigma_N \geq \sigma_1 \sqrt{\rho}$
- ✦ Limited accuracy reached for averaging over $O(\rho^{-1})$ stars.

Astrometric parameters

'Nuisance' parameters (from astrometric point of view):

- $\vec{a}$ attitude
- $\vec{c}$ calibration
- $\vec{g}$ global



Astrometric parameters

How to compute them?

- Least squares solution (with 10^9 parameters!)

$$\min_{\vec{s},\,\vec{a},\,\vec{c},\,\vec{g}} \sum_l \left[\frac{t_l - f_l(\,\vec{s}_i, \vec{a}, \vec{c}, \vec{g}\,)}{\sigma_l} \right]^2$$

t_l = measured observation time l (with uncertainty σ_l)

f_l = modelled observation time l

In this talk we ignore $\vec{c}$ and $\vec{g}$ since they are affected by large number of observations over large part of the sphere.

A direct solution is unfeasible (Bombrun et al. 2010), therefore solved iteratively using AGIS (Lindegren et al. 2010): Astrometric Global Iterative Solution

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Error characterization

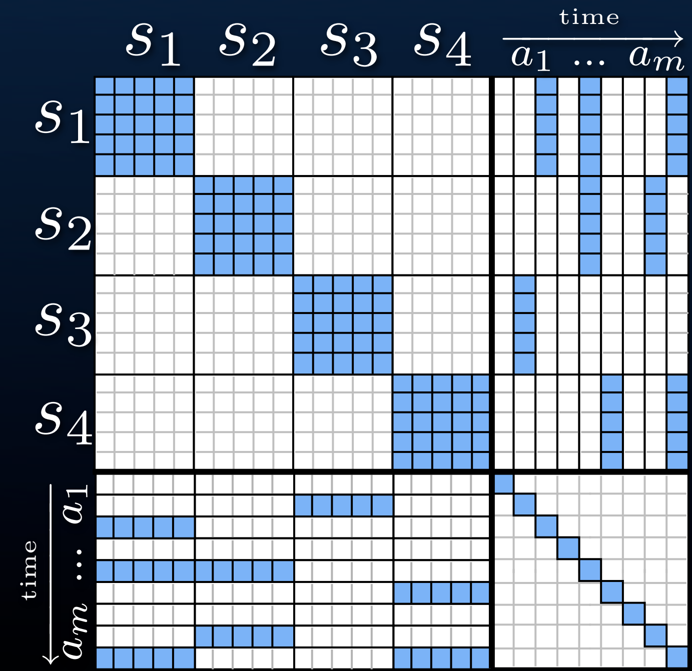
Least-squares estimation of $\vec{s}$ and $\vec{a}$

$$\forall l: \quad \vec{S}_l \, \Delta\vec{s}_i + \vec{A}_l \, \Delta\vec{a} = h_l$$

$$\vec{S}_l = (\partial f_l / \partial \vec{s}_i{}')/\sigma_l$$
$$\vec{A}_l = (\partial f_l / \partial \vec{a}')/\sigma_l$$
$$h_l = (t_l - f_l(\vec{s}_i, \vec{a}))\,/\sigma_l$$

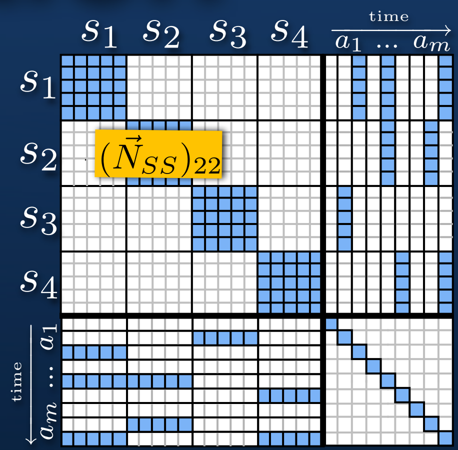
observation equations:
for each observations,
how to modify parameters
to get zero residual between
measured and modelled time.

$$\vec{N} = \begin{bmatrix} \vec{N}_{SS} & \vec{N}_{SA} \\ \vec{N}_{AS} & \vec{N}_{AA} \end{bmatrix} =$$



