Centroiding accuracy of bright stars

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Abstract

The proposition of using the spikes of bright stars (SAG-CUO-100), while improving the astrometric precision may lead to significant systematic errors, due to the asymmetry of the PSF, if the centroiding is done with a line-spread function. The baseline design, with the use of selectable gates, should not be used, since it does not allow calibrations for the brightest stars. We propose several options allowing to maintain a reasonable accuracy.

1 Introduction

In a previous document (AAEB-FACB-01), we expressed our support to the idea by Høg & Fabricius (SAG-CUO-100) of using the upper and lower spikes of bright stars. It was noted that the astrometric precision of the brightest stars could be as low as $\approx 1.5\mu$as, to be compared to $2.5 - 3\mu$as in the Gaia Study Report (GSR), assuming systematics for basic-angle variations and chromaticity corrections at the level of $\sqrt{2}\mu$as.

The new satellite design (as of April 23) simplifies the focal plane, but keeps the gates for bright stars. This lead us to check the sampling of bright stars with the accuracy performance in mind. For practical reasons however, we consider here only the GSR satellite design.

2 Astrometry with spikes

The standard positional measurement of individual observations makes use of the line-spread function (LSF), i.e. the one-dimensional PSF along-scan, of the patches obtained in the astrometric field. This is the case in the GSR, in the accuracy estimation section, and in GAIA-LL-032.

With patches covering most of the star flux, as is the case for stars fainter than 12^{m}, the centroiding error is negligible, as demonstrated in GAIA-LL-032, even when the left and right spikes are truncated, and even with a fitted LSF not strictly identical to the true LSF[1].

The sampling with the "spike method" as proposed in SAG-CUO-100.4 is different from the baseline method. Only upper and lower patches are kept, and the lines (along-scan) containing saturated pixels would not be downloaded. In this proposal, a window is made of 3+3 patches, with 16 samples of 4 pixels per patch.

The problem is that the PSF is not perfectly symmetric, as may be seen in Figure 1 (playing with the LUT). This PSF was computed at $V - I = 0$ for the astrometric field 09 (the worst case), using the program `syntpsf.f` given in SAG-LL-025. This PSF is given with a factor 4 oversampling (i.e. by steps of 0.25 pixel) and includes the smearing due to the transverse motion.

For simplification, we assume in what follows that the serial registers are designed such that the pixel saturation occurs before the sample saturation (which is not the case in the baseline design) and that the

[1] to give an example, at 12^{m}, a 10^{-3} relative error on the expected value of the central pixel gives a centroiding error 15 times smaller than the random error

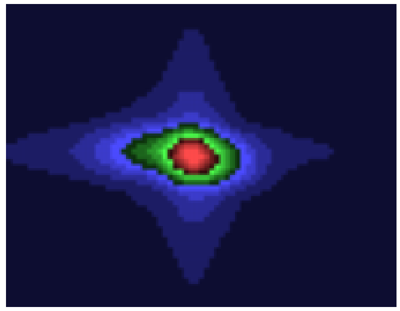

Figure 1: PSF *oversampled by a factor 4 for astrometric field point 09*

considered star is horizontally (along-scan) centered on a pixel. By “pixel saturation” we mean the beginning of non-linearity, not the full-well capacity. Assuming a saturation at 250ke$^-$ per pixel, the magnitude at which the first pixel becomes saturated occurs at $G \approx 10.8^{\rm m}$. Depending whether the star is vertically centered on a pixel or located across two pixels, the central line would be discarded or not, and the downloaded flux would be respectively $\approx 52\%$ or 96% of the total flux.

Let us focus what happens at e.g. 10.5$^{\rm m}$. Depending whether the star is vertically centered on a pixel (+0) or located across two pixels (+0.5), one or two central lines will be discarded. Between these two cases (± 0.25 pixel), one line only would be discarded. [2] The LSF of such a star, normalized at 1000 for the central pixel, is indicated for 3 different cases in Table 1.

