Can you infer dynamics from kinematics?

Jo Bovy & David W. Hogg
Center for Cosmology and Particle Physics, New York University

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summary

- we are developing *dynamical inference* methods for integrable, phase-mixed dynamical systems
  - snapshot in time of $(\mathbf{x}, \mathbf{v})$; what is the potential?
  - example: Solar System
  - similar to virial estimators but *much more precise*
- when phases aren't mixed, we are using anisotropies that develop in phase space to perform dynamical inference
  - cold stellar streams
  - example: 6-d map of the GD-1 stream
  - action space looks different from angle space
- when phases aren't mixed *and* potentials aren't integrable, phase-space structure is still highly informative

baby problem: Kepler problem

- Can you determine the mass of the Sun with a snapshot of eight positions and velocities?
- Can you determine the $(1/r^2)$ force law?
- Of course you cannot!
  - the equations of motion are independent of the initial conditions
  - any initial conditions are possible
  - is the system even *bound?*

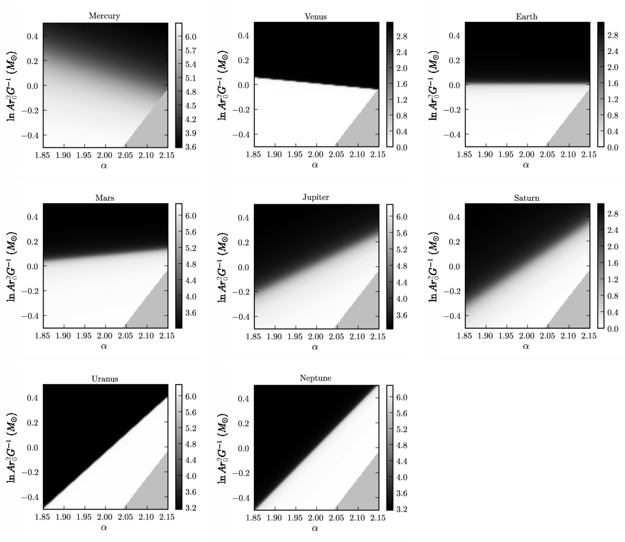
baby problem: Kepler problem

- Can you determine the mass of the Sun with a snapshot of eight positions and velocities?
- Can you determine the $(1/r^2)$ force law?

- Of course you *can!*
  - guess a force law
  - if the force law is too strong, all planets at aphelion
  - if the force law is too weak, all planets at perihelion
  - prefer masses and force laws for which the angles "look mixed"
- this problem is called "orbital roulette" (Beloborodov & Levin)

example: If Newton had only a snapshot...

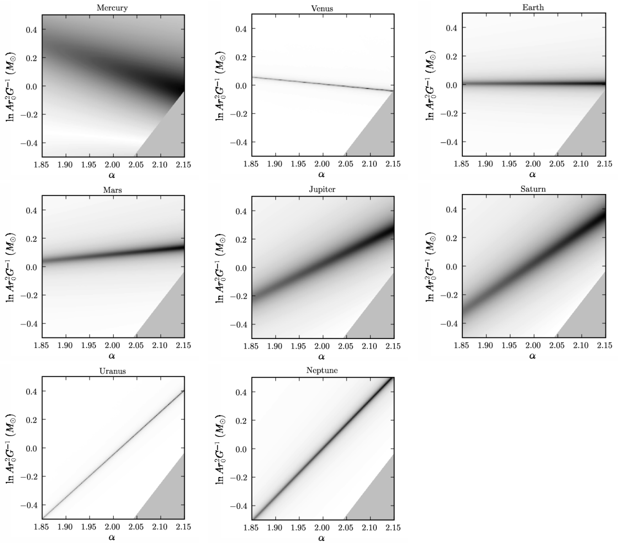
- take 8-planet ephemeris snapshot at 2009 April 1.0
- $\mathbf{a} = -A \left[\frac{r}{r_0}\right]^{-\alpha}$
- can we find α, marginalizing over all unknowns?
- Bovy, Murray, & Hogg, 2010, *ApJ* **711** 1704 (arXiv:0903.5308)



Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
ln $Ar_0^2G^{-1}$ ($M_\odot$)
α

Solar System generative model

- distribution function a function of constants of motion
  - $f_\theta(\mathbf{x}, \mathbf{v}) \propto |J|\, f_\theta(\ln\epsilon, e)$
  - must choose a *form* (or set of forms) for the distribution
  - θ is a large set of parameters we don't care about
- data (in 6-d) are a Poisson sampling of that distribution function
- Bayes theorem
  - priors over parameters $p(A, \alpha, \theta)$
  - prior times likelihood becomes posterior $p(A, \alpha, \theta|D)$
  - must *marginalize over* all distribution parameters
    $p(A, \alpha|D) = \int \mathrm{d}\theta\, p(A, \alpha, \theta|D)$
    $p(\alpha|D) = \int \mathrm{d}A\, p(A, \alpha|D)$
  - marginalization over *qualitatively diverse set* of distribution functional forms



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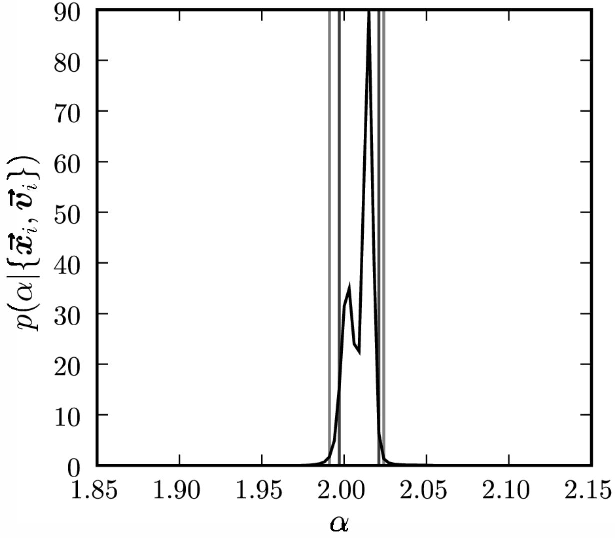
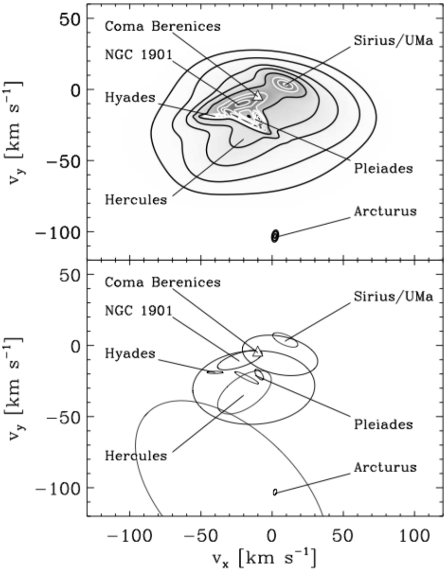


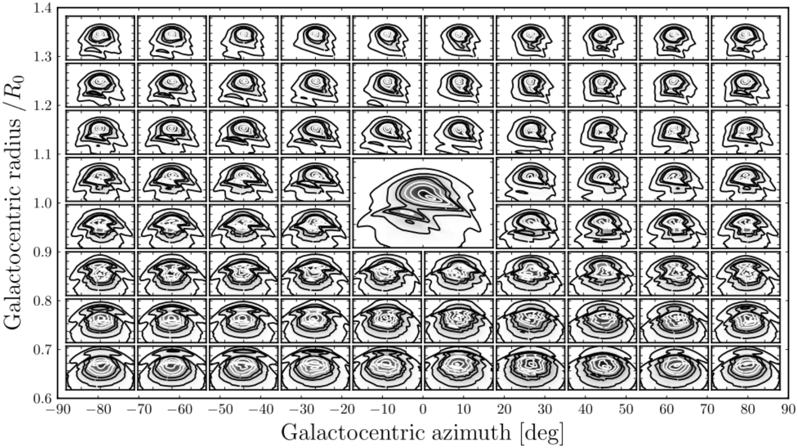
90
80
70
60
50
40
30
20
10
0
1.85
1.90
1.95
2.00
2.05
2.10
2.15
α
$p(\alpha|\{\vec{x}_i, \vec{v}_i\})$

