Crowding & info-recovery

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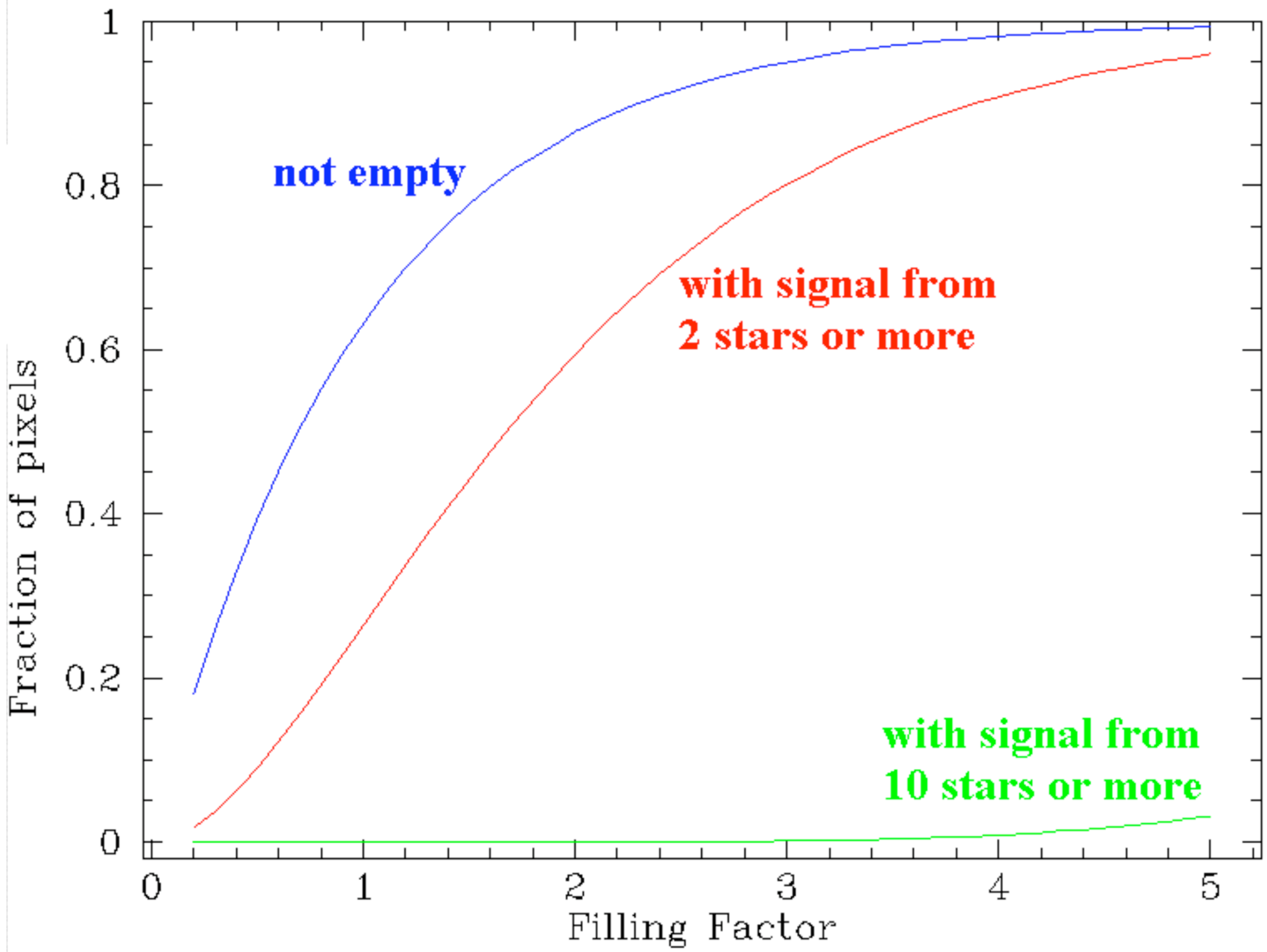
RV-VI, MSSL june 2003



not empty
with signal from
2 stars or more
with signal from
10 stars or more
Fraction of pixels
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0.4
0.6
0.8
1
0
1
2
3
4
5
Filling Factor

