ACCELERATION AND STOCHASTIC HIPPARCOS SOLUTIONS

Based on observations made with the ESA Hipparcos satellite

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Abstract. The Hipparcos astrometric parameters were obtained through the merging of abscissae data obtained by FAST and NDAC Consortia. Using a different weighting scheme, close but different astrometric solutions may be obtained. The resulting influence on acceleration (G) and stochastic (X) solutions is discussed; these solutions are studied from the point of view of the possible signature of astrometric binaries.

1. Introduction

The astrometric part of the main Hipparcos Catalogue (ESA, 1997) has been obtained using the data obtained by the Data Reduction Consortia (DRC) FAST and NDAC. These data are one-dimensional coordinates (abscissae) measured on reference great circles. Since the DRC used different reduction procedures, their results were not completely correlated, and a statistical improvement of the astrometric parameters was obtained from a merging of FAST and NDAC results.

The merging of the astrometric data, described in Arenou (1997), used these abscissae to calibrate the correlation coefficient between the DRC data. Using both the standard errors of the abscissae and the correlation between them, the covariance matrix of the observations was thus known and used by a least-square program, finally giving the astrometric parameters.

Apart from the standard 5-parameter model (position, parallax, proper motion), other models of star motion were needed (ESA 1997, Vol I, sect. 2.3).

Among them, acceleration solutions were produced for stars where the motion appeared significantly non-linear, and stochastic solutions were applied when other models failed to reproduce the scatter of the observations. The type of solution (G, X, ...) is indicated in Field H59 of the Catalogue. In both cases statistical tests were used to assess whether the solution was significant or not.

Using a weighted procedure instead, a different astrometric solution may be obtained. This procedure is described, together with its influence on G and X solutions.

The Hipparcos Intermediate Astrometric data (abscissae) are available in the CD-ROM 5 of ESA (1997),and described in vol I, section 2.8. Several applications of the intermediate data are described in van Leeuwen and Evans (1997). Another application, concerning the search for astrometric binaries, is shown section 4.

2. The weighted procedure

An alternative to the adopted merging procedure would have been to compute a solution using FAST data only, and another solution with NDAC data only. These solutions consist of the astrometric parameters, their standard errors and the correlations between them. Then, for each astrometric parameter a_i of a given star, a weighted mean of FAST and NDAC parameters may be computed:

$$a_{i\mathrm{W}} = w_i a_{i\mathrm{F}} + (1 - w_i) a_{i\mathrm{N}}$$

The optimal weight would be different for each star, and different for each of the astrometric parameters. The value w_i which minimises the variance $\sigma^2_{i\mathrm{W}}$ of $a_{i\mathrm{W}}$ is:

$$w_i = \frac{1 - \rho q_i}{1 - 2\rho_{i\mathrm{FN}} q_i + q_i^2} \tag{1}$$

where $q_i = \sigma_{i\mathrm{F}}/\sigma_{i\mathrm{N}}$ is the ratio of the standard errors and $\rho_{i\mathrm{FN}}$ the correlation coefficient between $a_{i\mathrm{F}}$ and $a_{i\mathrm{N}}$.

The standard error of the weighted mean parameter is then given by:

$$\sigma_{i\mathrm{W}} = \sqrt{\frac{1 - \rho^2_{i\mathrm{FN}}}{1 - 2\rho_{i\mathrm{FN}} q_i + q_i^2}}\,\sigma_{i\mathrm{F}} = q_i \sqrt{\frac{1 - \rho^2_{i\mathrm{FN}}}{1 - 2\rho_{i\mathrm{FN}} q_i + q_i^2}}\,\sigma_{i\mathrm{N}} \tag{2}$$

However, the correlation coefficient is unknown. It may be calibrated as follows, using the fact that the standard errors and correlations mainly depend from the magnitude.