Table 1: *Line-spread function for a* $G = 10.5^{\rm m}$ *star depending on the across-scan sub-pixel centering*

-0.25	13	18	27	53	107	183	324	894	1000	439	140	45	17	10	8	7	6
+0	14	20	30	60	121	209	361	930	1000	422	135	45	19	11	9	8	7
+0.25	15	20	32	64	129	232	388	961	1000	400	126	43	19	12	10	8	7

We use the program `centroiding.f` given in GAIA-LL-032, in order to compute the average centroiding bias and the centroiding precision. In this program we use an “adopted” LSF in subroutine `locest.f` while we introduce the “true” LSF in main program. We assume in what follows that the LSF adopted during the data reduction corresponds to the $+0$ case.

If this adopted LSF is the same as the true LSF, then the centroiding computed with the program given in GAIA-LL-032 gives a systematic centroiding error smaller than 0.00002 pixel, with a centroiding precision of about 0.0013 pixel. On the contrary, if the true LSF is the -0.25 case whereas the $+0$ case was assumed, then the systematic error is 0.06 pixel, 45 times larger than the random error.

The across-scan position will of course be known to better than 0.25 pixel. From the ASM measurements, the average across-scan positions of the 20$^{\rm m}$ stars are known to not better than 0.1 sample = 20 mas, as obtained when testing the APM and SWA detection algorithms in practice (IWG-OPM-002, p. 5). Using ASM1 and ASM3, the on-board attitude is thus known to better than $\approx 20\sqrt{2}/(\sqrt{2 \times 550} \times 2 \times 0.86{\rm s}) = 0.5$ mas/s since 550 stars are processed per second per instrument on the average [3]. Trying to do an across-scan centering on a 10.8$^{\rm m}$ bright star with the current 1×4 sampling would give at best a $0.00156 \times 4 \times 112 = 0.7$ mas precision (Sect. 4.2), about the same value. These two independent estimates suggest that a 1 mas precision for the across-scan position in each CCD is sensible, although a factor 2 or more (see end of Sect. 4.2) may still be gained. We thus scale the bias by the factor 1 mas$/(0.25 \times 112$ mas$)$ in what follows.

The absolute value of the bias and formal precision of the centroiding as a function of magnitude are shown Figure 2 left, for field point 09. Between 9$^{\rm m}$ and 10$^{\rm m}$, the factor 4 oversampling of the PSF was actually not sufficient to obtain a correct estimate of the bias, and it was needed to compute a PSF with a

[2]However, as the star may have to be observed the same way in all the AF fields, but will have different sub-pixel position in each of the CCDs, this may imply that the on-board sampling should be applied according to the worst case, meaning here two central lines discarded. For simplification purpose, we consider only the best case with one line discarded only.

[3]The GSR, p. 160, assumed 300 observations per instrument with a 2-10 mas individual across-scan precision

factor 40 oversampling using the `psf.f` program by L. Lindegren. For computational reasons, this was not done for other points of the astrometric field, bottom middle (02), and ”center” (01) and for $V - I = 4$, represented Figure 2 right. The bias is larger (up to $2\times$) than the random error between $8^{\rm m}$ and $10.5^{\rm m}$ on the whole astrometric field.

Concerning now the formal precision we represent the case where the number of suppressed lines is the minimum one, as noted above. Depending where the star is located on the pixel, the precision may vary, and is represented for a different location Fig. 3. Moreover, this precision is based on photon noise only: calibration errors (40μas per CCD crossing, GSR p. 261) could be the real limiting term for the astrometric precision of the stars brighter than $8^{\rm m}$. The indicated precision should thus not be used directly to compute the astrometric precision.