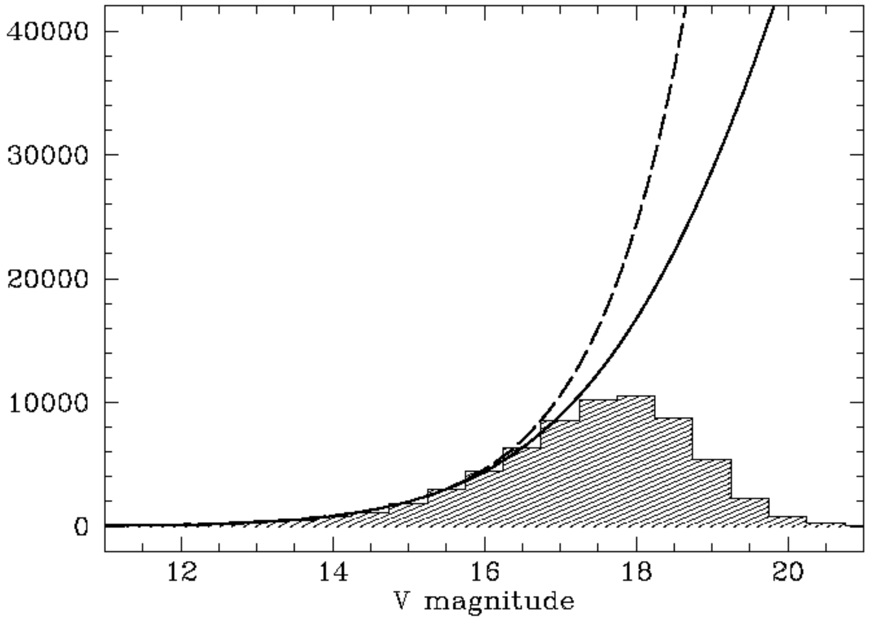


Figure 1. Luminosity function from 66 stellar fields with 76000 stars. Histogram represents magnitude distribution of V-magnitude measurements, solid line is the assumed true distribution (eq. 1), while dashed line is a potential fit ($dN/dV \propto 2.3^V$) to the bright stars ($V < 16$).

tribution of spectral types down to the GAIA faintness limit. A total of 75959 stars were analyzed.

The luminosity function for $V > 15$ is frequently described by a potential law. Here we adopt a heuristic law

$$dN/dV \propto 2.3^{V-0.05(V-15)^2}$$

Average star density of ($V < 17$) stars equals 15100 stars/degree^2 close to the Galactic plane ($|b| < 20^o$) and 1900 stars/degree^2 away from it ($|b| > 20^o$). This is somewhat larger than the corresponding values (6100 and 1200 stars/degree^2) from the Galaxy model (ESA-SCI (2000)4). This is probably a statistical anomaly; symbiotic stars tend to lie close to the galactic plane and selection effects may emphasize regions of higher star density. Even so only 10% of directions close to the Galactic plane ($|b| < 20^o$) reach the density of 40000 ($V < 17$) stars per degree^2. Spectral types of field stars cluster around an early K type at the bright end, reaching a mid-K for the faintest targets (Figure 2).

3. Crowding and the sampling law

The sampling law of the GAIA satellite guaranties that the arrangement of stars in the focal plane will be different for each of the $\sim$ 100 transits of a given star, providing that the spin and precession periods of the satellite are kept incommensurate. Two stars that badly overlap in one passage have non-overlapping tracings on the next pass. This is a simple consequence of the fact that the length of the spectral tracing in the dispersion direction is much larger than its width. There will be unfortunate cases, for example close optical doubles, with tracings overlapping in a significant fraction of observations. But such stars are rare and so of no interest to us here. Below we discuss the results on spectral overlaps for typical randomly positioned stars in the focal plane.

Two spectra overlap with a probability p if the length of the spectrum is larger than the free length

$$L = (ns)^{-1} \ln[(1-p)^{-1}] \tag{2}$$

where the star density equals n stars per degree^2 and the width of the stellar tracing is s arc-sec ($s \sim$ **1.5** arc-sec for the Astrium design of the GAIA spectrograph). The results are presented in Table 1.

Table 1. Severity of crowding following Eq. 2.

star density n (stars/deg^2)	average free length (arcsec)	probability (p) that the distance between spectral heads is smaller than L (arcsec)				
		$L = 3$	$L = 30$	$L = 300$	$L = 1500$	$L = 3000$
600	14400	0.000	0.002	0.021	0.099	0.188
1200	7200	0.000	0.004	0.041	0.188	0.341
3000	2880	0.001	0.010	0.099	0.406	0.647
6000	1440	0.002	0.021	0.188	0.647	0.875
20000	432	0.007	0.067	0.501	0.969	0.999
50000	171	0.017	0.159	0.824	1.000	1.000
100000	84	0.034	0.293	0.969	1.000	1.000

