Orbital characterisation of binaries from Monte Carlo inversion

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Introduction

Orbit determination is needed to have an initial guess of the orbital elements in a gravitationally bounded binary system. When the problem is poorly constrained it can be useless to provide a single solution; but on the other hand a bundle of solutions can be obtained from statistical inversion and Monte Carlo techniques. We present a method of Monte Carlo inversion based on the Thiele-Innes approach that enables to sample only one parameter instead of the full orbital phase-space. The method is adapted to the non-linear situations when a small number of observations is available, or also when observations are spread over a short arc. The method can be used for stellar resolved binaries as well as for solar system binaries. We present some results and prospects for trans-Neptunian binaries and binaries observed with Gaia.

1. Monte Carlo inversion

2.1 *Thiele-Innes*

One powerful and well-known method of orbit determination for visual (or, more generally, resolved) binaries is due to Thiele [1] completed by Innes and van den Bos. Starting with the relative position of the resolved pair (x_j, y_j, t_j) Thiele derived the fundamental relation relating the eccentric anomaly to the triangles for observations at time t_p and t_q:

$$\begin{aligned} &(t_q - t_p) \; - \Delta_{qp}/¢ = 1/n \, [\, E_q - E_p - \sin(E_q - E_p)] \\ &\Delta_{qp} = x_p \, y_q \text{ - } x_q \, y_p \\ &n = 2\pi/P \end{aligned} \tag{1}$$

where $¢ = \rho^2 \, d\theta/dt$ is the areal velocity and P the orbital period of the pair. The method is thoroughly described in the book of Aitken [2]. The apparent ellipse of coordinates (x, y) is given by the parameterization:

$$\begin{aligned} &X = \cos(E) \text{ - } e \\ &Y = \text{sign}(¢) \; (1\text{-}e^2)^{1/2} \; \sin(E) \end{aligned} \tag{2}$$

and the constants derived by Innes:

$$\begin{aligned} x &= A\,X + F\,Y \\ y &= B\,X + G\,Y \\ z &= C\,X + H\,Y \end{aligned} \tag{3}$$

to which we have added the equation for the radial component z, that—while irrelevant for the apparent projected ellipse—will be of particular use for the binaries in the Solar System described below. These Thiele-Innes constants (A, B, F, G, C, H) can be directly related to the classical elliptical elements (a, i, ω, Ω). Cid [3] has given the proof that 3 full observations (2D relative position and time, x_j, y_j, t_j) and knowledge of the areal constant or equivalently of the orbital period, are necessary and sufficient to derive—exception made of degenerate cases—the 7 parameter of the orbital solution. As reminded in [4], having a set of non-linear equations to solve in Eq. (1), some problem of non-convergence can occur depending on the initial value given. Nevertheless, the generalized least-squares solution, minimizing all the O-C for all observations at once, is not foreseen here; we are dealing instead with a heuristic (quasi)forward problem. Moreover the initial guess and system to solve is changing at each step of the Monte Carlo process described hereafter.

2.2 *Monte Carlo sampling of the orbital space*

In a way very much similar to statistical ranging technique of Virtanen et al. [5, 6] for deriving initial (heliocentric) orbit of solar system objects, and somewhat similar to the approach of Docobo [7] for binary star, we have derived a method for initial orbit determination of visual binaries [8]. Here the aim is not strictly to find the best fit or minimal solution, but to give from a heuristic approximation the bundle of possible solutions and explore the uncertainty region around a minimal (or preferably inferior) χ^2 fit solution. The basement of the algorithm is to choose randomly, among all available observations and considering realistic observational errors, 3 base points, and one guess for the orbital period from which a solution, if it exists, can be derived. In a second step, all observations are checked for the |O-C| norm with the proposed solution; if they pass the rejection criterion based again on some realistic observational error assumption, a possible solution is retained. The trial-and-error process is simply repeated in a Monte Carlo loop where the orbital period P can be sampled in the most largest way[1]. Note however that — thanks to the Thiele-Innes relation — only one dynamical parameter among the seven target parameters has to be sampled in this way to explore the full 7 dimensions orbital space phase. It is stressed also that the 3 base points should be in the same apparent orbital plane or observations block; this is important for solar system binaries as will be seen in next section. Such technique is adapted to modern computer and easily parallelised. In summary the algorithm is constructed on the following scheme:

- choose 3 base observation points and choose an orbital period;
- add noise to the available observational data;
- derive the orbital solution through Thiele-Innes algorithm;
- test for the |O-C| on all available observations[2]. If successful, keep the orbit; if not, reject the solution;
- repeat loop.

Testing the O-C for all observations necessitates to take into account the parallax of the system or the precession of the reference frame. Obviously when it is a binary in the Solar System the apparent ellipse is drastically changing over time, yielding a modification of the Thiele-Innes constants. These are computed from the change in the FOV direction by the matrix product:

$$\begin{pmatrix} A' & F' \\ B' & G' \\ C' & H' \end{pmatrix} = P_{FOV} \begin{pmatrix} A & F \\ B & G \\ C & H \end{pmatrix} \tag{4}$$

where the transition matrix P_{FOV} depends on the directions (α,δ) and (α',δ') of the FOV for each observing block.