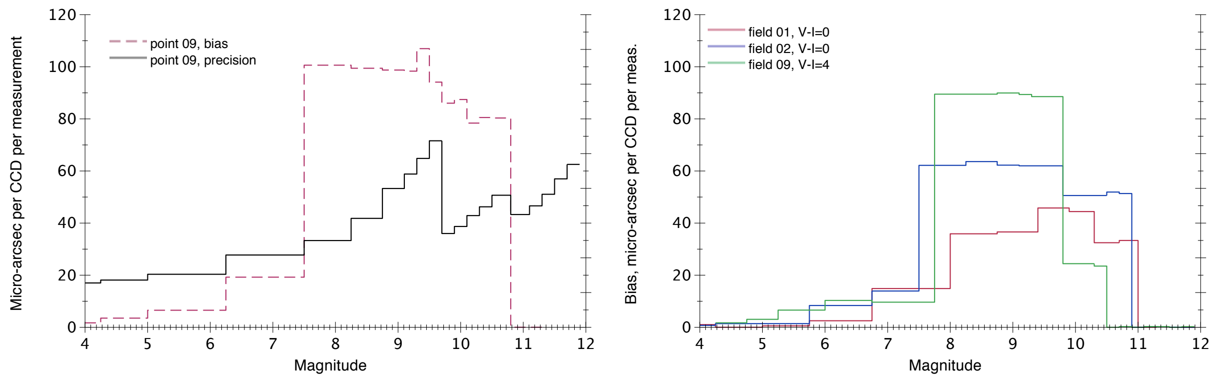

Figure 2: *Accuracy (assuming a 1 mas precision for the across-scan position) and precision of centroiding for astrometric field points 09 (left), bias for astrometric field 01 and 02 at $V - I = 0$ and for 09 at $V - I = 4$ (right)*

The question is also whether the systematic effect on centroiding translates as a systematic for the astrometric parameters or whether this would simply degrade the precision. Depending on the scanning law, the star will enter the focal plane at various ordinates for each focal observation epoch. This ordinate may be considered as random, so the bias will be reduced, like the random errors, by a factor $\approx \sqrt{2 \times 67} = 12$ on the average for the final astrometric parameters. However, for each field crossing, there are 16 CCD measurements, where the across-scan positions of the star from CCD to CCD are clearly correlated through the transverse motion and thus from the attitude knowledge. In general, negative biases on some CCDs will more or less be compensated by positive biases in the other CCDs. However, taking the extreme case of a transverse motion close to 0 (modulo one pixel/CCD), the bias would not be reduced by the average of the 16 measurements if the same LSF was used for the centroiding (which would thus be a bad idea), while the random errors would be reduced.

3 The baseline design

It would be tantalising to come back to the baseline design. The presence of the gates is such that a bright star will fit in a patch, and the systematic error exhibited above is not expected.

This does not mean that there are no problems here. There are two different ways to discuss the question of systematics, corresponding to what has been done respectively for the astrometric and photometric accuracy analysis in the GSR. The former evaluates the level of the various expected effects, the latter estimates at which precision the calibrations may be done to correct them (lower detectable bias).

We follow here this latter approach. For “faint stars”, the TDI is such that a line (along-scan) appears as a single entity. For bright stars, only a fraction of the line is used, depending on which gates are activated, this fraction being more and more small with the brightness of the star. This means that each of the parts between gates must be calibrated independently using stars of several range of magnitude.

In the worst case of the brightest stars, there are ≈ 500 stars brighter than $4^{\rm m}$, with ≈ 100 transits on the 10 across-scan CCDs. Assuming that all these stars may be used for calibrations (obviously not the case), this gives about 5000 observations per CCD for the whole mission. If calibrations occur every 6 months of a 5-year mission, there will be 0.25 observations on the average for each of the ≈ 2000 across-scan lines. This number translates to 5 observations for stars between $6^{\rm m}$ and $7^{\rm m}$. So, for stars brighter than $8^{\rm m}$, the number of observations will clearly not allow the CCD calibrations.

4 Proposals

From the previous discussion, it is clear that a spike method should be used but that the design would have to be improved in order to decrease or to suppress the systematic effects. We explore in turn three different solutions which are not exclusive and which have advantages and drawbacks. The first two try to reduce the bias level whereas the last one is conceptually better but is perhaps not technically feasible in practice.

Before that, it is worth mentioning the question of amplification. It could be sensible to make the saturation occur for the smallest number of stars, and for that purpose to manage that the saturation occurs first at the level of pixels, not at the sample level. An amplification amp=3 does not increase the dynamical range enough to make this complication worse it. However we may investigate the possibility of using an amplification of 1 that would increase the dynamical range to 1500ke^{-} (according to A. Holland, GAIA-LU-TN02). This would have a larger impact as this would allow to take samples much closer to the PSF center for the very bright stars: at $4^{\rm m}$ one can take a first sample of 3 pixels only after the $15^{\rm th}$ pixel with amp=6 while this can be done after the $10^{\rm th}$ pixel with amp=1. Also, with amp=1 the stars between $11^{\rm m}$ and $12^{\rm m}$ do not saturate at all (with amp=6 the sample would saturate but none of the pixels). This option was assumed in this document.

