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VELOCITY DISPERSIONS FOR GALACTIC GLOBULAR CLUSTERS

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ABSTRACT This appendix presents a bibliography for and summary of the velocity dispersion data available for 56 galactic globular clusters. Dispersions are now available for clusters spanning the whole observed range of properties, except for clusters of very low mass. The dispersion data are rather heterogeneous, so we have used them to produce a more homogeneous set of central velocity dispersions. These are used with a family of multi-component King-Michie models to estimate cluster masses, central densities, global mass-to-light ratios, and central mass-to-light ratios. Finally, we have also included a discussion of the maximum likelihood estimators for the mean velocity and dispersion of a set of velocities with differing measurement uncertainties.

INTRODUCTION

This appendix documents the tremendous progress that has been made in measuring the velocity dispersions of globular clusters since the pioneering efforts of Illingworth (1976), Da Costa *et al.* (1977), and Gunn & Griffin (1979) in the 1970's. The history and techniques behind the data and some of the conclusions drawn from it are discussed by Meylan & Pryor, these proceedings. It is probably

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fair to say that the initial survey of cluster dispersions has been completed, particularly after the large number of integrated-light measurements by Dubath *et al.* (1993). Table I lists dispersions for clusters with the whole observed range of metallicities, with the whole observed range of central concentrations, with distances from the galactic center between 0.9 and 99 kpc, and with absolute magnitudes between −4.8 and −10.4. While the current sample is not representative in a statistical sense, it does include at least some examples of nearly all types of clusters. Only those with the very lowest luminosity have escaped measurement because of their lack of bright stars and low expected dispersions.

While the imbalance between sophisticated theory and sparse observation that was apparent in 1984 at IAU Symposium 113 has been significantly redressed, new methods for precisely measuring the distribution of mass within clusters and the increasing sophistication of models for the dynamical evolution of clusters are demanding better velocity dispersion data. Fortunately, rapid progress can be expected in the next few years because of two technological innovations: multi-fiber spectrographs and narrow-band imaging using a Fabry Perot. These two complementary techniques are described in Meylan & Pryor, these proceedings (see also Gebhardt *et al.* and Reijns et al., these proceedings). Because of the current rapid progress in this field, the summary of data in Table I is not complete even at the time of this writing. We have included some unpublished data that was easily available to us. Our excuse for not including more is that there is too much of it! We have included references to older work that has been superseded by more recent studies. The reason for this, besides acknowledging the noble pioneers, is to facilitate searches for stars with variable radial velocities (such as binaries). Early studies provide very important first-epoch velocities.

The rest of this paper begins with a brief digression on the maximum-likelihood estimator used in this paper to calculate velocity dispersions from samples of radial velocities. It then describes how the data are presented in Table I and the methods used to calculate the derived cluster properties given in Table II.

MAXIMUM LIKELIHOOD DISPERSION ESTIMATOR

While the maximum likelihood estimator for the mean velocity, $\overline{v}$, and dispersion, σ_c, of a sample of velocities, v_i, are well-known when the uncertainties of the v_i are negligably small or all the same, there seems to be some confusion in the astronomical literature about the correct estimators when the uncertainties of the v_i are different. While the errors that have been made in the calculated dispersions are minor, we couldn't resist this opportunity to point out the correct procedure.

Begin by *assuming* that each velocity v_i is drawn from the normal distribution:

$$f(v_i) = (2\pi(\sigma_c^2 + \sigma_{e,i}^2))^{-1/2} \exp\left(-\frac{(v_i - \overline{v})^2}{2(\sigma_c^2 + \sigma_{e,i}^2)}\right), \quad (1)$$

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where $\sigma_{e,i}$ is the known measurement uncertainty of v_i. When the uncertainties are not known, the maximum likelihood method described by Godwin and Lynden-Bell (1987) can estimate both $\overline{v}$ and σ_c and the average measurement uncertainties from sets of velocities with some stars in common. Standard techniques for forming the likelihood of a set of N velocities and finding its maximum lead to the two equations

$$\sum_{i=1}^{N} \frac{v_i}{(\sigma_c^2 + \sigma_{e,i}^2)} - \overline{v} \sum_{i=1}^{N} \frac{1}{(\sigma_c^2 + \sigma_{e,i}^2)} = 0 \tag{2}$$

$$\sum_{i=1}^{N} \frac{(v_i - \overline{v})^2}{(\sigma_c^2 + \sigma_{e,i}^2)^2} - \sum_{i=1}^{N} \frac{1}{(\sigma_c^2 + \sigma_{e,i}^2)} = 0. \tag{3}$$