The Thiele-Innes method can be extended to binaries in the solar system as noted above; in this case and because of the typical scarceness of the observations available, the orbital period can be given as input parameter in Eq. (1) instead of the areal velocity. Moreover, the Thiele Innes method can be extended to the general case of conics [9] by making use of universal variables, so that widely

[1] There is a priori no limitation in the range of the orbital period, if the actual observations however cover almost

[2] Oulier observations can also be detected in such process, and subsequently removed, since they do not enter systematically in the solution computation.

separated systems with very long orbital period can be checked for being actually bounded or in a more simple gravitational interaction. The fundamental equation in the hyperbolic case can be written:

$$(t_q - t_p) \; - \Delta_{qp}/¢ = 1/\nu \; [\; F_q - F_p - \sinh(F_q - F_p)] \qquad (5)$$

where $\nu^2 = \mu/a^3$, and F_i is the hyperbolic anomaly. More generally, one has in universal variables

$$(t_q - t_p) \; - \Delta_{qp}/¢ = \mu \, \nu_3 \; [\; s_q - s_p \; ; \; h] \;) \qquad (6)$$

where h is the energy, $\nu_n \; [s \; ; \; h] = s^n c_n \; (-2hs^2)$ are the Stumpff functions, and negative orbital periods correspond by convention to hyperbolic orbits.

2. Binaries in the Solar System

While it has long been suspected since the beginning of the XX^th century with [10] that binary systems are present among asteroids [11, 12] (mainly from unusual lightcurves or secondary events during stellar occultations or binary craters), no clear evidence had been obtained before the direct imaging of the moonlet Dactyl orbiting around asteroid Ida made fortuitously by the space-probe Galileo en route to Jupiter. Later, during the second half of the '90^s, imaging with adaptive optics and radar techniques and clear observations of eclipsing events or non-synchronous systems from photometry revealed that binary systems are present in various dynamical classes of asteroids in the Solar System[3] (near-Earth objects, Mars crossers, main belt asteroids, Trojans of Jupiter, Centaurs and trans-Neptunian objects), see Table 1. These observations also revealed a large variety of binaries, with either low or large mass-ratio, quasi circular and equatorial or significantly eccentric orbits, binaries among dynamical families, multiple systems with several moonlets, etc. Besides, several formation scenarii have been proposed for the different classes of objects and are still under study.

Table 1: Number of binaries and systems detected among the different population of small bodies in the Solar System, and for different observing techniques ('Imaging' from ground- and space-based telescopes).

Type	Imaging	Radar	Lightcurve	Total
NEOs	0	23	14	37
Mar crossers	0	0	14	14
MBAs	21	0	54	76
Trojans	2	0	2	4
TNOs	74	0	1	75
Total	97	23	86	206

2.1. Trans-Neptunian objects

Solar system objects orbiting the Sun beyond Neptune, such as Pluto, are called trans-Neptunian objects (TNOs, or also Kuiper belt objects, KBOs); Centaurs are those crossing the orbits of the outer

[3] Not mentioning the Pluto system with Charon being discovered in 1987.

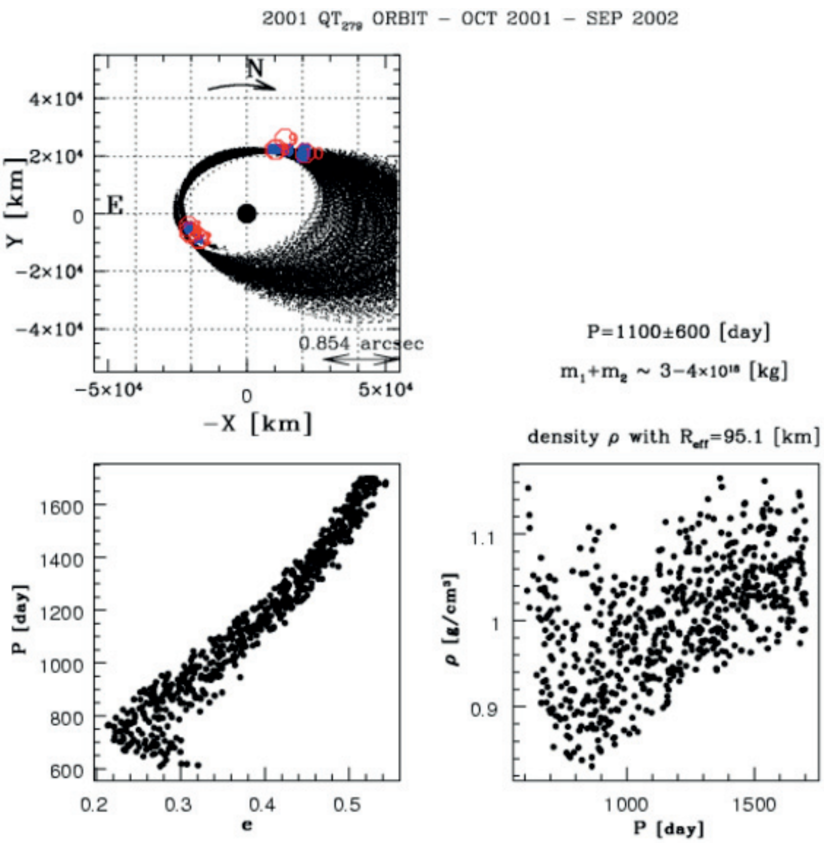

Fig. 1: An example of poorly constrained orbital solution. Nevertheless, one of the fundamental physical parameter—that is the total mass of the system—can be derived with sufficient precision to enable further analysis.