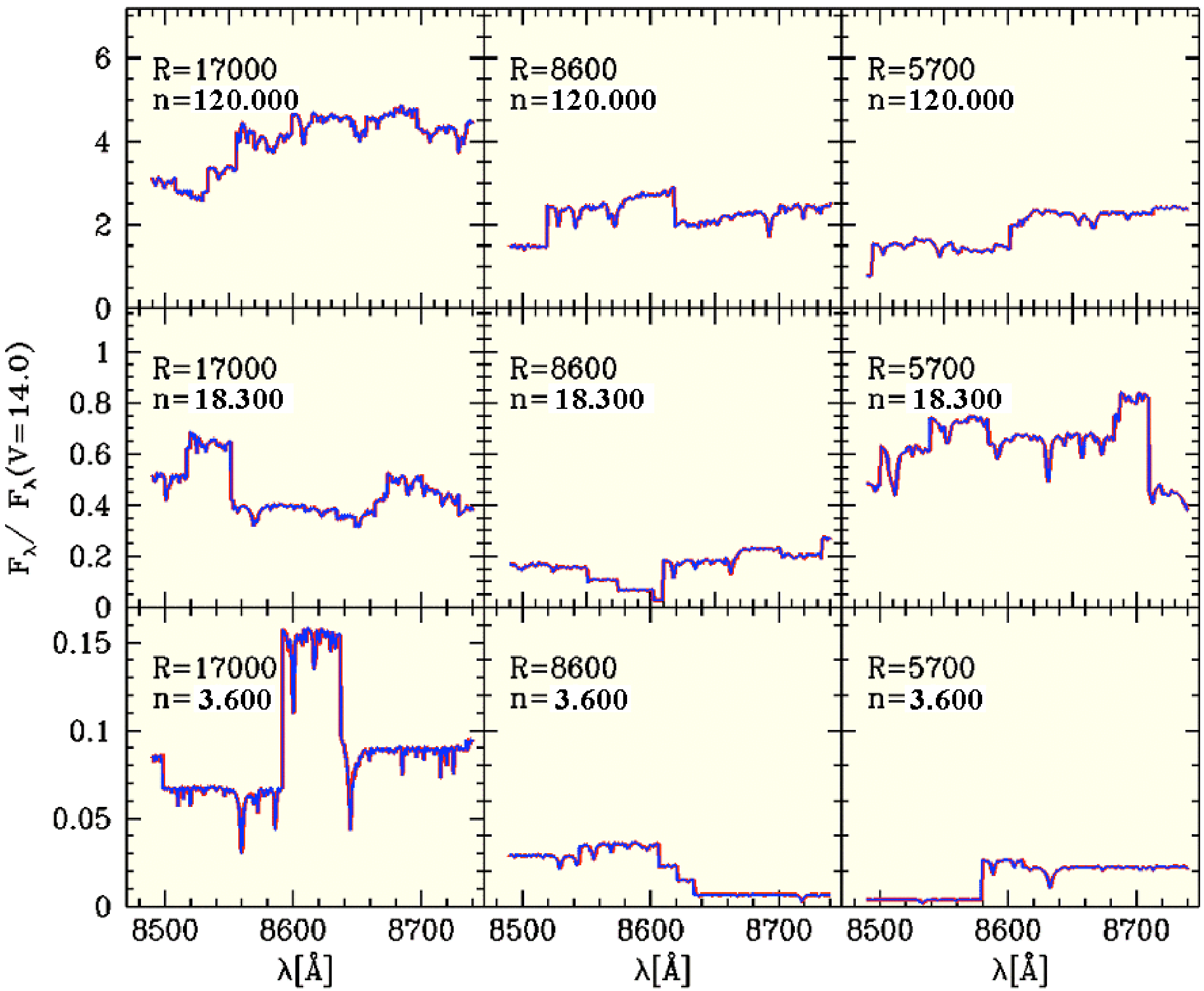
- Out of Galactic plane (n~1200) 4% of the spectra will have half the length (~350 pixels) overlapped with another V=17 star;
- close to the Galactic plane (n~6000) this fraction increases to 20%;
- in high density areas (n~50000) such overlaps in 90% of the cases.

We assume that:

- Stellar positions are accurately known. Star mappers as well as astrometry easily justify this assumption.
- Stellar magnitudes are accurately known. Typical errors will be ~ 0.001 mag for bright stars and ≤ 0.02 mag for stars of $V = 18$ (ESA-SCI 2000(4)). This is well below the spectroscopic shot noise, so this assumption is justified. Contemporaneous star mapper flux measurements can supply the required information in case of variable stars.
- Stellar types are roughly known. A mismatch of 250 K in temperature and 0.5 in $\log g$ and [Fe/H] was assumed. This is some 3-times larger than expected typical final-mission stellar classification errors from photometry (Jordi 2002). So this assumption is justified even for mid-mission analysis. Simulated star types clustered around K1 V, i.e. a typical spectral type of background stars, and their luminosity function followed results of Zwitter & Henden (2002).
- Stellar radial velocities are roughly known. The assumed errors were

$$\sigma(v_r) = v_o \; 2.51^{V-14.0} \quad , \; v_o = 0.2\,\mathrm{km/s} \tag{1}$$

  where $\sigma(v_r)$ is the standard deviation of the difference between assumed and true radial velocities and V is the visual magnitude of the star. This is compatible to mission averaged results for a K1 V star (Zwitter 2002). It turns out that radial velocity spread of background stars is not critical for the final results, so this assumption is justified.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R=8600
n=120.000
R=5700
n=120.000
R=17000
n=18.300
R=8600
n=18.300
R=5700
n=18.300
R=17000
n=3.600
R=8600
n=3.600
R=5700
n=3.600
$F_\lambda / F_\lambda(V=14.0)$
λ[Å]
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8600
8700