Gunn & Griffin (1979) derived a very similar set of equations for determining the velocity parameters of King-Michie models. These equations must be solved numerically for $\overline{v}$ and σ_c, but this can be easily done by iteration. If the measurement uncertainties are all the same constant σ_e, eqs. (2) and (3) reduce to the usual

$$\overline{v} = \frac{1}{N} \sum_{i=1}^{N} v_i \quad \text{and} \quad \sigma_c^2 = \frac{1}{N} \sum_{i=1}^{N} (v_i - \overline{v})^2 - \sigma_e^2. \tag{4}$$

These equations remind us that maximum likelihood dispersion estimators are biased. Monte Carlo simulations show that the bias is larger than the usual $(N/(N-1))^{1/2}$ when the $\sigma_{e,i}$ are not equal to zero. The bias grows as N decreases or the $\sigma_{e,i}$ increase relative to σ_c. For example, it is roughly twice the usual value when $\sigma_{e,i} \simeq \sigma_c/2$ for N = 20 – 50. However, the bias is always much less than the uncertainty in the dispersion. Those who wish to remove the bias should perform Monte Carlo simulations modeled on their data.

The simulations mentioned above can also be used to determine the uncertainties in $\overline{v}$ and σ_c. However, it is convenient to have a simpler procedure to estimate these uncertainties. For large samples, the estimators of the two model parameters have a bivariate normal distribution with a variance-covariance matrix that is the inverse of the information matrix (e.g., see Sarazin 1980 and Strong 1985 for a discussion and references to the extensive statistical literature). As long as the covariance is small compared to the variances, the variances are reasonable estimates of the uncertainties of the parameters. In our experience, this is always the case when the velocity measurement uncertainties are smaller than the cluster dispersion and the sample consists of more than several stars. Some tedious algebra shows that the variances and covariances are

$$\text{var}(\overline{v}) = I_{22}/(I_{11}I_{22} - I_{12}^2) \tag{5}$$

$$\text{var}(\sigma_c) = I_{11}/(I_{11}I_{22} - I_{12}^2) \tag{6}$$

$$\text{covar}(\overline{v}\sigma_c) = -I_{12}/(I_{11}I_{22} - I_{12}^2), \tag{7}$$

where

$$I_{11} = \sum_{i=1}^{N} \frac{1}{(\sigma_c^2 + \sigma_{e,i}^2)} \tag{8}$$

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$$I_{22} = \sum_{i=1}^{N} \left(\frac{1}{(\sigma_c^2 + \sigma_{e,i}^2)} - \frac{(v_i - \overline{v})^2 + 2\sigma_c^2}{(\sigma_c^2 + \sigma_{e,i}^2)^2} + \frac{4\sigma_c^2 (v_i - \overline{v})^2}{(\sigma_c^2 + \sigma_{e,i}^2)^3} \right) \tag{9}$$

$$I_{12} = \sum_{i=1}^{N} \frac{2\sigma_c (v_i - \overline{v})}{(\sigma_c^2 + \sigma_{e,i}^2)^2}. \tag{10}$$

When all the $\sigma_{e,i}$ are the same constant, σ_e, the covariance is zero and the above equations reduce to the intuitively reasonable

$$\mathrm{var}(\overline{v}) = (\sigma_c^2 + \sigma_e^2)/N \tag{11}$$

$$\mathrm{var}(\sigma_c) = (\sigma_c^2 + \sigma_e^2)^2/(2N\sigma_c^2). \tag{12}$$

Monte Carlo simulations show that the uncertainties given by eqs. (5) and (6) are slightly conservative for large N and accurate to better than about 5% for N as small as 20. CP has a Fortran program that calculates $\overline{v}$, σ_c, and their uncertainties that he will send to anyone who requests it.