Assuming that $\rho_{i\mathrm{FN}}$ and q_i are constant for a given magnitude, and developing $\mathrm{Var}((a_{i\mathrm{F}} - a_{i\mathrm{N}})/\sqrt{\sigma^2_{i\mathrm{F}} - 2\rho_{i\mathrm{FN}}\sigma_{i\mathrm{F}}\sigma_{i\mathrm{N}} + \sigma^2_{i\mathrm{N}}}) = 1$, the correlation coefficient may be estimated by:

$$\rho_{i\mathrm{FN}} \simeq \left\langle \frac{1 + q_i^2}{2q_i} \right\rangle \left[1 - \mathrm{Var}\left(\frac{a_{i\mathrm{F}} - a_{i\mathrm{N}}}{\sqrt{\sigma^2_{i\mathrm{F}} + \sigma^2_{i\mathrm{N}}}}\right)\right] \tag{3}$$

Each of two terms of the product in this equation may be calibrated as polynomials of Hp magnitude. Introducing this $\rho_{i\mathrm{FN}}(Hp)$ into Equations 1 and 2 gives the weighted astrometric parameter and associated standard error for a star of magnitude Hp.

In order to get the full covariance matrix of the weighted astrometric parameters, there remains however to compute the correlation between two different parameters. The covariance between two parameters $a_{i\mathrm{W}}$ and $a_{j\mathrm{W}}$ is:

$$\begin{aligned} \langle a_{i\mathrm{W}} a_{j\mathrm{W}} \rangle &= w_i w_j \langle a_{i\mathrm{F}} a_{j\mathrm{F}} \rangle + (1 - w_i)(1 - w_j) \langle a_{i\mathrm{N}} a_{j\mathrm{N}} \rangle + \\ &\quad + w_i (1 - w_j) \langle a_{i\mathrm{F}} a_{j\mathrm{N}} \rangle + (1 - w_i) w_j \langle a_{i\mathrm{N}} a_{j\mathrm{F}} \rangle \end{aligned} \tag{4}$$

The covariances $\langle a_{i\mathrm{F}} a_{j\mathrm{F}} \rangle = \rho_{ij,\mathrm{F}}\sigma_{i\mathrm{F}}\sigma_{j\mathrm{F}}$ and $\langle a_{i\mathrm{N}} a_{j\mathrm{N}} \rangle = \rho_{ij,\mathrm{N}}\sigma_{i\mathrm{N}}\sigma_{j\mathrm{N}}$ have been obtained through the least square astrometric solution of each respective DRC. The covariance $\langle a_{i\mathrm{F}} a_{i\mathrm{N}} \rangle = \rho_{i,\mathrm{FN}}\sigma_{i\mathrm{F}}\sigma_{i\mathrm{N}}$ has also been estimated, as explained above. Although the respective DRC do not use the same abscissae, the secondary covariance $\langle a_{i\mathrm{F}} a_{j\mathrm{N}} \rangle = \rho_{ij\mathrm{FN}}\sigma_{i\mathrm{F}}\sigma_{j\mathrm{N}}$ occurs since the same photons were used by both DRC. It may be computed using the approximation $\rho_{ij\mathrm{FN}} \approx \rho_{i,\mathrm{FN}}\rho_{ij,\mathrm{N}} \approx \rho_{ij,\mathrm{F}}\rho_{j\mathrm{FN}}$.

The above method has been applied to the parallaxes of all Hipparcos single stars. The median precision $\sigma_{i\mathrm{W}}$ is drawn in Figure 1. Concerning the standard errors, the weighted method seems roughly equivalent to the merged solution. It is less rigorous than the adopted merging procedure. Using the same basic data, it gives however a different solution, of course not significantly, which is used below.

3. The influence on G and X solutions

The Hipparcos G solutions were obtained using a statistical test equivalent to a Gaussian two-sided 3σ test, which was applied to a merged 7-parameter solution on the whole Catalogue: without a real acceleration, the $g_{\alpha *}/\sigma_{g_{\alpha *}}$ and $g_\delta/\sigma_{g_\delta}$ normalised acceleration components are expected to follow a Gaussian G(0,1), so that the unit-weight variance of the bi-dimensional Gaussian should follow a $\chi^2(2)$ distribution.

However, given the level of significance (.0027) and the number of concerned stars, several hundreds of stars may have been selected by chance only. This hypothesis is reinforced by the fact that the variance of the normalised accelerations may also have been slightly underestimated. It is then of interest to also apply a significance test to the weighted solution.