Error characterization

normal $\vec{N} = \begin{bmatrix} \vec{N}_{SS} & \vec{N}_{SA} \\ \vec{N}_{AS} & \vec{N}_{AA} \end{bmatrix} =$



covariance $\vec{C} = \begin{bmatrix} \vec{C}_{SS} & \vec{C}_{SA} \\ \vec{C}_{AS} & \vec{C}_{AA} \end{bmatrix} = \vec{N}^{-1}$ unfeasible!

$$\vec{C}_{SS} = \vec{N}_{SS}^{-1} + \vec{N}_{SS}^{-1}\vec{N}_{SA}\left(\vec{N}_{AA}^{-1}\right)\vec{N}_{AS}\vec{N}_{SS}^{-1} + \vec{N}_{SS}^{-1}\vec{N}_{SA}\left(\vec{N}_{AA}^{-1}\vec{N}_{AS}\vec{C}_{SS}\vec{N}_{AS}\vec{N}_{AA}^{-1}\right)\vec{N}_{AS}\vec{N}_{SS}^{-1}$$

Cov when only estimate source parameters.

Cov due to the non-perfect estimation of attitude parameters:

$\sigma_a^2 \simeq 1/k \qquad k = \#\text{stars observed per att param}$

For uniform sky density and all stars same magnitude:

$$(\vec{C}_{SS})_{ij} \simeq \begin{cases} (\vec{N}_{SS}^{-1})_{ii} + \frac{1}{k}(\vec{N}_{SS}^{-1})_{ii} + (?) & \text{if } i = j \\ \vec{0} + \frac{f}{k}(\vec{N}_{SS}^{-1})_{ii} + (?) & \text{if } i \neq j \text{ within} \sim \text{FOV} \end{cases}$$

param cov of one star

param cov between different stars

Error characterization

Estimating covariances

- Check accuracy of expansion statistically using Monte-Carlo techniques.

  For M runs with different observation noise realizations:

$$\mathrm{Cov}[x_u, x_v] = \mathrm{E}[e_u e_v] \simeq \frac{1}{M} \sum_{m=1}^{M} e_u^{(m)} e_v^{(m)}$$

$$\rho_{uv} \simeq \frac{1}{M} \sum_{m=1}^{M} e_u^{(m)} e_v^{(m)} / \sqrt{\sum e_u^{2(m)} \sum e_v^{2(m)}}$$