Solar System: conclusions

- we *can* infer the force law from a snapshot
  - bodes well for Milky Way modeling with *Gaia*
- crushed the virial theorem
  - planets with small $\mathbf{x} \cdot \mathbf{v}$ dominate
- (outperformed frequentist orbital roulette)
- precision is good but *much worse* than measurement precision
  - measurements are 10^{-8}, result is 10^{-2}
  - most of the information in the data goes into *modeling the distribution function*
  - artificial restriction of the distribution function is *not a good idea*
  - bodes ill for Milky Way modeling with *Gaia*
- *incredibly* computationally intensive
  - 8 planets: 1 CPU-week)
- Bovy, Murray, & Hogg, 2010, *ApJ* **711** 1704



Bovy, Hogg, & Roweis 2009 *ApJ* **700** 1794–1819



Bovy 2010 arXiv:1006.0736

but angles are *never* mixed: theory

- the Milky Way has been around for 10^2 dynamical times
- stars are formed in groups with velocity widths of a few $\mathrm{km\,s^{-1}}$
- Milky Way orbits are 200 $\mathrm{km\,s^{-1}}$
- angle-mixing will be complete only after much more than 10^2 dynamical times
- *and* new stars are forming and accreting all the time

angles are *never* mixed: observation

- substructure and satellites
- tidal streams
- velocity structure in the disk

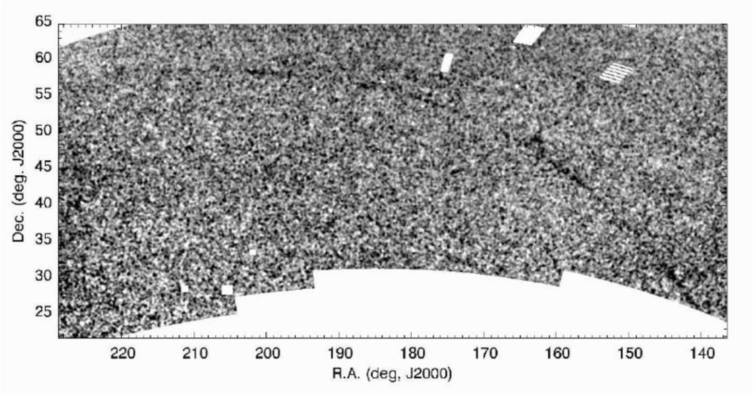
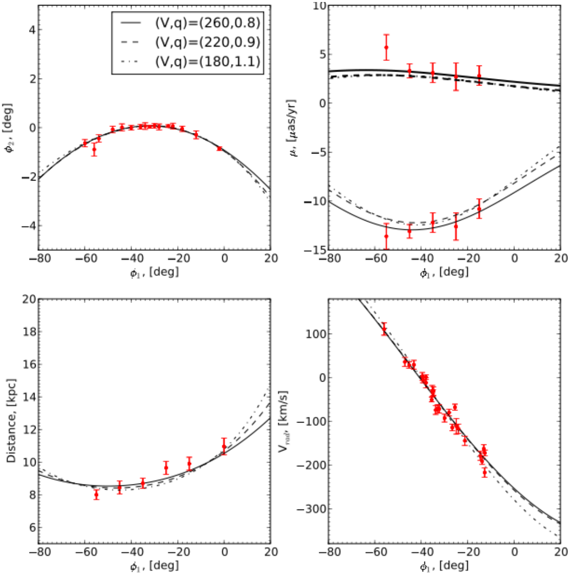


Fig. 1.— Smoothed, summed weight image of the SDSS field after subtraction of a low-order polynomial surface fit. Darker areas indicate higher surface densities. The weight image has been smoothed with a Gaussian kernel with $\sigma = 0.2°$. The white areas are either missing data, or clusters, or bright stars which have been masked out prior to analysis.