A 7-parameter astrometric solution has been computed using FAST data only, and one for NDAC only. A weighted solution has then been obtained for the two acceleration components. The correlation coefficients $\rho_{g_{\alpha *}\mathrm{FN}}$ and $\rho_{g_\delta \mathrm{FN}}$ were found almost identical, and the same calibration against magnitude was applied.

The normalised accelerations were computed. There mean was not different from 0 and their standard deviation was about 1.1. The standard deviation of the normalised differences $(g_\alpha * - g_\delta)/(\sigma^2_{g_{\alpha *}} - 2\rho_{g_{\alpha *} g_\delta}\sigma_{g_\alpha *}\sigma_{g_\delta} + \sigma^2_{g_\delta})$ is also 1.09, which suggests that the correlation coefficient has also been correctly estimated. It is about 35% smaller on the average than the correlation coefficient between abscissae obtained with the adopted merging procedure.

Applying the $\chi^2(2)$ test mentioned above to this weighted solution gives 2632 stars with a significant acceleration. This is roughly equivalent to the 2622 G solutions in the Hipparcos Catalogue, but with only 2182 common solutions. This shows the expected fluctuation on the detection of accelerated stars, suggesting that some of the accelerations of the G solutions may not be significant.

It may be noted that when using a level of significance equivalent to a 4σ Gaussian test, and taking into account the 1.1 unit-weight error of accelerations, only 920 significant accelerations are found. The latter have a smaller distance, as expected from selection effects (Figure 2).

The stochastic solutions were obtained with a 5-parameter model, where a supplementary scatter ('cosmic error") was quadratically added to the abscissae. The astrometric parameters are often more reliable than those obtained with a standard 5-parameter solution. Because of bad abscissae configurations, the parameters are also sometimes unreliable, which is however reflected in the given error bars; some of these stars had their astrometric parameters improved after the finalisation of the Catalogue, the new parameters being indicated in a note.

A stochastic solution has been computed for FAST and NDAC data only. Applying the same level of significance ($\epsilon > 5\sigma_\epsilon$ corresponding to about a 3σ Gaussian two-sided test), only 330 stars would need a stochastic solution for both DRC. This is to be compared to the 1561 X entries in the Hipparcos Catalogue. It is then possible that part of these solutions were selected because of some bad observations only.

4. Astrometric binaries

New astrometric binaries were found during the data reduction process and their orbits were determined or improved. There are a total of 235 O (orbital) solutions in the Catalogue.

In the case of most G solutions, the significant curvature of the motion of the photocentre is likely the manifestation of the binary nature of the concerned stars; the situation concerning X solutions is less obvious. It is however of interest to check whether orbital binaries could be among these stars. This may be done by searching for a quasi-sinusoidal signature in right ascension and declination. Another approach using periodograms has been attempted for the detection of binarity (Bernstein and Bastian, 1996) and applied to planetary candidates (Perryman et al., 1996).

The problem is however complicated on one hand by the short time span of the Hipparcos mission. On the other hand, the abscissae are one-dimensional data and the precise position of the star at a given observation epoch is then not determined directly. Moreover, the observations are not too numerous and not evenly distributed in time.

The position may however be inferred from the combination of some observations at a few different epochs: for each of these mean epochs, the Hipparcos parallax and proper motion components were held fixed, and the position components were obtained by a weighted least-square. In order to get precise positions, enough degree of freedom are needed, so that abscissae were combined over up to 100 days, which does not help to find short period binaries.

Among all X and G solutions, some new suspected astrometric binaries are shown in the Figures below. HIP 39903, a suspected spectroscopic binary (Murdoch and Hearnshaw, 1993), is confirmed. The positional residuals are shown in the following Figures.

A simultaneous fit of the astrometric and orbital parameters has been done, using the same program which was applied for the O solutions. However, the problem being non-linear, a good initial value is needed in order to converge to a plausible solution. In most of the cases, other ground-based data is needed in order to confirm the suspected binarity and to give a reliable orbit.

References

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