THE VELOCITY DISPERSION DATA

Table I presents a summary of the velocity dispersion data available for galactic globular clusters. The first column gives the designation of the cluster in IAU coordinate format and the next two columns give various cluster names. The fourth column is a number identifying the reference from which the rest of the data on that line were derived. Column five contains "int" if the dispersion was derived from the Doppler broadening of an integrated-light spectrum and the number of stars in the sample if it was derived from individual radial velocities. GM has assembled most of the integrated-light data and CP most of the individual velocity data. Whenever the individual velocities were actually available, and this was in most cases, CP reviewed the data to exclude non-members and stars with variable radial velocities. The next three columns pertain to the individual velocity data and give the range of radii covered by the sample (in arcminutes), the average velocity uncertainty (in $\mathrm{km\,s^{-1}}$), and the mean cluster velocity and its uncertainty (in $\mathrm{km\,s^{-1}}$). When the velocities were available, the mean velocity, the dispersion, and their uncertainties have been calculated using the estimators described in the previous section. No corrections for dispersion bias have been applied. The dispersion and its uncertainty are given in the final column. For integrated-light data, these values have been taken directly from the study referenced.

We emphasize that these data, and the individual velocity data in particular, come from heterogeneous sources. We have made no attempt to remove zero-point differences between the different sets of radial velocities, which are likely present at the $\lesssim 1\ \mathrm{km\,s^{-1}}$ level. These don't affect the dispersion, but the quoted uncertainties in the mean velocities are statistical uncertainties only. The differences between the radial ranges of the stellar velocity data for different clusters also means that the quoted dispersions should be used cautiously.

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DERIVED CLUSTER PARAMETERS

We have derived from the data in Table I a more homogeneous set of central projected velocity dispersions, with the goal of aiding those wishing to pursue statistical studies of globular clusters properties (though note the discussion in the Introduction of the properties of the cluster sample). When only an integrated-light dispersion was available, we adopted that value as the central dispersion. When only individual stellar velocities were available, we derived the central projected dispersion by fitting a King model to the data, as described below. When there were multiple sets of data, we generally used the dispersion with the smallest statistical uncertainty. We caution that, particularly for clusters with collapsed cores, King models may not correctly predict the central dispersion from velocities at larger radii (for example, see Peterson et al. 1989).

Since the most common use of an homogeneous set of dispersions will be to calculate cluster masses and mass-to-light ratios (M/L's), we have calculated these using a family of isotropic King models having five mass components. The mean masses of the stars in each component are 1.2, 0.71, 0.50, 0.32, and 0.20 $M_\odot$. A two-piece power-law mass function with an exponent of 1.35 (about the Salpeter value) above a mass of 0.32 $M_\odot$ and an exponent of 0.0 below determines the numbers of stars and stellar remnants in each component. The mass function is truncated at 0.16 $M_\odot$ and (for the calculation of the number of remnants) at 8 $M_\odot$. For more details on these models and the fitting procedures, see Pryor et al. (1989, 1991). Combining this mass function with a mass-luminosity relation appropriate for globular cluster stars gives the models a "population" global M/L_V of 2.0. The population central M/L_V varies from 1.8 for the low-concentration models with little mass segregation to a minimum of 1.2 for c around 1.7 and then increases to 4.5 for c=3.0. Here concentration is $c=\log(r_t/r_s)$, where r_t and r_s are the model tidal and scale radii, respectively. M/L is given in solar units. The central M/L, $(M/L)_0$, used here is the central mass density divided by the central surface brightness. Only at concentrations larger than 2.2 does this $(M/L)_0$ differ from the central mass density divided by the central luminosity density by more than 0.3. In our experience, this family of models generally does a good job of fitting globular cluster surface brightness profiles and matching the population M/L with the "dynamical" M/L derived by fitting the model (e.g., Pryor et al. 1989, 1991; Meylan 1987, 1989).

Many data besides the dispersion are needed to calculate a cluster mass and M/L. We have taken cluster distances and reddenings from the compilation by C. Peterson, these proceedings. The central surface brightnesses, core radii, and concentrations come from Trager et al., these proceedings, except for a few cases where we have taken values from the literature or our own unpublished data. For clusters with collapsed cores, we have used the Trager et al. radius at which the surface brightness falls to half its maximum value as a core radius and have taken tidal radii from Webbink (1985). The "best-fitting" model for each cluster was determined by selecting the model in a grid with c = 0.4, 0.5, ..., 3.0 which had the closest match to the observed ratio of r_t and the radius, r_c, at which the surface brightness has dropped to half its central value. Note that r_s and r_c are not identical in the family of models used here. The Trager et al. data were used to set the model's central surface brightness and to determine r_c. The ratio of r_s

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and r_c for the model selected then determines r_s. This simple procedure ensures that the model surface brightness profile resembles that of the cluster, but does not guarantee that the model is a good fit to the cluster profile, particularly for large concentrations. Thus the cluster properties listed in Table II should not supersede those from individual detailed studies. Finally, the velocity scale of the model is set either by fitting the individual stellar velocities using the Gunn & Griffin (1979) maximum-likelihood technique or by matching the observed central projected velocity dispersion.