AGISLab
Investigating the Astrometric Global Iterative Solution

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Results

AGISLab

Investigating the Astrometric Global Iterative Solution

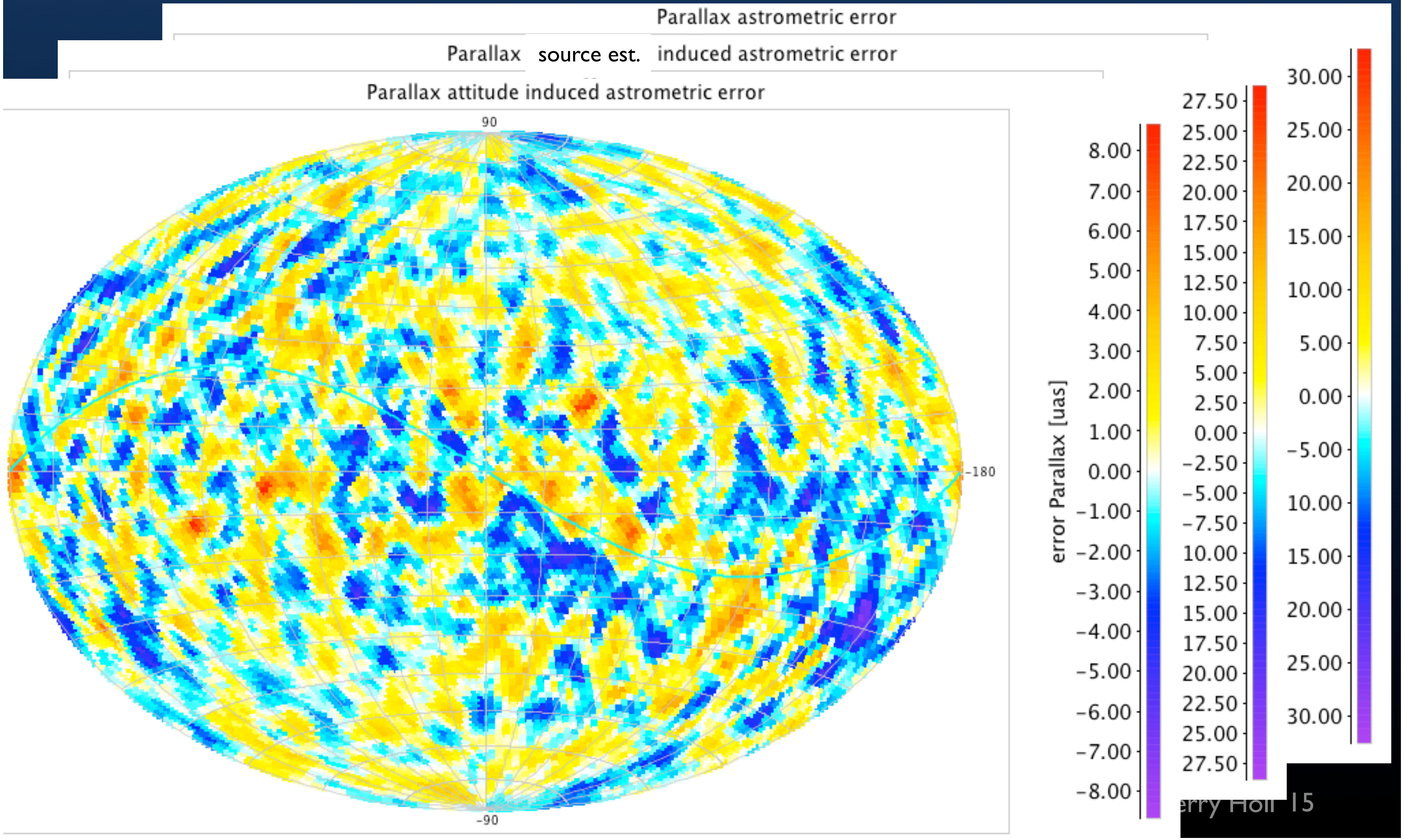
Simulations properties:

- Stars are uniformly distributed on the sky.
- All stars have the same magnitude (G-band mag 11.8).