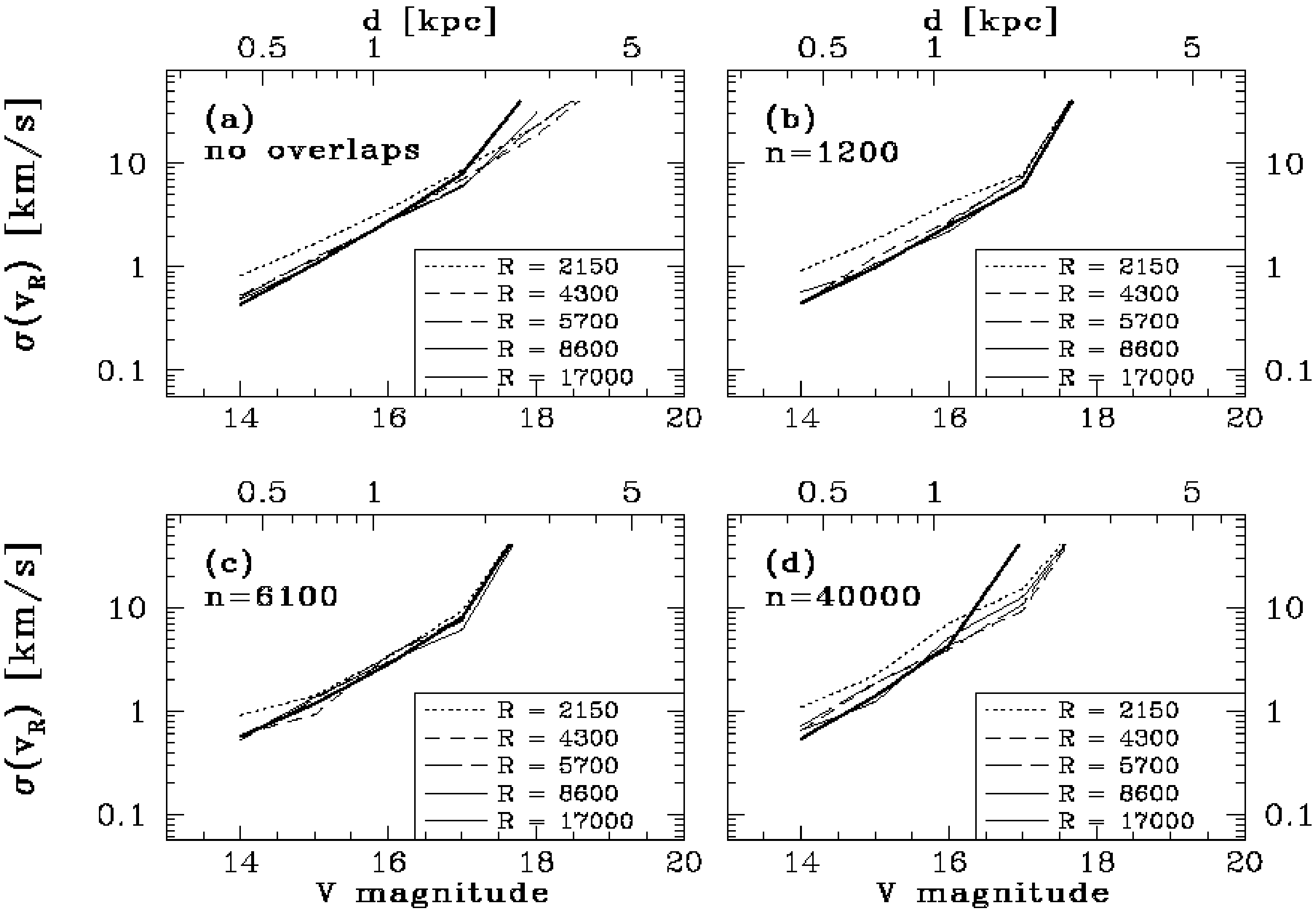

Figure 2. Radial velocity errors for a mission averaged spectrum of a K1 V star at different resolutions *R*. (a) no spectral overlaps, (b) observations at star densities characteristic for high Galactic latitudes, (c) for the Galactic plane, and (d) for a high density environment.

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Overlaps degrade RV accuracy if S/N per 1Å bin □ 3.