Table II lists the results of the above procedure. The first column is the cluster NGC number. The next three columns give the cluster distance, the reddening, and the central V-band surface brightness used. Next comes the central projected velocity dispersion adopted for the cluster and used in the modeling. This is followed by the concentration of the model used and the fitted r_s. The final four columns are the logarithm of the central density, the $(M/L_V)_0$ (note again that this is defined here to be the central mass surface density divided by the central surface brightness), the logarithm of the cluster mass, and the global M/L_V.

We stress that the total masses and the global M/L's given in Table II *are very model-dependent.* As was first pointed out by Gunn & Griffin (1979), adding large numbers of low-mass stars has little effect on the model fits since mass segregation implies that most of these stars are at large radii (see Meylan & Pryor, these proceedings, for further discussion of why surface brightness and velocity dispersion profiles do not uniquely determine the cluster mass). Thus we could have found much larger global M/L's by taking a smaller low-mass cutoff for the mass function. The exception to this is clusters with $c \lesssim 0.8$, where mass segregation should be less important. These clusters suggest that our global M/L's are approximately correct (e.g., Pryor et al. 1991), but low-concentration clusters may have evolved dynamically so as to have lost most of their low-mass stars. Richer et al. (1991) have suggested, based on deep luminosity functions, that globular clusters do contain large numbers of low-mass stars, though this conclusion depends strongly on the uncertain mass-luminosity relation for lower-main sequence stars (see Hesser, these proceedings).

The central densities and $(M/L_V)_0$'s listed in Table II are less model-dependent than the global cluster properties. For clusters with collapsed cores, however, these quantities should be treated as approximate averages for the region within a few r_s of the cluster center.

Using biweight estimators (Beers et al. 1990; these estimators are insensitive to outliers) with the entire sample of 56 clusters yields a mean M/L_V of 2.3 and a dispersion about the mean of 1.1. Similarly, the mean $(M/L_V)_0$ is 1.7 and the dispersion is 0.9. The three clusters with the most extreme M/L's, NGC 6256, NGC 6325, and NGC 6366, have very high reddenings, or uncertain velocity dispersions, or both. However, there appear to be real variations in M/L within the sample. For example, both the central and global M/L of ω Cen (NGC 5139) are about a factor of 3.3 larger than those of M71 (NGC 6838) and this difference is much larger than would be expected from the uncertainties in the cluster dispersions, density profiles, and luminosities. The models used to

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derive the M/L's can be questioned, but at least the difference cannot be blamed on different models. Changing the M/L of an old stellar population by a factor of 3.3 requires a dramatic change in the mass function, such as changing the low-mass cutoff from 0.4 $M_\odot$ to 0.06 $M_\odot$ (e.g., Pryor et al. 1991).

An understanding of the variations in globular cluster M/L's is an important goal of cluster studies in the next 5–10 years. Using the values in Table II, the global M/L does not correlate significantly (absolute value of the correlation coefficient $|r| < 0.22$) with distance from the galactic center, distance from the galactic plane, metallicity, concentration, or half-mass relaxation time (for this and the following discussion the three clusters with the most deviant M/L's have been excluded from the sample). Global M/L is weakly correlated with the total mass ($r = 0.31$). Independence of these two quantities can be rejected at better than 95% confidence, but this conclusion and the correlation coefficient are compromised by the correlation between the errors in mass and M/L. Mandushev et al. (1991) have found similar results with a sample of 32 clusters. Whether M/L really tends to increase with increasing mass is still uncertain.

The $(M/L)_0$ correlates positively with cluster concentration ($r = 0.40$) and more weakly with mass ($r = 0.28$). The correlation with concentration is caused by somewhat higher $(M/L)_0$'s for clusters with $c \gtrsim 2.5$, i.e., clusters with collapsed cores. Such clusters are preferentially found near the galactic center (Chernoff & Djorgovski 1989) and this probably explains the anti-correlation of $(M/L)_0$ with distance from the galactic center ($r = -0.38$). A large $(M/L)_0$ at high concentration is a feature of the multi-component King-Michie models used in the fitting and thus this result might not actually be required by the data. However, most of the velocity dispersion data come from close to the cluster center and fitting single-component King-Michie models still tends to yield larger M/L's for clusters with $c \gtrsim 2.5$. Thus we probably are seeing the increase in the $(M/L)_0$ expected because of mass segregation in very centrally-concentrated clusters. Grabhorn et al. (1992) present evidence for a large $(M/L)_0$ in the collapsed-core cluster M15 based on an analysis using Fokker-Planck models.