4.1 The across-scan attitude

Whereas the along-scan attitude will be largely improved on ground, this not seem the case for the across-scan attitude. The downloaded 1×8 samples in the astrometric field give no resolution across-scan, and the 5×5 ASM3 samples alone do not help much. One way to improve the across-scan attitude precision is thus to make use of the length of the astrometric field.

For this purpose, we have several possibilities impacting on the focal plane design and on the telemetry:

- two options are possible, each needing a $\times 2$ sampling: either to add a sky mapper at the end of the field, giving a factor 12 improvement on the attitude, or to design differently the central samples of AF17, with a factor 10 improvement;
- either the attitude is simply improved on-board, or a patch is downloaded for an on-ground improved attitude reconstruction by comparison with ASM3. In the first case, more on-board resources are needed (running a centroiding algorithm for each object in another CCD). The latter choice is probably preferable since a better centroiding algorithm using the adapted PSF, and a better background knowledge will be available on-ground. This means however a $\approx 12\%$ increase of telemetry per astro instrument if 25 more samples per object are downloaded.

In summary, the bias could be reduced to less than 20% of the random error, the current sampling design for bright stars would not have to be changed, the attitude improvement would benefit to all stars, and it can be noted that the supplementary measurements could be useful for NEO measurements too.

4.2 Sampling of bright stars

What is needed to decrease the bias is the across-scan position of the bright star on a given CCD. The above section could suggest that the bias is a matter of calibration, whereas it is simply due to a censorship on observed data with an estimator which does not take this into account. In the previous section we relied on an indirect position through the attitude, here on the direct position obtained with the bright star alone on the CCD.

This would imply to use the best resolution as possible across-scan, i.e. one pixel per sample. This sampling would remove the need for the amplification, and the drawback of a $\approx 13\%$ increase of the telemetry compared to the baseline, assuming the 16×24 SAG-CUO-100.4 design, not taking into account the increase of telemetry for stars fainter than 12 as described in SAG-CUO-100.4.

A bi-dimensional PSF fitting should replace the LSF fitting. The worst precision can be computed when the flux is minimum, when the first line is suppressed because of saturation, at 10.8^{m}, although this depends on the exact location of the star on the pixel.

The fitting algorithm with censorship on saturated pixels is still to be written. In order to have an estimate of the precision, we assume that the same precision would be obtained with a vertical centroiding as with the horizontal centroiding. In this case, which is optimistic, the centroiding algorithm would give a precision of 0.00156 sample for a 10.8^{m} star. The across-scan position of the star would thus be known at the level of $0.00156 \times 112 = 0.175$ mas per observation, and the bias would be reduced by a factor 5.7. The covariance between the abscissa and ordinate resulting from the simultaneous determination of both is however not taken into account.

In turn, the on-ground across-scan attitude knowledge would benefit from the across-scan position of bright stars: with 2 million stars brighter than 12^{m}, i.e. $2 \times 67 \times 2$ million field transits with 16 measurements over 4 years means 34 measurements/s on the average, which gives 0.03 mas/s, assuming the optimistic 0.175 mas estimation for the across-scan position. Incidentally, if the 1×4 sampling of SAG-CUO-100.4 is conserved, then the attitude may be known at the level of 0.12 mas/s, still on the very optimistic side.

4.3 Use of horizontal spikes

If, instead of upper and lower spikes, the left and right spikes were used, the systematics would not occur. There are three questions here: a) can a centroiding be done on a LSF with central columns missing, b) would there be enough flux to achieve the same precision, c) will the left and right spikes be contaminated by the saturated pixels.

Concerning the first point, there are no reasons why such a centroiding could not be done, since the censorship can be taken explicitly into account into the maximum likelihood analysis, although a larger number of centroiding failure may occur. A quick modification of `centroiding.f` and `locest.f` has been done, discarding the samples $> 1500\mathrm{ke}^-$ or containing a saturated pixel. Only one standard patch of 6 (high) samples on the left and one on the right of the saturated pixels were used, which has a negligible impact on the telemetry.