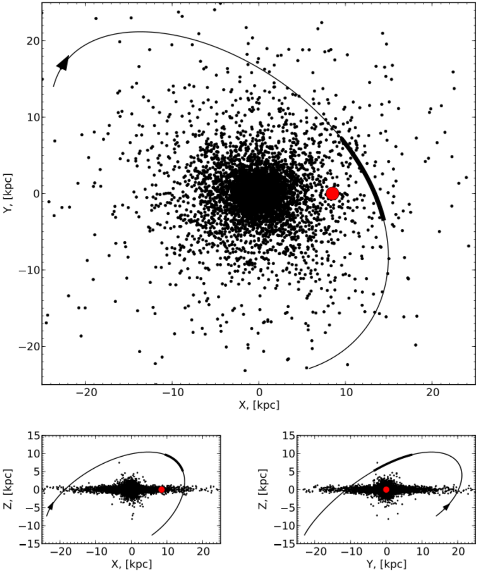
Grillmair & Dionatos 2006 *ApJL* **643** L17–L20.

GD-1 Stream generative model

- foreground–background modeling
  - *no* star is a member of the stream at high probability
  - space is (RA, Dec, proper motion, distance, RV)
- mixture of a smooth background and a cold stream
- data are a Poisson sampling of the model
- fit for the amplitude and all properties of the stream
- *never* subtract data from data
  - arithmetic operations on data (on-source minus off-source) are a bad idea
  - at low signal-to-noise this matters



Koposov, Rix, & Hogg 2010 *ApJ* **712** 260–273



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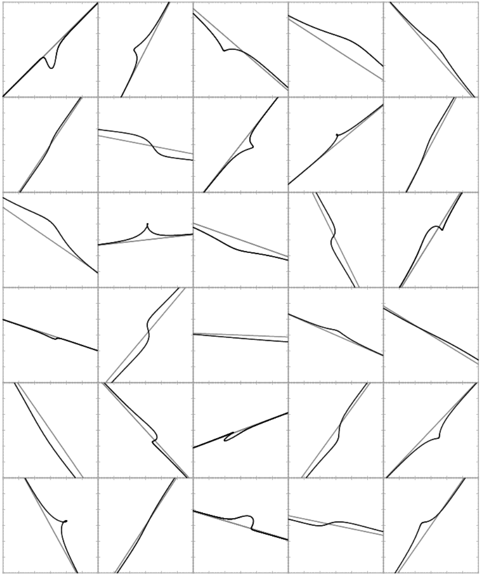
cold stream map: conclusions

- We have a 6-d map of the GD-1 stream.
  - overcame low per-star significance
- well-fit by a test-particle orbit in the Milky Way
- Milky Way halo appears (roughly) spherical
  - detected oblateness consistent with effect of the disk
- Koposov, Rix, & Hogg, *ApJ* **712** 260 (arXiv:0907.1085)

potentials are never (naively) integrable!

- Milky Way is filled with substructure
- Milky Way is accreting satellites and mass continuously
- no time-independence
- no axisymmetry
- no known symmetries of any kind
- Open question: *Is the outcome of integrable-model fitting a useful approximation when the system is dynamically chaotic?*



will we have lots of streams?

- yes!
- in *SDSS* there are a handful of cold streams each containing hundreds of stars
- *SDSS* is a 2.5d map of the stellar distribution
- *Gaia* is a 4.5d or 5.5d map
- streams with only tens of stars will be *easily* detectable

the general problem

- can we perform inference if we *can't* assume phase mixing *or* integrability?
- yes, of course! The keys:
  - stars form in cold clumps in phase space
  - co-eval stars (ought to) show chemical similarities
  - Milky Way forms by gravitational collapse from a homogeneous neighborhood
  - fit dark-matter initial conditions (including phases) and the birthrate (as a function of phase-space through time)
- (no-one said it would be easy)
- relates to other *comprehensive modeling* problems in astrophysics and elsewhere

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