Correlations such as those discussed above are a promising tool for understanding globular cluster stellar populations. Today we strongly suspect that some globular clusters do differ in M/L, with the principal stumbling block to converting that suspicion into a certainty being the insensitivity of current mass-estimation techniques to low-mass stars. We still have little knowledge of how widespread these differences are and of their cause. These questions will be answered when the correlation and mass-estimation tools that are now becoming available are applied to the best of the currently available, and soon to be available, data.

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TABLE I Velocity data for galactic globular clusters

IAU	Cluster NGC		Ref	N	R (′)	⟨unc⟩ (km s^{-1})	⟨V_r⟩ (km s^{-1})		σ_{obs} (km s^{-1})	
0021−723	104	47 Tuc	13,15	217	0.0–14.3	0.6	−19.48	0.67	9.84	0.47
			14	272	0.2–14.4	0.6	−18.80	0.60	9.60	0.40
			3	135	...	3.0	...	...	...	...
			10	int	...	...	...	...	10.50	0.40
			15,6	int	...	...	−18.75	0.20	9.30	0.30
0050−268	288		31	30	0.1–5.8	0.8	−46.20	0.53	2.79	0.39
			33,34	64	0.1–8.6	0.8	−45.77	0.33	2.54	0.25
0100−711	362		7	208	0.1–3.5	...	+223.28	0.45	6.34	0.32
			10	int	...	...	...	...	7.50	0.90
			6	int	...	...	+223.20	0.20	5.60	0.40
0512−400	1851		10	int	...	...	...	...	7.90	0.70
			6	int	...	...	+320.50	0.60	10.40	0.20
0522−245	1904	M79	23,34	53	0.0–2.8	0.9	+205.92	0.68	4.81	0.49
			6	int	...	...	+200.80	0.50	3.50	1.10
0734+390	2419		23,17	12	0.2–4.1	1.6	−20.03	0.90	2.66	0.76
0911−646	2808		10	int	...	...	...	...	14.20	1.30
			37	int	...	...	...	...	13.40	2.60
1015−461	3201		1	421	0.0–36.	...	+494.60	0.20	4.30	0.15
1207+188	4147		29	8	0.0–0.7	1.0	+183.01	0.97	2.54	0.74
1236−264	4590	M68	32	18	0.5–3.3	0.9	−95.06	0.61	2.39	0.46
1313+179	5053		35	24	0.9–8.6	0.6	+42.82	0.27	1.15	0.22
1323−472	5139	ω Cen	14	318	0.3–23.4	0.9	+232.20	0.70	12.20	0.50
1339+286	5272	M3	9,27	112	0.0–15.5	0.5	−147.07	0.51	5.39	0.36
			6	int	...	...	−146.30	0.40	4.20	0.90
1343−511	5286		6	int	...	...	+57.10	0.80	8.00	1.10
1403+287	5466		21	12	0.7–6.3	1.0	+107.25	0.55	1.62	0.47
			31	25	0.7–6.3	1.0	+107.67	0.35	1.42	0.31
			35	36	0.7–6.3	0.6	+106.58	0.29	1.63	0.22
1436−263	5694		6	int	...	...	−142.80	0.60	5.50	1.10
1500−328	5824		37	int	...	...	...	...	12.80	1.50
			6	int	...	...	−26.10	0.40	10.50	0.50
1516+022	5904	M5	36	46	0.0–9.5	1.6	+53.13	0.76	4.91	0.56
			6	int	...	...	+54.60	0.40	5.90	0.80

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TABLE I cont. Velocity data for galactic globular clusters