planets. About thousands of such bodies have been discovered since the progress achieved in observational surveys, starting in the '90s. Due to their distance from Sun and Earth and relative small sizes they are faint, with magnitudes typically in the range V≈20–24, and more difficult to observe in ground-based telescopes. Among those bodies a significant fraction is made of binaries showing generally significant mass ratio. In the case of trans-Neptunian binaries (TNBs) their faintness and small apparent separation, implies the use of large ground-based telescopes equipped with Adaptive Optic systems (Keck, VLT, …) or space-based HST to obtain the relative astrometry of the orbiting pair, and hence orbital parameters estimation.

In the case of ground-based observations with the ESO/VLT at Cerro Paranal, one will still need a guide star (either natural NGS, or from laser LGS) to enable the AO system loop operation, since the target itself is not bright enough. Even if making use of the LGS, a tip-tilt start must be available, so that in any case a star needs to be close enough of the scientific target. These constraints make the observations of TNB possible only during an appulse with a star that are computed at IMCCE, which appulse star must be bright enough and with good observational condition. Moreover the ephemeris of the TNO needs to be known with enough accuracy to ensure successful target acquisition in the small field of view at NACO or SINFONI cameras. In our observing program at VLT, observations with NACO (S13 or S27 camera with pixel scale of 13 and 27 mas, respectively) are used to derive one additional astrometric point completing the one already published (mainly from HST) and colour-photometry in the near-infrared (J,H,K), completing the one obtained from HST in the visible. Observation with the spectro-imager SINFONI camera also enables imaging of the pair and derivation of an astrometric point. The main objective in this case being to extract the spectra for both components; it is nevertheless only feasible for the brightest target because of strong SNR limitations, and even so exposure time needs to be long [14, 15].

Due to the inherent difficulties and need of large facilities to obtain resolved observations of binaries in the Solar System, data for such system are often scarce and one needs to derive the full information or constraint, or ensure that any least-squares solution (or maximizing some merit function) is free from local minima effects. The Thiele-Innes method, which is dynamical in essence, is more powerful than geometrical methods because of this observations scarcity. Since only one parameter is sampled (the orbital period) and the algorithm is easily parallelised, computation of a number density of the solution space can be reasonably fast. Even if the orbit is not fully constrained, knowledge of the system's mass and subsequently its density can be of high value to shed some light on the body's composition (see Fig. 1).

Being distant objects, trans-Neptunian have small parallax, yet much larger than stellar systems, so that after several years the apparent orbit will change. It is hence possible to derive the full true orbit and remove the typical inclination ambiguity [16]. This is of particular interest to derive accurate predictions of mutual events (occultation and eclipse), or prediction of stellar occultations by the secondary [14], and to test formation of binary systems [17].

2.2. Other prospects

The Gaia mission will enable modest imaging, yet free of seeing and with very stable and well calibrated PSF. Although Gaia will provide some additional constraints on the Pluto/Charon system improving indirectly the mass determination of the small satellites Nix and Hydra [18], TNBs will be generally too faint to be detected. On the other hand Main belt asteroids, Trojans and Centaurs will be monitored during the 5 years missions providing approximately 60 observations per target. The mass determination of known binaries will be drastically improved together with the primary's quadrupole J_2 and other perturbative parameters. Some binary and multiple systems might be discovered directly from the imaging, however it is stressed that observations in the highest resolution are 1-dimensional only, with a pixel size of 60 milli-arcsec; while observations within the star mapper are 2-dimensional but with a 2×2 pixel binning. If not resolved, high accuracy astrometry can put into evidence the wobble of an astrometric binary. This can be performed directly from the Gaia observation of a solar system body, when the relative orbit frequency and spin frequency of the primary are clearly different — excluding hence systems that are double synchrone (i.e. when orbital period and spin of each component are equal), and also when the combination of the couple (mass-ratio, separation) is adequate to allow an astrometric signature at the milli-arcsec level. Indirectly, the Gaia stellar catalogue will also provide some improvements in the study of astrometric binaries. Presently astrometric binaries among TNOs should be numerous since the number of binaries is increasing with decreasing separation [19]. First attempt to detect such signature has successfully been performed on a known system with modest telescope and local astrometric catalogue [20].

Conclusion

A method for orbit determination based on Monte Carlo inversion and Thiele Innes algorithm has been presented. It is adapted for exploring the full space of solutions and to derive the bundle of orbits based on few scarce and sparse data. Applications for trans-Neptunian binaries are given as well as prospects for astrometric binaries and binaries observed with the space mission Gaia.

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