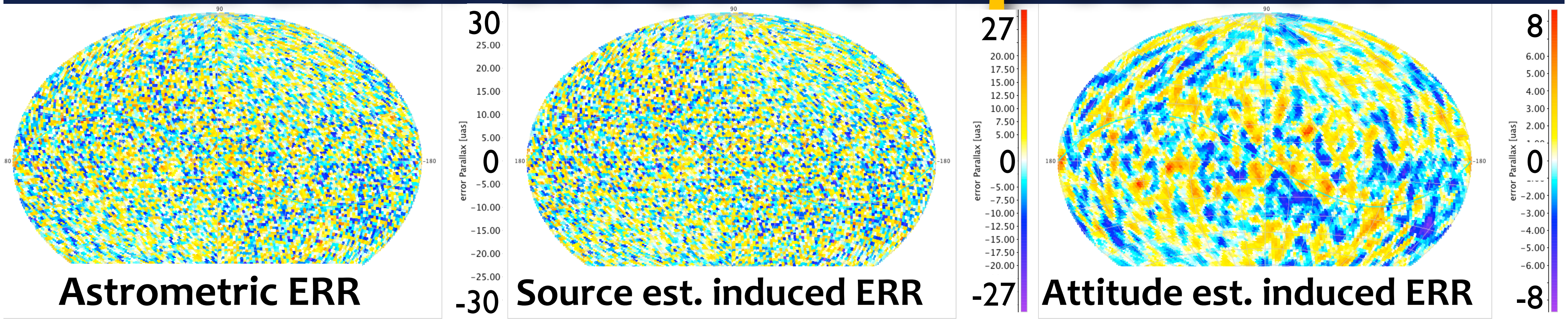
Results: error composition

$$e_{\varpi} \equiv e_{\mathcal{S}\varpi} + e_{\mathcal{A}\varpi}$$

$\mathrm{FOV} = 6.9^{\circ}$ $\quad$ $\#\mathrm{stars}/\mathrm{FOV} = 12$



Results: error composition



$$e_{\varpi} \equiv e_{\mathcal{S}\varpi} + e_{\mathcal{A}\varpi}$$

$$\sigma^2_{\varpi} = \sigma^2_{\mathcal{S}\varpi} + \sigma^2_{\mathcal{A}\varpi} + \sigma^2_{\mathcal{S}\mathcal{A}\varpi}$$

$$(\vec{C}_{SS})_{ij} \simeq \begin{cases} (\vec{N}^{-1}_{SS})_{ii} + \frac{1}{k}(\vec{N}^{-1}_{SS})_{ii} + (?) & \text{if } i = j \\ \quad + \frac{1}{k}(\vec{N}^{-1}_{SS})_{ii} + (?) & \text{if } i \neq j \text{ within} \sim \text{FOV} \end{cases}$$

param cov of one star

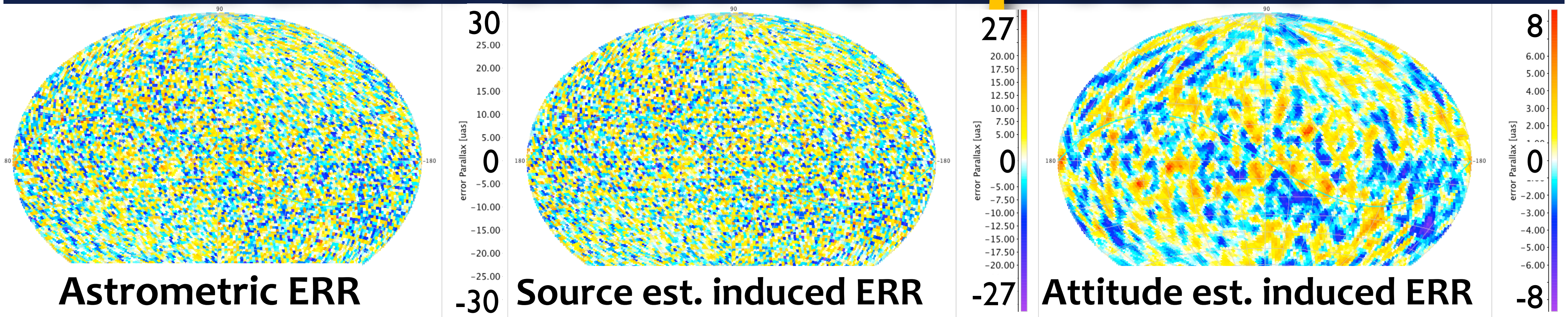
Source est. induced COV.

Attitude est. induced COV.

$k =$ mean #stars observed per att param

Statistical estimate: $\sigma^2_{\mathcal{A}} = \mathrm{E}[e^2_{\mathcal{A}}] \simeq \frac{1}{M}\sum_{m=1}^{M} e^{2(m)}_{\mathcal{A}}$