IAU	Cluster NGC		Ref	N	R (′)	⟨unc⟩ (km s^{-1})	⟨V_r⟩ (km s^{-1})		σ_{obs} (km s^{-1})	
1531−504	5946		6	int	...	...	+129.00	2.00	3.70	<6
1614−228	6093	M80	10	int	...	...	...	...	12.50	2.50
			37	int	...	...	...	...	12.40	2.50
			6	int	...	...	+7.70	1.60	13.30	0.60
1620−264	6121	M4	21	19	1.8–11.2	0.6	+70.02	0.90	3.88	0.64
			36	23	0.0–9.9	1.1	+69.67	0.96	4.44	0.71
			20	182	0.2–14.6	1.0	+70.86	0.27	3.50	0.20
1629−129	6171	M107	29,25	67	0.1–6.4	1.1	−34.23	0.47	3.67	0.36
			5	4	...	1.1	−33.80	2.60	...	...
1639+365	6205	M13	12	136	0.0–13.0	1.0	−246.44	0.57	6.62	0.41
1644−018	6218	M12	29,32	64	0.1–9.6	0.6	−42.62	0.50	3.97	0.36
			36	16	...	1.4	−42.65	1.00	3.73	0.74
1654−040	6254	M10	36	25	0.6–5.7	1.5	+75.50	1.14	5.48	0.84
1656−370	6256	Ter 12	6	int	...	...	−104.70	3.50	6.50	<10
1657−004	Pal 15		22	4	...	2.3	+68.91	1.12	...	...
1658−300	6266	M62	10	int	...	...	...	...	13.70	1.10
			6	int	...	...	−71.90	1.50	14.30	0.40
1701−246	6284		6	int	...	...	+27.40	1.60	6.20	0.90
1707−265	6293		6	int	...	...	−148.00	1.80	7.60	1.40
1714−237	6325		6	int	...	...	+30.90	1.70	5.80	1.40
1715+432	6341	M92	11	49	0.0–6.0	1.2	...	...	5.00	0.50
1718−195	6342		6	int	...	...	+117.90	1.50	4.60	0.90
1725−050	6366		5,38	6	1.8–8.2	0.9	−122.64	0.54	0.93	0.52
1726−670	6362		33	24	0.0–3.2	0.7	−13.30	0.58	2.76	0.43
1732−447	6388		10	int	...	...	...	...	18.90	0.80
1736−536	6397		4	11	2.2–9.5	1.5	+19.40	1.11	3.29	0.80
			16	127	0.2–11.8	...	+18.80	0.10	3.50	0.20
			6	int	...	...	+15.00	1.60	2.00	<4
1746−370	6441		10	int	...	...	...	...	17.60	0.80
			6	int	...	...	+16.00	1.50	18.00	0.20
1800−300	6522		6	int	...	...	−10.40	1.50	6.70	0.70
1801−003	6535		25,33	26	0.1–2.1	1.8	−215.27	0.54	2.15	0.45
1804−437	6541		37	int	...	...	...	...	8.20	2.10

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TABLE I cont. Velocity data for galactic globular clusters

IAU	Cluster NGC		Ref	N	R (′)	⟨unc⟩ (km s^{-1})	⟨V_r⟩ (km s^{-1})		σ_{obs} (km s^{-1})	
1807−317	6558		6	int	...	...	−198.90	1.50	2.90	1.10
1820−303	6624		30	19	0.0–0.6	1.1	+54.10	1.22	5.21	0.88
			37	int	...	...	...	...	8.90	1.80
			18	71	0.0–2.4	1.2	+54.36	0.66	5.40	0.48
1821−249	6626	M28	30	20	0.1–1.0	1.1	+15.53	1.86	8.23	1.32
1833−239	6656	M22	21	12	1.4–4.8	0.8	−150.77	2.39	8.23	1.70
			32	35	0.2–6.2	0.8	−147.98	1.43	8.43	1.02
			2,19	130	1.0–7.0	1.0	−148.77	0.59	6.64	0.42
1840−323	6681	M70	30	17	0.0–0.6	1.1	+218.81	1.23	4.94	0.89
			37	int	...	...	...	...	9.90	3.10
			6	int	...	...	+223.30	1.50	9.20	0.50
1850−087	6712		8	49	0.1–2.0	0.9	−107.48	0.59	4.03	0.43
1851−305	6715	M54	10	int	...	...	...	...	14.20	1.00
1906−600	6752		6	int	...	...	−32.10	1.50	4.50	0.50
1914−300	6779	M56	32	20	0.0–1.8	1.0	−135.85	0.86	3.70	0.62
1936−310	6809	M55	31	20	0.6–4.0	1.0	+176.58	0.87	3.76	0.64
			33	73	0.5–7.8	1.0	+174.45	0.51	4.24	0.37
1951+186	6838	M71	21	15	0.3–3.2	0.7	−22.51	0.65	2.40	0.48
			36	11	0.3–2.6	1.4	−23.22	0.57	1.32	0.64
			26	89	0.0–3.2	0.6	−23.16	0.24	2.16	0.17
2003−220	6864	M75	10	int	...	...	...	...	10.30	1.50
2031+072	6934		32	10	0.1–0.6	0.9	−412.22	1.58	4.91	1.14
2127+119	7078	M15	24	120	0.0–4.6	1.1	−107.09	0.82	8.95	0.59
			24	int	...	...	...	...	25.00	7.00
			6	int	...	...	−111.40	1.50	14.00	0.30
2130−010	7089	M2	28,32	68	0.1–7.4	0.6	−3.11	0.90	7.39	0.64
2137−234	7099	M30	37	16	0.1–0.8	1.0	−186.95	0.98	3.78	0.71
			37	int	...	...	...	...	6.00	0.60
			18	24	0.1–1.3	1.3	−186.97	1.12	5.32	0.82
			6	int	...	...	−183.10	1.50	5.10	0.80