The result is shown Figure 3 and shows that the needed precision is obtained for stars fainter than 8.5$^{\mathrm{m}}$. The reason is that more flux remains when one column is removed (left/right spikes method) than when one line is removed (upper/lower spikes method), simply because the diffraction pattern represents 1.81 pixels along scan versus 1.46 pixels across-scan. The influence of the shallow slopes of the horizontal spikes occurs mainly for stars brighter than 7. There, the centroiding procedure fails in a large (still $< 3\%$) number of cases and the upper/lower spikes method is more precise. However, in terms of scientific return, the point is that the number of stars brighter than 8.5$^{\mathrm{m}}$ is negligible compared to the number of stars between 8.5$^{\mathrm{m}}$ and 12$^{\mathrm{m}}$ where the precision with the left/right spikes is comparable to the upper/lower spikes.

The last question is the major criticism against the use of horizontal spikes, not mentionning the charge transfer inefficiency problem. However, in the input to the CCD study, the antiblooming efficiency is asked to allow 2000 times the full well capacity. We need here only a factor 20 (between 7.5$^{\mathrm{m}}$ and 11$^{\mathrm{m}}$), if the upper/lower spikes method is used for the brighter stars.

The question here is whether the antiblooming is efficient both horizontally *and* vertically, and whether it is 100% efficient. If the saturated pixels alone are affected, the result above can be achieved. If one pixel left and right of the saturated pixels would also be affected, the uncertainty would increase by about 70%.

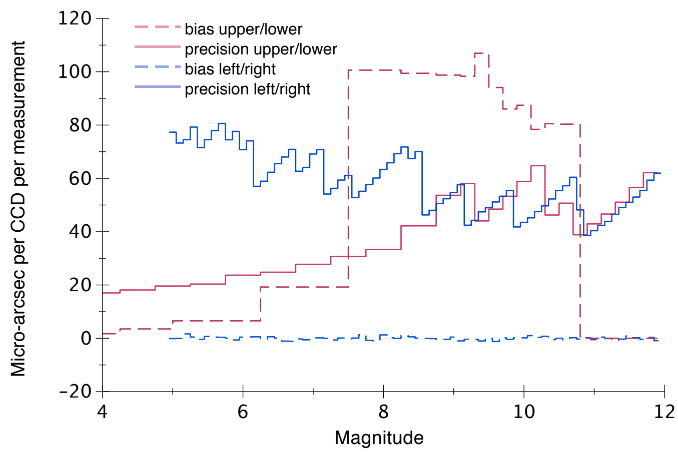

Figure 3: *Same legend as Fig. 2 left, using either the upper/lower or left/right spikes*

While this method may appear unrealistic, it should be noted that, with the two other methods, a ≈ 0.1 mas precision on the across-scan position is useless if the "theoretical" comparison LSF is not known precisely by steps of $\approx 1/1000^{\text{th}}$ across-scan pixels. It is not clear whether this may be easily achieved, because we may use only the sampling of bright stars for this purpose.

5 Conclusion

Bright stars are of large importance for the outcome of the mission, and it must be ensured that their accuracy is preserved and that their precision is improved. For the moment, the centroiding bias of bright stars is smaller but comparable to the random error. To be conservative until more realistic tests are performed, one could consider to further decrease the bias by mixing the alternatives suggested above, namely to improve the across-scan attitude using a measurement at the end of the astrometric field, or changing the sampling of bright stars, to download left and right spikes (small telemetry extra-cost) and to adapt the centroiding algorithm.

It should be noted that, due to the longer integration in the astrometric CCDs of the new satellite design, the saturation per sample will occur for stars ≈ 0.9 mag fainter than in the GSR design, while the transverse motion will limit the saturation per pixel, but will not reduce the asymmetry. Moreover, it appears that the full well capacity would occur at 190ke$^-$, pushing again the saturation towards fainter stars. What has been described in this document is thus still relevant with the new design.

6 Acknowledgment

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