Results: error composition

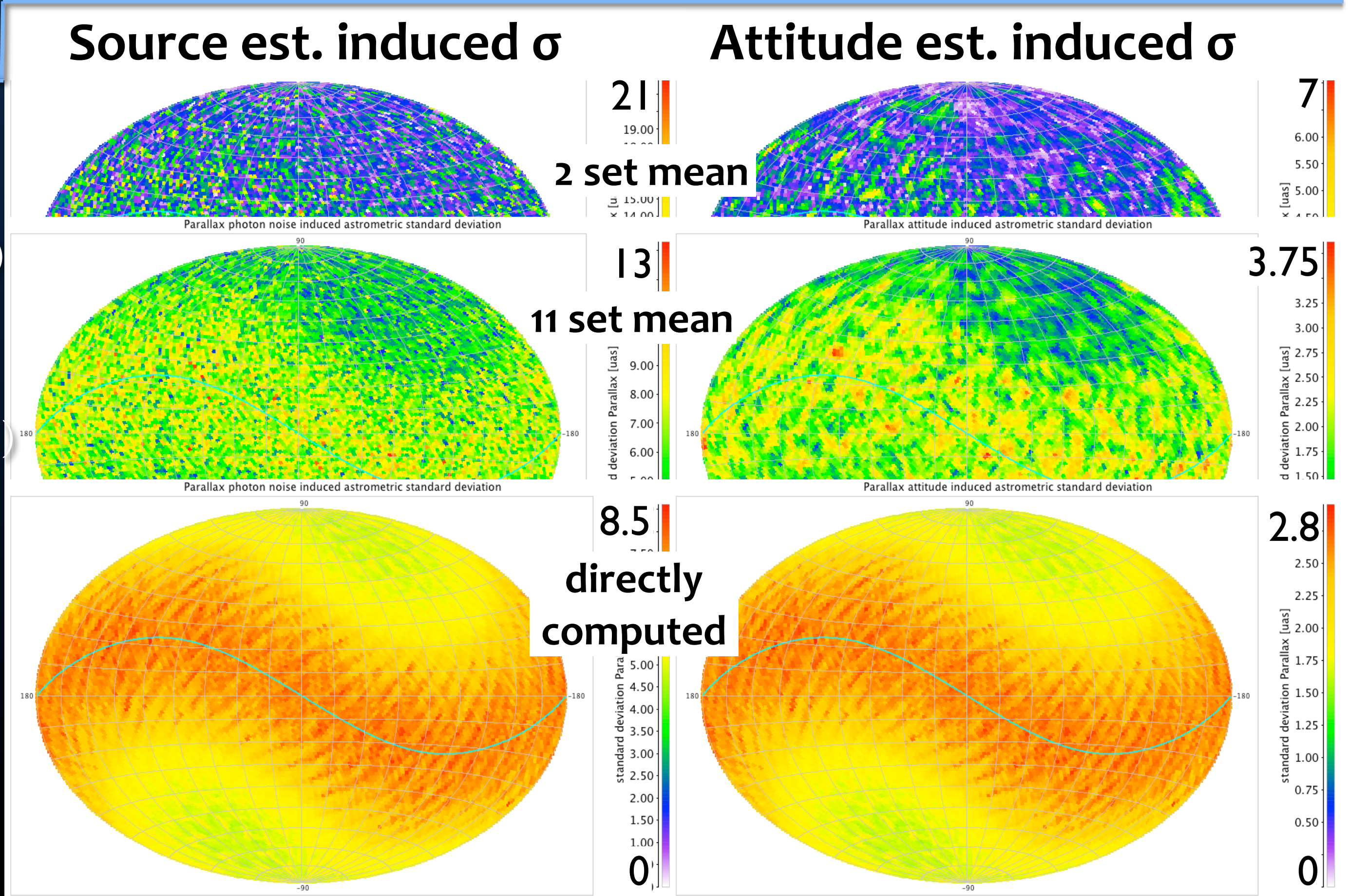


$$e_{\varpi} \equiv e_{\mathcal{S}\varpi} + e_{\mathcal{A}\varpi}$$

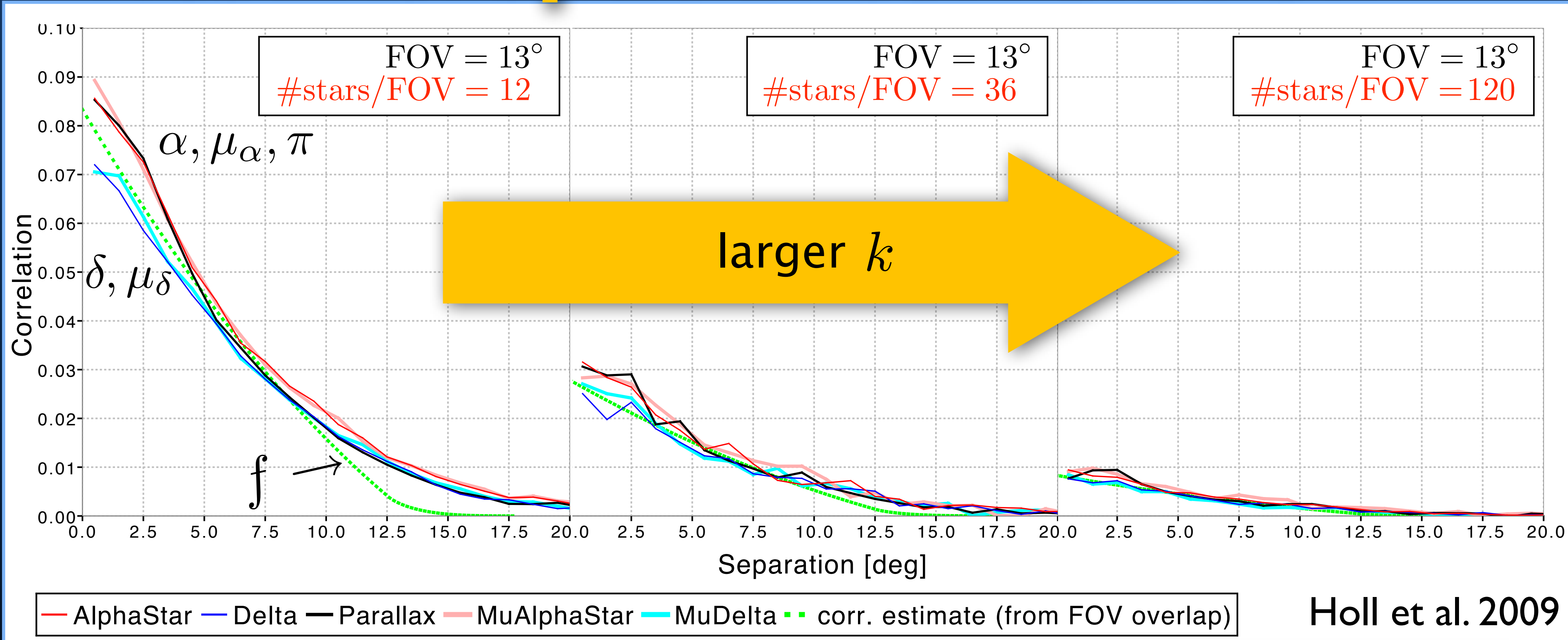
Statistical estimates:

$$\sigma^2_{\mathcal{S}} = \mathrm{E}[e^2_{\mathcal{S}}] \simeq \frac{1}{M}\sum_{m=1}^{M} e_{\mathcal{S}}^{2(m)}$$

$$\sigma^2_{\mathcal{A}} = \mathrm{E}[e^2_{\mathcal{A}}] \simeq \frac{1}{M}\sum_{m=1}^{M} e_{\mathcal{A}}^{2(m)}$$



Results: spatial correlation



Holl et al. 2009

Source est. induced COV.

Attitude est. induced COV.

$$(\vec{C}_{SS})_{ij} \simeq \begin{cases} \ldots + (?) & \text{if } i = j \\ \vec{0} + \frac{f}{k}(\vec{N}_{SS}^{-1})_{ii} + (?) & \text{if } i \neq j \text{ within} \sim \text{FOV} \end{cases}$$

param cov between different stars

f = fraction of times i and j observed together

k = mean #stars observed per attitude param

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Future work

What next:

- Publish paper on covariance model & Monte-Carlo estimates for uniform sky density and single magnitude (Holl et al. 2010, in prep.).
- Study and publish covariances for (realistic) non-uniform sky densities and mixed magnitudes.
- Study effect of biassed observations on our estimated astrometric parameters (e.g. from CCD radiation damage).
  Collaboration with Thibaut Prod'homme (Leiden).

Conclusions

Remember:

- Knowledge of the errors in the catalogue parameters is essential for answering any astrometric ‘uncertainty’ question.
- In principle all ‘nuisance’ parameters will give a contribution to the covariances of the source parameters. We are able to characterise the largest contribution, coming from the attitude.
- Covariances between all (astrometric) parameters can be estimated using Monte-Carlo techniques and/or approximated using our model.

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