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TABLE II Derived parameters for galactic globular clusters

NGC	d (kpc)	E(B–V)	$\mu_{0,V}$	σ_0 (km s^{-1})	c	r_s (pc)	$\log(\rho_0)$ ($M_\odot/pc^3$)	$(M/L_V)_0$	log(M) ($M_\odot$)	M/L_V
104[1]	4.6	0.04	14.42	11.5	2.2	0.34	5.1	1.4	6.1	2.0
288	8.4	0.03	20.00	2.9	0.9	3.83	2.1	2.2	4.9	3.0
362[2]	8.6	0.04	14.79	6.4	2.1	0.30	4.7	0.7	5.5	1.1
1851	12.2	0.02	14.17	10.4	2.5	0.16	5.7	1.9	6.0	1.9
1904	13.0	0.01	16.18	5.2	1.8	0.49	4.2	1.3	5.2	2.2
2419	83.5	0.06	19.77	3.0	1.4	8.05	1.4	0.7	5.6	1.2
2808	9.2	0.24	15.17	13.4	1.9	0.56	4.9	1.5	6.2	2.4
3201[3]	5.0	0.21	18.96	5.2	1.3	2.09	3.0	2.6	5.5	4.1
4147	18.8	0.02	17.39	2.6	1.9	0.43	3.7	1.1	4.7	1.8
4590	9.3	0.05	18.83	2.5	1.7	1.64	2.5	0.9	5.0	1.6
5053	16.1	0.01	22.03	1.4	0.8	11.62	0.6	1.3	4.7	1.6
5139[4]	4.9	0.15	16.81	16.0	1.2	3.75	3.5	2.3	6.6	3.6
5272	10.1	0.01	16.39	5.6	2.0	1.12	3.5	0.8	5.8	1.2
5286	9.3	0.24	16.21	8.0	1.5	0.72	4.3	1.2	5.5	2.1
5466	15.8	0.00	21.60	1.7	1.5	8.20	0.8	1.3	5.2	2.3
5694	32.8	0.10	16.48	5.5	2.0	0.44	4.3	1.5	5.4	2.4
5824	32.4	0.14	15.12	11.6	2.7	0.27	5.3	2.3	6.6	1.6
5904	7.6	0.03	16.05	5.7	2.0	0.67	4.0	0.9	5.6	1.4
5946	10.5	0.56	17.19	3.7	2.3	0.15	4.8	0.9	4.9	1.1
6093	8.4	0.20	15.38	12.4	2.1	0.26	5.4	3.4	6.0	5.2
6121	2.0	0.40	17.88	4.2	1.6	0.44	4.1	1.5	4.8	2.6
6171	6.4	0.33	18.89	4.1	1.5	0.92	3.5	2.3	5.1	3.9
6205	7.2	0.02	16.77	7.1	1.5	1.68	3.4	1.3	5.8	2.2
6218	5.6	0.18	18.00	4.5	1.4	1.02	3.5	1.7	5.1	2.8
6254	4.3	0.28	17.70	6.6	1.4	1.01	3.8	2.1	5.4	3.5
6256	10.2	0.80	17.89	6.5	2.8	0.03	6.6	10.1	5.3	6.2
6266	5.5	0.50	15.19	14.3	1.8	0.24	5.7	2.0	5.8	3.4
6284	11.8	0.28	16.35	6.2	2.3	0.16	5.2	2.5	5.4	3.1
6293	8.5	0.36	15.72	7.6	2.8	0.05	6.3	4.3	5.6	2.6
6325	6.0	0.88	17.56	5.8	2.8	0.03	6.7	6.2	5.1	3.8
6341	7.5	0.02	15.46	5.9	1.9	0.41	4.4	1.0	5.3	1.6
6342	12.0	0.44	16.97	4.6	2.5	0.09	5.4	2.5	5.1	2.4
6366	4.0	0.65	21.13	1.3	0.9	2.27	1.9	0.4	4.0	0.5
6362	7.7	0.10	19.28	2.8	1.1	3.09	2.2	1.0	5.0	1.5
6388	11.0	0.34	14.50	18.9	1.8	0.33	5.7	2.1	6.2	3.6

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TABLE II cont. Derived parameters for galactic globular clusters

NGC	d (kpc)	E(B–V)	$\mu_{0,V}$	σ_0 (km s^{-1})	c	r_S (pc)	log(ρ_0) ($M_\odot$/pc^3)	$(M/L_V)_0$	log(M) ($M_\odot$)	M/L_V
6397[5]	2.2	0.18	16.66	4.5	2.8	0.10	5.3	3.3	5.4	2.0
6441	10.7	0.42	14.83	18.0	2.0	0.27	5.8	2.3	6.2	3.7
6522	7.2	0.50	15.75	6.7	2.6	0.06	6.1	2.2	5.3	1.9
6535[6]	6.9	0.33	19.20	2.4	1.4	0.48	3.6	2.0	4.2	3.2
6541	6.6	0.13	15.41	8.2	2.3	0.16	5.5	2.8	5.6	3.5
6558	8.8	0.43	16.48	2.9	2.5	0.05	5.6	1.2	4.4	1.2
6624	8.1	0.28	15.32	5.4	2.5	0.09	5.6	1.3	5.2	1.2
6626	5.9	0.38	15.99	8.6	1.7	0.35	4.9	1.5	5.4	2.5
6656	3.0	0.36	17.40	9.0	1.3	1.21	4.0	2.0	5.7	3.3
6681[7]	9.2	0.06	14.82	5.1	3.0	0.03	6.5	3.6	5.2	1.6
6712	6.8	0.46	18.83	4.3	0.9	1.99	3.0	0.9	5.0	1.3
6715	21.2	0.15	14.75	14.2	2.0	0.50	5.0	1.6	6.3	2.6
6752	4.2	0.04	15.20	4.5	2.5	0.12	5.2	1.1	5.2	1.1
6779	9.8	0.20	17.91	4.0	1.4	1.00	3.4	1.2	5.0	2.0
6809	4.8	0.11	19.40	4.9	0.7	4.49	2.5	2.8	5.3	3.4
6838[8]	3.9	0.28	18.91	2.3	1.3	1.18	2.8	0.7	4.5	1.1
6864	18.5	0.16	15.53	10.3	2.0	0.39	4.9	2.1	5.9	3.4
6934	14.9	0.11	17.33	5.1	1.6	0.96	3.6	1.5	5.3	2.5
7078[9]	10.5	0.05	14.21	12.0	2.8	0.10	6.2	3.5	6.3	2.2
7089	11.9	0.02	15.78	8.2	1.9	0.92	4.0	1.1	6.0	1.9
7099	7.4	0.05	15.35	5.6	2.7	0.06	5.9	3.5	5.3	2.5

Notes: 1. The σ_0 comes from Meylan (1989) and Gebhardt et al. (1993). 2. The σ_0 comes from Fischer et al. (1993). 3. The σ_0 comes from Côté et al. (1993). 4. The σ_0 comes from Meylan (1987). 5. The σ_0, r_c, and $\mu_{V,0}$ come from Meylan & Mayor (1991). 6. The r_c comes from Peykov (1988) and the $\mu_{V,0}$ comes from an unpublished measurement by CP. 7. The σ_0 comes from the individual stellar velocities. The integrated-light measurements yield a very high M/L. 8. The r_c and $\mu_{V,0}$ come from unpublished modeling by CP. 9. The σ_0 comes from Gebhardt et al. (1993).

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